LESSONS

IN

ELEMENTARY PRACTICAL PHYSICS

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BY

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PREFACE.

The second volume of our work on Practical Physics is based on the same lines as those of its predecessor. We have adhered to the plan of subdivision into a series of Lessons, each descriptive of something to be done by a definite method with definite apparatus.

It has been thought desirable to make the first three chapters introductory to the rest of the work. Many teachers will, we doubt not, agree with us that the advantages resulting from such a preliminary course are very considerable, for not only does the student become practically acquainted with some of the leading principles of the science, but he has also his inventive and constructive faculties developed. These three first chapters, taken in connection with some elementary text-book on Electricity and Magnetism, and combined with attendance at a course of elementary lectures, will enable the student to grasp without difficulty the less elementary chapters that follow.

Chapter IV. deals with the methods of *Measurement of Resistance.* The importance of this subject justifies the extended treatment it has received.

Chapter V. treats of the *Tangent Galvanometer,* its practical applications, and the methods of determining its constants.

The general use of the magnetometer and dip circle of the Kew pattern at British Observatories has induced us to describe fully in Chapter VI. the manipulation of these particular instruments, and the corrections necessary for the accurate determination of the *Magnetic Elements.*

We would especially direct attention to Chapter VII., on *Electro-Magnetism and Electro-Magnetic Induction.* In this chapter the plan of giving qualitative experimental work has been extensively introduced, for we find that only by such experiments can students really grasp the meaning of such things as *lines of force* and their application.

Chapters VIII. and IX. deal respectively with the *Condenser* and the *Electrometer.*

We have supplemented the work with several Appendices. The more important of these contain the application of Kirchhoff's laws and an elementary account of Potential and Lines of Force.

An examination of this volume will show (1.) That much of the apparatus described is of a simple character, such as can be readily made in the work-

shop of a physical laboratory without special appliances. (2.) That we have restricted ourselves, as a rule, to a few typical methods and typical instruments. (3.) That copious references to fuller sources of information have been given. This last point we esteem to be one of considerable importance in a science which is rapidly changing, being especially valuable to the English reader, who has not the advantage of reference to a general treatise such as Wiedemann's *Elektricität.* (4.) That additional exercises are given, calculated to lead the student on to some branch of original work. (5.) That theoretical explanations have been entered into, more especially where it was necessary that the ordinary work should be supplemented.

For help in developing the methods we are much indebted to several students in our laboratory, especially to Messrs. H. Holden, B.Sc.; E. J. Okell; J. H. Hume-Rothery, B.Sc.; W. Armistead; J. Shepherd; C. H. Lees, B.Sc.; and R. W. Stewart. For the special services of preparing a number of diagrams and the verification of numerical work we must thank Mr. E. J. Okell and Mr. C. H. Lees respectively. Messrs. Holden, Lees, and Stewart have also assisted in the correction of proofs.

Much of the apparatus has been constructed by the mechanical assistants of the Owens College Physical Department — Messrs. E. Binyon and James Griffiths; principally by the latter, who also has

had charge of the preparation of photographs from the apparatus.

We must again thank Mr. J. D. Cooper for the care he has taken with the engravings.

Finally, we must express our acknowledgment of the service of Professor T. H. Core in correcting proofs, and for the suggestions received from several other colleagues, especially from Dr. Schuster, who has been good enough to read through several parts of the book.

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CONTENTS.

CHAPTER I.

ELEMENTARY ELECTROSTATICS.

CHAPTER II.

MAGNETISM—FUNDAMENTAL LAWS AND EXPERIMENTS.

CHAPTER III.

VOLTAIC ELECTRICITY—FUNDAMENTAL LAWS AND MEASUREMENTS.

CHAPTER IV.

THE MEASUREMENT OF RESISTANCE.

CHAPTER V.

THE TANGENT GALVANOMETER.

CHAPTER VI.

DETERMINATION OF THE MAGNETIC ELEMENTS.

CHAPTER VII.

ELECTRO-MAGNETISM AND ELECTRO-MAGNETIC INDUCTION.

CHAPTER VIII.

THE CONDENSER.

CHAPTER IX.

THE ELECTROMETER.

APPENDIX.

LIST OF TABLES.

ADDITIONAL TABLES, APPENDIX G.

PRACTICAL PHYSICS.

CHAPTER I.

ELECTROSTATICS—ELEMENTARY PHENOMENA AND LAWS.

Lesson I.—Electrification by Friction and Conduction.

1. *Apparatus.*—(1.) Two pieces of glass tubing about 350 mm. long by 15 mm. in diameter. Each must be closed at one end by the blowpipe. The tubes must be thoroughly clean and dry. The open end should be closed by a cork to keep out dust. (2.) Several ebonite penholders. (3.) A stirrup of copper wire covered with gutta-percha, suspended by two narrow silk ribbons. Fig. 1 shows the method of making the stirrup. (4.) A pad of good silk about 150 mm. square. (5.) Electrical amalgam (see Appendix) mixed with a little tallow. (6.) A piece of catskin or other fur.

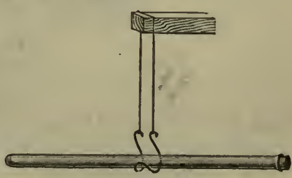


Fig. 1.

(7.) Two small gold-leaf electroscopes. Fig. 2 shows a convenient form of these. Here A is a Florence flask of four-ounce capacity, provided with an india-rubber cork, through which passes a short ebonite rod, *e*. The ebonite rod is perforated so as to admit the passage through it of

a brass rod having a brass disc soldered at one end, while the other end is filed so as to make a knife-edge. Two gold leaves are attached to this end of the brass rod. (The method of manipulating gold-leaf is described in the Appendix.) A hole should be drilled through the brass disc at *b*, to serve for the attachment of wires. The flask must be thoroughly clean and dry. It should be well washed, the final washing being with distilled water, and then dried before the fire. The cork, with its fittings, should be inserted when the instrument is still warm. (8.) A tin can about 10 cm. long by 7 cm. wide. (9.) A block of paraffin wax. (10.) Several mètres of No. 32 B. W. G. copper wire. (11.) Several mètres of silk thread.

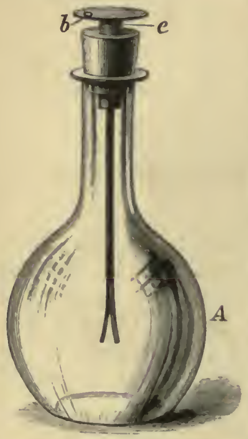


Fig. 2.
THE GOLD-LEAF ELECTROSCOPE.

Experiment I.—Electrification by Friction.—Warm both of the glass tubes, and rub one with dry warm silk on which amalgam has been spread. In absence of amalgam dry warm silk alone will answer. The tube so rubbed must be placed so that it is supported at the middle by the stirrup. Next, take the other glass tube and rub it in the same way. On approaching the rubbed portion of the second tube to the rubbed portion of the first the latter will be repelled. All this must be done quickly, otherwise the charge may be lost. Next, rub or excite an ebonite penholder by means of warm dry fur or warm dry flannel, and replace the glass tube on the stirrup by this penholder. Excite another penholder in the same way. On approaching the excited portion of the second penholder to the excited portion of the first the latter will be repelled.

It thus appears that excited glass repels excited glass, while excited ebonite repels excited ebonite. In a precisely similar manner it may be shown that excited glass attracts excited ebonite, and excited ebonite excited glass. We thus see that the state produced in the ebonite by excitation is different from that produced in the glass. Excited glass is said to be *positively* and excited ebonite *negatively* electrified. Here the words positive and negative are merely convenient expressions, and do not imply that there is anything essentially positive in the physical state of excited glass, or essentially negative in that of excited ebonite.

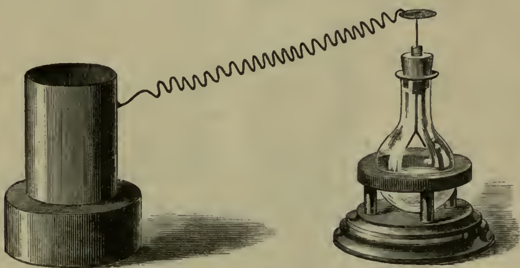


Fig. 3.—ELECTRIFICATION BY CONDUCTION.

Experiment II.—Electrification by Conduction.—Place the tin can on the block of paraffin, then connect the tin can with the plate of the electroscope by means of a copper wire about two mètres in length (see Fig. 3, where the electroscope is shown supported on a wooden stand). The wire must not touch anything as it passes from the one vessel to the other. Excite an ebonite penholder and rub it upon the tin can. The two gold leaves of the distant electroscope will immediately repel each other and fly apart. Here the electroscope becomes electrified by *conduction*, the copper wire being a conductor of electricity. Next, substitute a silk thread for the copper wire, and it will be

found that the electroscope will now remain unaffected, the silk being an *insulator* or *non-conductor*. Wet the silk with water and repeat the experiment. The wetted silk will now be found to be a conductor. In using the electroscope care must be taken that it does not receive too great a charge, for in this case the leaves might be torn.

If we examine a sufficiently large number of bodies we shall find that there is no essential difference between conductors and insulators, the difference being rather in the *degree* of conducting power which the various substances possess.

The following table gives a list of substances in their approximate order of conductivity :—

TABLE A.

ORDER OF CONDUCTORS.

Good Conductors . .	Metals, carbon, acids, saline solutions, water.
Semi-Conductors . .	The body, cotton, dry wood, paper.
Non-Conductors (or Insulators) . .	Oils, porcelain, wool, silk, sealing-wax, sulphur, resin, gutta-percha, india-rubber, shellac, paraffin, ebonite, glass, gases.[1]

Experiment III.—All Substances of a Different Nature may be Electrified by being rubbed together.—In order to electrify a metallic substance or other conductor it must be furnished with an insulating support. Place, for instance, the tin can on the block of paraffin, and connect the former with the electroscope by means of a copper wire. Beat the tin can with warm dry fur. The leaves of the electroscope will diverge, showing that the tin can has been excited.

LESSON II.—Electrification by Induction.

2. *Apparatus.*—That of the previous lesson with the addition of the following :—Two brass knobs (ordinary

[1] Air charged with aqueous vapour is also a non-conductor, but of course, when the vapour is permitted to deposit itself as moisture upon the supports, it destroys their insulation.

door handles will do very well) mounted on ebonite penholders, supported by wooden stands (see Fig. 4).

Experiment I.—Use of Electroscope.—When an electrified body is brought near the gold-leaf electroscope the leaves separate. This shows that electrification may be produced by the influence of an electrified body acting through the air. This is called *electrification by induction.* Let us proceed to study this phenomenon as it appears in the electroscope, learning at the same time the correct mode of using that instrument for testing the nature of an electric charge.

Fig. 4.—Induction Apparatus.

(1.) Let us first of all give the electroscope a positive charge by touching the brass plate with a stick of excited glass, which we then withdraw. A certain divergence of the gold leaves will be caused by this charge. Now, if we bring from above towards the plate of the electroscope either this charged stick of glass or another similarly excited, it will be noticed that the leaves diverge more and more widely as the positively charged glass continues to approach the plate.

(2.) If we next cause a stick of excited ebonite to approach the plate of the positively charged electroscope, we shall find that the negative charge which the former has will cause the leaves

to collapse. If, however, this negative charge be very strong, on bringing it still nearer, the leaves will again open out. When such a charge is quickly brought near the electroscope it is possible that the first collapse of the leaves may escape the notice of the observer.

(3.) If a conducting body, such as the hand, be approached towards the plate of the charged electroscope, the gold leaves will tend to collapse.

We see from this experiment that the slow approach to the plate of the charged electroscope of a similarly charged body will cause the leaves to open out, while the slow approach of a body charged with the opposite electricity will cause the leaves to fall together, and, if it be strong enough, as the approach is continued, afterwards to open out.

We see also that the approach of a neutral conductor whose parts are at varying distances from the plate will tend to make the gold leaves collapse.

Experiment II.—Charging by Induction.

(1.) Having discharged the electroscope, excite a stick of ebonite and bring it near to the plate; the leaves will separate. Whilst the ebonite is kept in this position (near the plate) touch the plate for a moment, and then withdraw the finger; the leaves will now fall together. Remove the ebonite, and the leaves will again open. If we test the character of the charge it will be found to be positive, or opposite to that of the ebonite.

(2.) Had we employed a stick of glass instead of one of ebonite, the charge would have been negative. This method of charging an electroscope is called *charging by induction*, and is usually better than the other method, or that by conduction.

Experiment III.—Study of Induction.—Let us now proceed to inquire further into the nature of induction.

(1.) Let us take the two brass knobs mounted on ebonite penholders, and place the edges of the knobs in contact with each other. Then let us bring an electrified rod, for instance, an ebonite rod charged with negative electricity, near one of the knobs, but not touching it. Whilst the ebonite rod is in this position, separate the knobs from one another and test their charges. They will be found to be charged with opposite kinds of electricity, the one nearest the electrified ebonite rod being positively charged.

(2.) Repeat the experiment, but, when the electrified ebonite rod is near, instead of separating the knobs, touch either of them momentarily with the finger; both knobs will now be found to have a positive charge.

(3.) Make the experiment as in (2), but instead of touching the knobs with the finger touch them with the plate of the electroscope; the electroscope will be found to have received a negative charge.

We learn from these various experiments that when electrification by induction takes place, both kinds of electricity are produced, or rather separated from each other, in the neutral conductor, that of the same name as the charge of the inducing body having a tendency to escape. It is therefore said to be *unbound* or free, whilst that of the opposite name is said to be *bound* as long as the inducing charge is present.

The student should now be able to understand the preceding experiments. For instance, in Experiment I., we see why the slow approach to the plate of a charged electroscope of a similarly charged body should cause the leaves to open out, inasmuch as the approaching body may be imagined to decompose the neutral electricity of the electroscope, attracting or binding that of an opposite name

to itself, and thrusting that of the same name as far away as possible—that is to say, to the leaves which consequently diverge.

In like manner it might be shown that the slow approach of a body charged with the opposite electricity will cause the leaves to fall together, and to open out afterwards with an opposite charge as the approach is continued.

When a neutral body, such as the hand, is placed near the plate of a charged electroscope, the leaves will tend to collapse, because the electricity of the instrument will act upon the hand, decomposing (as it were) its neutral electricity, pulling that of the opposite name as near to it as possible, and thrusting that of the same name through the body to the earth. In this manner part of the charge will become *bound*, and, being withdrawn from the gold leaves, these will tend to collapse.

Again, it is manifest that when the electroscope is charged by induction, the office of the finger when it touches the plate is to take away the free electricity, or that of the same name as the charge of the inducing body. What is then left is the charge of this body, and a nearly equal amount of electricity of the opposite name in the plate of the electroscope. Both these are practically bound, and therefore do not influence the gold leaves; withdraw, however, the inducing body, and the electricity of the electroscope is now free, and acts therefore upon the leaves.

It is unnecessary to discuss the other experiments.

Lesson III.—The Electrophorus of Volta.

3. *Apparatus.*—(1.) A simple electrophorus (Fig. 5). A convenient form consists of an ebonite disc—*the plate*—about 60 mm. in diameter, having a metal disc, termed *the sole*, of the same size screwed to its under surface. The upper surface of the ebonite is well polished. A separate brass disc with smooth edges, somewhat smaller than the

plate, is provided with a rod of ebonite as a handle, and forms *the lid* of the instrument. An electrophorus constructed in this manner is a very satisfactory instrument. A simpler variety may be made by melting ordinary sealing-wax in the lid of a round tin canister, so as to form a smooth plate. A disc of tin, with a handle of sealing-wax, will serve as a lid. (2.) An electroscope. (3.) Fur or flannel.

Fig. 5. —Electrophorus.

Use of the Instrument.—The plate must first be excited. A few whisks with the fur of a cat will serve to electrify the polished ebonite or sealing-wax strongly. If this cannot be procured, a piece of hot flannel will do instead. Next place the lid upon the plate, and touch the metal of the lid momentarily with the finger. On raising the lid it will be found to be charged and capable of giving a spark. As often as this process is repeated the lid becomes charged, provided that the plate is freshly excited occasionally. The electrophorus thus forms a simple electrical machine. The labour of touching the disc may be avoided if a metal pin be passed through a hole in the plate, so that when the lid is in position the sole may be in connection with the lid.

Theory of the Instrument.—This may be studied by performing the following experiments :—(1.) Find the nature of the charge of the ebonite ; this will be found to consist of negative electricity. (2.) Find the nature of the charge of the lid after it has first been touched and then removed from the electrophorus ; this will be found to consist of positive electricity. (3.) Place a charged electrophorus, with its lid (untouched), upon the plate of the electroscope ; on touching the lid the gold leaves will fly apart, and will

be found to be charged with positive electricity. (4.) Now, retaining the arrangement of (3), withdraw the lid from the electrophorus, when the gold leaves of the electroscope will immediately collapse.

The electrophorus thus acts by means of induction. The ebonite, when struck with the fur or flannel, is negatively electrified, and this negative electricity decomposes the neutral electricity of the sole, pulling the positive to itself and thrusting the negative into the earth. The action of the positive of the sole upon the negative of the ebonite serves to bind the latter into the substance of the ebonite. When the lid is put on, the electricity of the ebonite is not communicated by contact to the lid, the ebonite being a non-conductor, and only touching the lid in a few points. But, the lid being acted on inductively, when it is touched with the finger the free negative of the lid is thrust through the body of the operator into the earth, at the same time releasing the bound positive electricity of the sole. The lid, if carried away, will thus be found to be positively electrified.

When the electrophorus is placed upon an electroscope, and in that position the lid is touched, the positive electricity of the sole, being released, is permitted to go to the gold leaves, which, in consequence, diverge. When, however, the lid is carried away, this positive is recalled into the sole, and the gold leaves collapse.

Lesson IV.—Faraday's Ice Pail Experiments.

4. *Apparatus.*—(1.) Two tin cans, one 10 cm. deep by 7 cm. wide, the other 7 cm. deep by 5 cm. wide. The larger can has a layer of paraffin wax covering the bottom; the smaller is furnished with an ebonite penholder as a handle (Fig. 6). (2.) A block of paraffin to serve as an insulating support. (3.) A small electrophorus. (4.) Two electroscopes. (5.) Connecting wires. (6.) It may be convenient

to use an electrophorus lid smaller than that mentioned in the previous lesson, such, for instance, as a halfpenny attached to an ebonite penholder (Fig. 7).

Experiment I.—Place the larger tin can on the block of paraffin, and connect the tin with an electroscope. Charge the lid of the electrophorus, and lower it into the tin can, without allowing it to touch the latter. The electroscope leaves will diverge as the lid is lowered, but when it is a little way inside the can this divergence will reach a maximum, and then remain unaltered. Now make the lid to touch the metal near the bottom of the can; no alteration will be produced by this in the amount of divergence of the electroscope leaves. The charge of the electroscope will be found to be of the same kind as was that of the electrophorus lid. Remove the lid, and test it by a second electroscope; it will be found to be perfectly discharged.

Fig. 6.

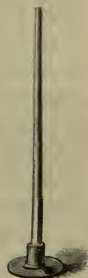


Fig. 7.

Experiment II.—Repeat the previous experiment, but when the lid is near the bottom of the can (not having been in contact with the metal), touch the outside of the can with the finger, so as to withdraw the external charge. The leaves of the electroscope will now collapse, but if the electrophorus lid be removed without allowing it to touch the can, the leaves will again separate to as great an extent as before. Test the charge of the electroscope, and it will be found to be of the kind opposite to that of the electrophorus lid.

Experiment III.—Place the smaller tin within the larger, so as to make it rest upon the layer of paraffin at the bottom of the latter. Introduce within the smaller tin

the electrified electrophorus lid. This will give rise to a condition of electrification in which the inner sides of the two tins will be negatively and the outer sides positively charged.

Now make contact between the two cans. The inner surface of the inner can will now be negatively and the outer surface of the outer can positively electrified. Next let the lid be removed, but not discharged, the inner vessel removed and discharged, and then both replaced. The outer vessel may now be made to have a double charge by repeating the above process, and a small initial electrification may thus be multiplied as many times as we please.

Explanation of these Experiments.—When the charged electrophorus lid has been lowered sufficiently far into the can (as in Experiment I.), it acts inductively upon the can, attracting to the inside a quantity of electricity equal in amount but opposite in character to that of the lid, and repelling to the outside a quantity equal in amount but similar in character to that of the lid. When the lid touches the inside of the can its electricity combines with that equal and opposite charge which has been induced on the inside, leaving the outside electrification altogether unaffected. The lid will now be found to have no charge, because (see Lesson VI.) it has come from being in contact with the interior of a conductor, the charge of which clings to the outside.

In Experiment II. the touching of the outside of the can carries off to the earth electricity equal in amount and similar in character to that of the lid, thus causing the leaves of the electroscope to collapse. When, however, the charged lid is removed from the inside, the electricity of the inside of an opposite character to that of the lid which was formerly bound by the lid, is now free to influence the gold leaves.

In Experiment III., after contact has been made between the two cans, the outer one is positively and the

inner one negatively charged. If the inner one be now removed, discharged, and then replaced, we shall of course have positive in the outer and nothing in the inner. But if now the charged lid be reintroduced into the inner can, and contact made as before between the two cans, there is no reason why a second positive charge should not be given to the outer can.

Indeed for this purpose there is no necessity for the two cans, for if the lid, after being discharged, as in Experiment I., be recharged and introduced into the can after contact with the inside, a double charge will be given to the outside.

Lesson V.—Electrification by Friction— (*Continued from Lesson I.*)

5. *Apparatus.*—(1.) Glass rod, ebonite, electroscope, etc. (2.) A number of different insulators, such as flannel, sealing-wax, paraffin, gutta-percha, etc. (3.) The small electrical machine described below.

Experiment I.—Both kinds of Electricity are produced by Friction.—The rubber of amalgamed silk or fur is usually not a good insulator, so that its charge is generally lost when held in the hand before its electrification can be tested. To exhibit the electrification of the rubber, place the pad of silk or fur on the cap of the electroscope, and rub the silk or fur by means of the glass rod or rod of ebonite. Examine the nature of the electricity with which the electroscope becomes charged. It will be found to be of the opposite kind to that of the glass or ebonite. Test in this manner the quality of the electrification produced with different materials, and verify the following table, in which the substances are arranged in such an order that any substance in the list becomes negative if rubbed with a body that precedes it, but positive if rubbed with a body that follows it in the list.

TABLE B.[1]

SHOWING ORDER OF ELECTRIFICATION.

Catskin.	Sulphur.	Resin.
Glass.	Flannel.	Gutta-percha.
Silk.	Cotton.	Metals.
The hand.	Shellac.	Gun-cotton.
Wood.	India-rubber.	

Experiment II.—Both kinds of Electricity are produced by Friction in Equal Amounts.—This may be shown by rubbing two bodies together within an insulated chamber connected with an electroscope. There should then be no external sign of electrification. A simple apparatus, such as is shown in Fig. 8, may be used for this purpose. A is a tin

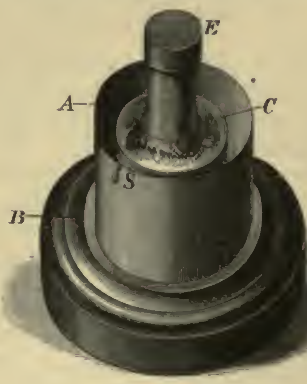


Fig. 8.

can embedded in a block of paraffin (B) that is protected by the wooden base. Within A is a smaller can (C), not necessarily metallic, but for purposes of convenience made of tin, cemented to the bottom of A by means of paraffin.

[1] The order in the above table is liable to change, depending upon the exact composition and the nature of the surface of the substance.

The inside of C is lined with fur. A metal rod is soldered to the bottom of C. An ebonite cylinder (E) is supported by the metal rod so that the ebonite may easily be rotated, rubbing against the fur as it does so. The outer tin can is connected with an electroscope by means of the hook S. On rotating E *no effect* is observed until it is withdrawn to the outside of the outer vessel, when the electroscope will indicate positive electricity.

Here the positive electricity developed on the fur decomposes the neutral electricity of the outer can, pulling the negative to itself and sending the positive away to the electroscope.

Lesson VI.—Effect of a Conducting Enclosure.

6. *Apparatus.*—(1.) A tin can sufficiently large to contain an electroscope. It should be provided with slits, opposite to each other, to enable an electroscope to be observed when placed within. (2.) Two electroscopes. (3.) Block of paraffin, conducting wire, etc.

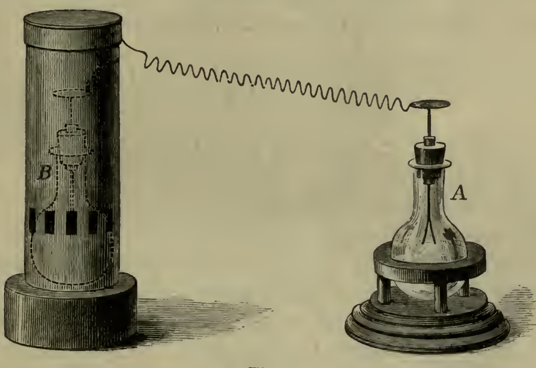


Fig. 9.

Experiment I.—There is no Electrification within a Conductor.—Place the small gold-leaf electroscope within the

tin can. Carry a wire from the electroscope to the inner surface of the can. The tin must be placed on a block of paraffin and have its outer surface connected with a second electroscope (Fig. 9). Now electrify the tin can, when the electroscope A will immediately show the presence of electricity, while the electroscope B will not be affected. It will be found that however intense the electrification of the outer vessel may be, the electroscope B will show no signs of electrification.

Experiment II.—Protection from External Influence.—Disconnect the wire from the inner surface of the tin can. Give the electroscope B a charge by means of an electrophorus. Now electrify the tin. No further effect will be perceived on the electroscope within the can. In this way it is shown that a body within a metallic enclosure has its electric state uninfluenced by electrifying the enclosure from without. And further, if you bring an electrified body near the outside of the can it can likewise be shown that the electroscope in the interior will be quite uninfluenced by its inductive action. This has important practical applications, as we shall afterwards see.

7. *Summary of Laws.*—By aid of the preceding experiments the student will be enabled to illustrate the following laws.[1]

I. "*The total electrification of a body or system of bodies remains always the same, except in so far as it receives electrification from or gives electrification to other bodies.*"

The more we improve the insulation of a charged body the longer does the charge remain, and it is therefore assumed that an absolutely isolated charge remains constant. A charge may be retained for years in a chemically dried, hermetically sealed glass vessel.

II. "*When one body electrifies another by conduction the*

[1] *Elementary Treatise on Electricity*, by J. Clerk Maxwell.—Clarendon Press.

total electrification of the two bodies remains the same, that is, the one loses as much positive or gains as much negative electrification as the other gains positive or loses negative electrification."

This may be proved by bringing two unequally and differently charged bodies into contact within an insulated enclosure connected with an electroscope, when it will be found that the divergence of the leaves of the electroscope will be the same after contact as before it.

III. "*When electrification is produced by friction or by any other known method, equal quantities of positive and negative electricity are produced.*"

This is illustrated by Experiment II., Lesson V.

IV. "*If an electrified body or system of bodies be placed within a closed conducting surface, the interior electrification of this surface is equal and opposite to the electrification of the body or system of bodies.*"

In the case of an electrified body placed in the laboratory or other room where the experiment is performed, the floor, walls, ceiling, etc., take a charge equal and opposite to that of the body.

We see this from the analogy of Experiments I. and II., Lesson IV., in which the inside of the tin can was found to contain electricity opposite in character but equal in amount to that of the electrophorus lid.

V. "*If no electrified body is placed within the hollow conducting surface, the electrification of this surface is zero. This is true not only of the electrification of the surface as a whole, but of every part of the surface.*"

This is seen from Experiment I., Lesson VI., in which no effect is produced upon the electroscope within a tin can by electrifying the outside of the can.

8. *Fundamental Quantitative Law.*—If we add to the above five fundamental laws of electric phenomena a sixth quantitative law, the student will be placed in possession of all that is necessary to explain elementary electro-

statics. Suppose that at a point A there are m units of positive electricity, and at B m' units of the same kind of electricity, and let the distance between A and B be d centimètres, then it is found that the force f of repulsion may be represented thus,

$$f = \frac{mm'}{d^2} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

If the electrification at A or B be negative, this is indicated by putting a minus sign before the symbol of quantity. Thus if A be charged with $+3$ units and B with -6 units at a distance apart of 2 cm., then

$$f = \frac{(+3) \times (-6)}{2^2} = -4\tfrac{1}{2}.$$

A negative sign, therefore, denotes attraction. The proof of the very important law in formula (1) is experimentally difficult. It was first attempted by Coulomb by using a torsion balance.

9. *Definition of the electrostatic unit of quantity of electricity.* —The above expression (1) will enable us to define the electrostatic unit of quantity. Let $f = 1$, $d = 1$, and $m = m'$, then must $m = 1$; in other words, when A and B are each charged with an electrostatic unit of electricity, and are placed one centimètre apart, then will these points tend to separate with the force of one dyne.

Further consideration of electrostatics will now be postponed until electrometers are discussed.

CHAPTER II.

MAGNETISM—FUNDAMENTAL LAWS AND EXPERIMENTS.

10. A Magnet, for the purpose of this work, may be considered to be a piece of steel or iron that is capable of attracting steel or iron. Every magnet has two *poles* or points of apparent maximum magnetisation. The line joining these is called the *magnetic axis*. A freely-suspended magnet balanced so as to swing horizontally sets itself in such a manner that its axis will lie in a direction known as the *magnetic meridian*. This direction makes, at a given place and time, a definite and generally small angle with the geographical meridian. With reference to this position assumed by the magnet, that pole which points nearly north is generally called the north pole, and that which points nearly south the south pole; but other names have been given to these poles, as will be seen from the following table:—

Table C.

Names of Poles.

Pole that points to the North, or North-seeking.	Pole that points to the South, or South-seeking.	
North	South	Ordinary usage.
Austral	Boreal	French usage.
Marked	Unmarked	Faraday.
Red	Blue	Sir G. Airy.
True South	True North	Sir Wm. Thomson.
Positive (+)	Negative (−)	Mathematical usage.

It thus appears that there is great confusion in the ordinary terminology. We shall in this volume use the letter N. or the sign + to denote the north-seeking, and the letter S. or the sign – to denote the south-seeking pole of the magnet.

Lesson VII.—Fundamental Experiments.

11. *Apparatus.*—Bar and horse-shoe magnets with their poles marked, some thin knitting needles, pieces of watch-spring, soft iron nails, silk fibre, test tubes with corks, blowpipe, sealing-wax varnish, steel filings, two bars of very soft iron, No. 20 iron wire, a strip of tinned iron, various specimens of steel and iron, and two brass clamps.

First fit up a delicate instrument for indicating the presence of magnets and magnetic bodies. We shall call this instrument the *magnetoscope.* Let us first heat a piece of watch-spring in the blowpipe flame to bright redness, and then plunge it quickly into a beaker of cold water. The watch-spring will now be found to be hard and brittle. Next break off a piece a little shorter than the breadth of the test tube, and then proceed to magnetise it by rubbing it always in the same direction on one of the poles of the bar magnet. Fit into the open end of the test tube a cork provided with a glass tube terminating in a crook, as shown in Fig. 10. Now proceed to attach a fine silk fibre to the small piece of magnetised watch-spring, which may be done with the help of a little beeswax. The magnet should hang horizontally when suspended by the silk fibre. In order to secure

Fig. 10.
The Magnetoscope.

this position a little sealing-wax varnish may be put on one end—the varnish serving the double purpose of perfecting the horizontality of the magnet and of distinguishing the one end from the other. Attach the other end of the fibre to the glass hook, and place the suspended system in the test tube. The test tube may be supported by a large cork, through which a hole has been bored to admit the end of the tube.

With the aid of this instrument and the above materials let the student perform the following experiments:—

Experiment I.—Show that like poles repel and unlike poles attract each other—the north pole of the experimental magnet being that end which points to the north.

Experiment II.—Magnetise a piece of the brittle watch-spring by drawing it across the N. pole of a magnet. Notice that the end of the piece of watch-spring which last leaves the pole of the magnet is an S. pole. Show that when broken into two parts each portion of this remains a perfect magnet. Show also that however often the process of breaking is repeated, the above result will ensue, each portion remaining a perfect magnet.

Experiment III.—Show that a test tube filled with filings or turnings of hard steel may be magnetised just as if it were a steel bar. Then show that if the turnings are taken out, mixed together, and replaced, they are no longer a magnet. To magnetise the filings a strong magnet should be used.

Experiment IV.—Anneal a piece of soft iron wire by heating it to redness and allowing it to cool slowly, then show that this wire attracts both ends of the magnet. A body which attracts both ends of the magnet is said to be a *magnetic* body.

Experiment V.—Attempt to magnetise the soft iron wire by drawing it across the pole of a magnet; when withdrawn it will be found that at the most only a very feeble magnet is produced.

Experiment VI.—Show, however, that while the soft iron remains in contact with or near the pole of the magnet the former becomes temporarily a magnet, being capable of attracting iron filings; show also that the portion of the soft iron in contact with the magnetic pole possesses a magnetism opposite in name to that of the pole, and that portion of it farthest from the magnetic pole a magnetism the same as that of the pole by magnetising a piece of steel by its means. This is called *magnetisation by induction.*

Experiment VII.—Magnetise a piece of watch-spring. Wind a piece of wire round it so that the watch-spring may be held in the blowpipe flame, where it should be heated to a bright redness. The piece of watch-spring if tested when cold will be found to be no longer a magnet.

Experiment VIII.—Heat a bar of iron to a bright redness in a good fire, and show that it will not now, on being placed in a horizontal position near the magnetoscope, affect the latter. Watch the needle, and notice that when the bar reaches a certain temperature there will be a sudden deflection. A piece of wood should be placed between the bar and the magnetoscope to prevent the radiated heat burning the suspending thread.

Experiment IX.—Hold a bar of soft iron in a vertical position and smartly tap the upper end with a hammer. Whilst *still vertical* test the bar, when it will be found to be a magnet, the lower end being the north-seeking pole. Reverse the bar and repeat the experiment, the polarity will be found to be reversed, the now lower end of the magnet being still the north pole. Here the bar becomes magnetised by the inductive action of the earth, which acts like a large magnet.

Experiment X.—Instead of a bar of soft iron take a long and somewhat wide slip of ordinary tinned iron, and placing it vertical as in the last experiment, mechanically

disturb its particles, causing the iron to make a noise. It will now be a magnet, the lower end being the north pole. If carried gently away and applied to the magnetoscope, it will probably be found to retain its magnetised state; but if once more mechanically disturbed when horizontal and pointing east and west it will be found to be no longer a magnet, being now devoid of all polarity. In this state it will of course attract equally both poles of the suspended needle.

Experiment XI.—Heat a piece of steel and allow it to cool in the close neighbourhood of a magnet. The steel will be found to be magnetised. Heat also two bars of iron, but while cooling place one bar so as to lie magnetic north and south, and the other so as to lie magnetic east and west—all magnets being removed to some distance away. The bar lying north and south will alone be magnetised. Of course care must be taken that the bars are not shaken.

Experiment XII.—Obtain a large knitting needle and clamp it firmly at the ends by brass clamps. Violently twist the wire when in the close neighbourhood of a magnet. The knitting needle will be found to have become permanently magnetised.

From these experiments we may learn various things.

In the first place we see that the difference between hard and soft iron consists in this, that the former is capable of retaining its magnetised condition when withdrawn from the exciting cause, while, however, the latter is unable to do so, losing all or nearly all its magnetism when withdrawn.

Now a body which, owing to molecular rigidity, does not readily lose its magnetisation is, from the same cause, less susceptible of acquiring this condition. To increase its susceptibility in this respect we promote a certain molecular freedom, either by heat or mechanical disturbance, while the body lies in a position favourable to magnetic

influence. The magnetisation is thus allowed to enter, and when entered is kept there, for when the body is cooled, or when the mechanical disturbance has ceased, the particles are once more in a rigid state. A kind of trap is thus laid for the magnetisation, this being invited to enter through an open door, which is immediately shut, so that the guest is converted into a prisoner. This property of hard iron is called *coercive force*, and from Experiment X. we may gather that soft iron is not entirely devoid of this property, possessing it, however, to a very much smaller extent than hard iron.

LESSON VIII.—The Magnetic Field.

12. *Exercise.*—To obtain and fix magnetic curves.

Apparatus.—Bar and horse-shoe magnets, pieces of soft iron, a piece of ferrotype iron, a paraffin bath, sheets of thin writing paper, iron filings, a piece of fine muslin.

Method.—Melt the paraffin wax in the bath, and soak in it a sheet of writing paper. Lift the paper out of the bath by one corner and allow the melted paraffin to drain off. Suspend the paper by one corner until the paraffin has set hard. Coat several sheets in this way. Now place closely over a horizontal bar magnet a sheet of the prepared paper, which should be supported so that the surface is level by means of pieces of wood. Scatter through the fine muslin iron filings over the paper from about a foot above it. Tap the paper until the filings set themselves along lines which are called *magnetic curves*. Next pass the flame of a Bunsen's burner over the paraffin paper so as to melt the paraffin, when the filings will sink into the melted wax. On removing the flame the paraffin will soon solidify, and the filings will be retained permanently in the position which they occupied before melting — that is to say, ranged along magnetic curves. These curves may best be studied by holding the preparations up against

the light, when the forms assumed by the particles of iron will be distinctly seen, owing to the translucency of the paper.

Let the student thus obtain the following curves :—

(*a*) Curves—from simple bar magnet.
(*b*) " " horse-shoe magnet.
(*c*) " " 2 bar magnets with like poles together.
(*d*) " " 2 bar magnets with unlike poles together.
(*e*) " " bar magnet with a piece of soft iron in its field.
(*f*) " " bar magnet near a thin disc of iron.
(*g*) " " end of bar magnet.

It will thus be seen what is meant by the *magnetic field.* This expression merely denotes that space all round a magnet through which it is capable of exercising an influence upon soft iron or other magnets. Again, the magnetic curves represent, in the first place, ropes or chains more or less continuous, into which the iron filings arrange themselves when they are rendered free to turn by the influence of tapping. Now, had we used instead of iron filings a series of very small needles free to move, these would have similarly arranged themselves along the magnetic curves, and the direction of the force acting on any one such needle would be along the tangent to the curve at that point. The needle would, in fact, place itself so that this force would pass along its axis, that is, it would constitute itself a tangent to the curve or be a virtual portion of the curve. A magnetic curve is therefore a line or path such that if we walk along it with a little needle in our hand this needle will always point along the path.

A magnet has not always its magnetism symmetrically distributed along its length. To prove this, let us obtain a

long knitting needle, then, starting at a point $\frac{1}{4}$ of the whole length from one end, let us draw the N. pole of a magnet several times towards this end. Next, with the same pole of the magnet, and starting from the same point, let us perform the same process towards the other end. In this way *consequent points* or *poles* will be produced. These will be revealed when we obtain magnetic curves from such a magnet by the process indicated above. Such a peculiar disposition of magnetism is generally, however, a source of trouble, and our object is to avoid it rather than court its production.

Using as a pencil one pole of a strong magnet, draw a pattern upon a thin steel plate, such as the blade of a saw, going over the pattern several times. If we then obtain magnetic curves, these may be found to be very complicated. Such figures are known as *Haldat's Figures.*[1]

✓13. *Quantitative Relations.*—By no process can a north pole be separated from a south pole. This may be inferred from the fact (Art. 11) that when a magnet is broken each piece becomes a separate and complete magnet, so that a magnet with one pole is not possible. It is, however, customary in magnetic observations to assume that a magnet acts as if its power were concentrated at two points near its ends called poles. This assumption is the more nearly true the longer and thinner is the magnet. Let us therefore suppose that we have two thin magnets each 2 feet in length, and a suitable torsion balance. We could arrange our apparatus so that we might neglect the forces exerted between the poles that are most distant from each other, confining our attention to those that are near together. By this means we might investigate the law of attraction or repulsion between these near poles, and show that it is probably "that of the in-

[1] See Treatise on Magnetism from the 7th edition of the *Encyclopædia Britannica*, by Sir D. Brewster, chap. ii.

verse square." We shall, however, adopt a different method of procedure. We shall begin by assuming that the poles are at the ends of the magnet, and that the law of force is that of the "inverse square," and with this assumption we shall investigate the action of one magnet upon another in certain special cases. The formulæ deduced will then be submitted to experimental verification, which will prove their truth, and along with it the truth of "the law of the inverse square," upon which the formulæ were built.

The force of attraction or repulsion between two magnetic poles is expressed in the C. G. S. system in dynes. If f and f' be the magnetic strength of two poles, and d the distance between them, then the whole force of mutual repulsion or attraction will be

$$F = \pm \frac{ff'}{d^2} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

the upper sign being used when there is repulsion, and the lower sign when there is attraction.

The magnetic force exercised by the earth at a given time and place on a unit pole is called the *Intensity of the Earth's Magnetic Field* at that time and place. If we have a magnetic needle so placed that it is free to oscillate in a vertical plane, its horizontal axis of motion passing strictly through its centre of gravity, and if the vertical plane in which it oscillates be that of the magnetic meridian, then the needle will point in an inclined position, and the total intensity of the earth's magnetism will act upon it. Under these circumstances, if f be the magnetic strength of the poles, we shall have the whole force acting on a pole represented as follows:—

$$\text{Whole force acting on magnet} = \pm f\mathrm{T} \quad . \quad . \quad . \quad (2)$$

(Here T corresponds to $\frac{f'}{d^2}$ of formula (1). For although we are entitled to regard the earth as a magnet, we cannot treat it as an ordinary magnet of great size, the two poles

of which are at definite and measurable distances from the locality of our observation.)

If the above magnet be now supported so that it is compelled to swing in a horizontal plane, it will point in the direction of the magnetic meridian, but it will not now have the whole of the earth's magnetic force acting upon it, but only the horizontal component of that force. If we call this H we shall now have

$$\text{Force acting on magnet} = \pm f\text{H} \quad . \quad . \quad . \quad (3)$$

This force is entirely of the nature of a *couple*, that is to say, if the needle be turned in a direction at right angles to the magnetic meridian, we shall have an attractive force acting on the N. pole of the needle at right angles to its length tending to pull this pole to the north, and an opposite force acting on the S. pole of the needle tending to push it from the north or pull it southwards. And inasmuch as the length of the needle is quite insignificant compared to the size of the earth, so that both ends of the needle may be looked upon as identically situated with respect to the earth's magnetic field, the one of these forces will be precisely equal to the other. Hence the earth's action on the magnet will be merely directive, and the magnet will neither be attracted nor repelled by the earth as a whole. Hence also a magnetised body does not experience any increase or diminution of weight as a consequence of its magnetisation.

The couple which thus acts on a horizontally suspended needle may be termed *The Horizontal Terrestrial Magnetic Couple*. Its moment is found by multiplying the above force by the arm of the couple, that is to say, by the distance between the two poles. If λ be the distance between one of the poles and the centre of the needle, then the distance between the two poles will be 2λ, and the moment of the horizontal terrestrial magnetic couple will be $2\lambda f\text{H}$.

Let us now proceed to study the action of a magnet upon a needle. For this purpose conceive nOs (Fig. 11) to be a small horizontally-suspended magnetic needle which is displaced from its position of rest, MOR in the magnetic meridian, so as to make the angle α (which we shall suppose to be small) with this meridian.

Let H denote the horizontal intensity of the earth's magnetism; let f' be the strength of a pole, and 2λ the distance between the poles, then the couple urging the needle back to its position of rest will be

$$2\lambda f' \mathrm{H} \sin \alpha \quad . \quad . \quad . \quad (4)$$

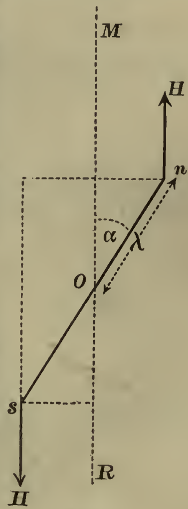


Fig. 11.

Suppose now that the needle nOs is kept deflected by a powerful horizontally-fixed permanent magnet NS (Fig. 12), placed with its axis in a line that is perpendicular to the magnetic meridian, and that passes through the centre of suspension of the magnetic needle. Let $\pm f$ be the strength of the poles of the fixed magnet, $2l$ the distance between its poles, and d the distance of its centre from the centre of the needle.

If we suppose λ to be very small compared to the distance d, and if the angle α is not great, then the distance of the pole S from n or s will be approximately represented by $d - l$, while that of the pole N from n or s will be approximately represented by $d + l$.

We thus find (assuming the law of force to be that of the inverse square)

$$\text{Attraction of S upon } n = \frac{-ff'}{(d-l)^2}.$$

$$\text{Repulsion of N upon } n = \frac{+ff'}{(d+l)^2}.$$

Hence total attractive action upon n

$$=ff'\left\{\frac{1}{(d+l)^2}-\frac{1}{(d-l)^2}\right\},$$
$$=\frac{-4ff'ld}{(d^2-l^2)^2}.$$

In like manner the total repulsive action upon s

$$=\frac{+4ff'ld}{(d^2-l^2)^2}.$$

Bearing in mind that this force makes approximately an angle $(90-\alpha)$ with the length of the needle, we thus see

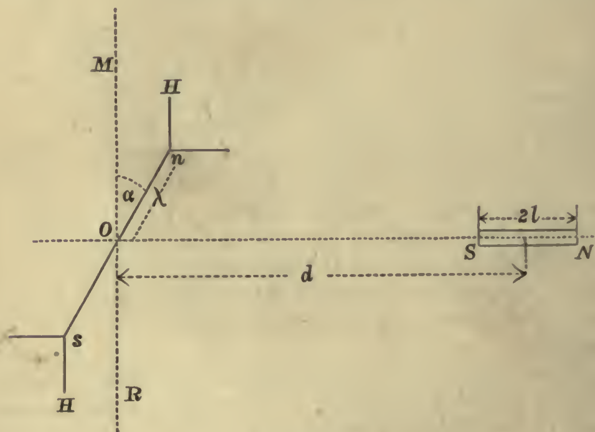


Fig. 12.

that the needle is acted upon by a couple whose moment is

$$\frac{8ff'l\lambda d\cos\alpha}{(d^2-l^2)^2}.$$

Now this moment must (since there is equilibrium) be equal to that of the earth's magnetic couple; hence

$$\frac{8ff'l\lambda d\cos\alpha}{(d^2-l^2)^2}=2f'\lambda\mathrm{H}\sin\alpha\,;$$

or

$$\frac{2fl}{\mathrm{H}}=\frac{(d^2-l^2)^2}{2d}\tan\alpha.$$

It will be seen that $2fl$ is the strength of the one pole of the permanent magnet multiplied by the distance between the two poles; this is called the *moment of the magnet*. If we designate this moment by M we have

$$\frac{\mathrm{M}}{\mathrm{H}}=\frac{(d^2-l^2)^2}{2d}\tan\alpha \quad . \quad . \quad . \quad (\mathrm{I}_b)$$

and if d be very great compared with l this will become

$$\frac{\mathrm{M}}{\mathrm{H}}=\frac{d^3}{2}\tan\alpha \quad . \quad . \quad . \quad (\mathrm{I}_a)$$

In a similar manner the relation between M and H can be ascertained when the magnet is placed *broadside on*, as in B, Fig. 13. These two positions we shall call the A and B Tangent Positions of Gauss.

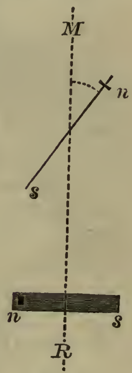


Fig. 13.

In the following table are given the first, second, and third approximations to the value of $\frac{\mathrm{M}}{\mathrm{H}}$ for the two cases A and B.

Table D.

Formulæ for the Tangent Positions A and B of Gauss.

Position.	1st Approximation (a).	2d Approximation (b).	3d Approximation (c).
A	$\frac{\mathrm{M}}{\mathrm{H}}=\frac{d^3\tan\alpha}{2}$	$\frac{\mathrm{M}}{\mathrm{H}}=\frac{(d^2-l^2)^2\tan\alpha}{2d}$	$\frac{\mathrm{M}}{\mathrm{H}}\left\{1+\frac{1}{d^2}(2l^2-3\lambda^2+15\lambda^2\sin^2\alpha)\right\}=\frac{d^3\tan\alpha}{2}$,
B	$\frac{\mathrm{M}}{\mathrm{H}}=d^3\tan\alpha$	$\frac{\mathrm{M}}{\mathrm{H}}=(d^2+l^2)^{\frac{3}{2}}\tan\alpha$	$\frac{\mathrm{M}}{\mathrm{H}}\left\{1-\frac{1}{d^2}(\tfrac{3}{2}l^2-6\lambda^2+\tfrac{45}{2}\lambda^2\sin^2\alpha)\right\}=d^3\tan\alpha.$

If these formulæ be examined it will be observed that

in the first approximation the length of the magnet is neglected as being small compared to the distance between the magnet and the needle. In the second approximation, however, this is taken into account, but the poles are regarded as being placed at the extremities of the magnet. But in the third approximation regard is had to the length of the small needle. What we here obtain is virtually an expression of the following form, if we take position A :—

$$\frac{M}{H}\left(1+\frac{K}{d^2}\right)=\frac{d^3 \tan a}{2} \quad . \quad . \quad . \quad (1)$$

where K is a constant, inasmuch as the variable expression involving $\sin^2 a$ is exceedingly small. The best method of determining this constant is by varying the distance, which we shall now suppose to become d', while the angle of deflection becomes a'. Hence

$$\frac{M}{H}\left(1+\frac{K}{d'^2}\right)=\frac{d'^3 \tan a'}{2} \quad . \quad . \quad . \quad . \quad (2)$$

Between (1) and (2) the constant K may be eliminated, and we obtain

$$\frac{M}{H}=\frac{d^5 \tan a - d'^5 \tan a'}{2(d^2 - d'^2)} \quad . \quad . \quad . \quad . \quad (3)$$

LESSON IX.—Action of a Magnet on a Magnet.

14. *Exercise.*—To prove the formulæ of the preceding paragraphs experimentally.

Apparatus.—A compass box consisting of a small magnetic needle (Fig. 14) pivoted at the centre of a circular card, which is graduated. The needle has a pointer pp' of brass wire placed at right angles to the magnetic needle. To avoid parallax in reading the position of the pointer the bottom of the compass box is provided with a mirror, which is indicated by the shaded portion of the figure.

The compass box is placed on a scale graduated in millimètres. A short but powerfully magnetised bar magnet will likewise be required.

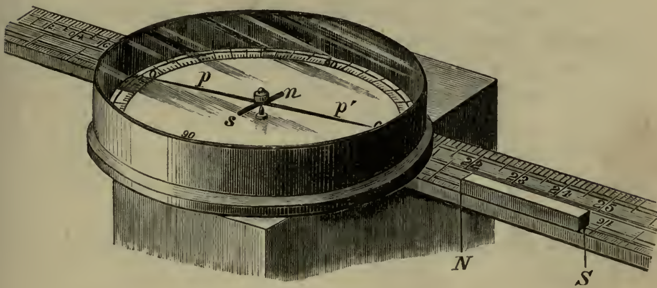


Fig. 14.—Apparatus for Proof of Laws.

Method.—Arrange the apparatus for the A position of Gauss. See that both pointers are at zero. Place the bar

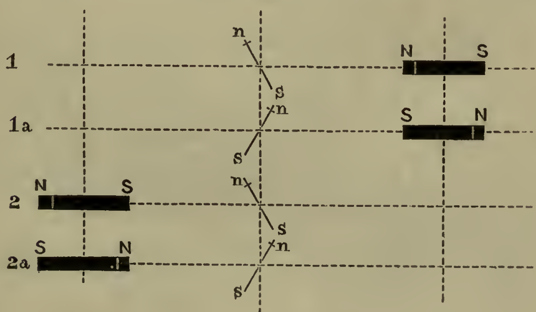


Fig. 15.

magnet on the east limb of the instrument with its N pole west, and note the deflection produced by means of the pointers. Note at the same time the exact distance be-

tween the centres of the two magnets. Then turn the magnet *end for end*, so that while its centre preserves the same position its south pole is now nearest the needle, and again read the deflected position of the pointers. Next take the magnet to the other limb of the instrument, leaving its distance from the centre of the needle the same as before, and obtain a series of deflections similar to those already described. Fig. 15 shows the various positions. Take the mean of the deflections in order to obtain the angle α. Lastly, repeat the observations with different distances, and then calculate the value of $\frac{M}{H}$ with the aid of the preceding formulæ.

Repeat the experiment for the B position of Gauss.

Example.—

A POSITION.

	Position.	Distance of Magnet from Compass Box.	Deflection (α).
Experiment I.—			
	1	20 cm.	11°·25
	2	,,	11°·0 —Mean, 11°·125
	3	,,	11°·0
	4	,,	11°·25
Experiment II.—			
	1	10 cm.	30°·00
	2	,,	30°·00—Mean, 30°·19
	3	,,	30°·50
	4	,,	30°·25
Experiment III.—			
	1	5 cm.	49°·00
	2	,,	53°·00—Mean, 51°·06
	3	,,	53°·00
	4	,,	49°·25

Length of magnet = 10·5 cm. Diameter of compass box = 18·0 cm.
Length of compass needle = 28·5 mm.

The difference between the values in Experiment III.

would seem to indicate that the magnet was too near the compass box, hence we shall reject the result.

		Exp. I.	Exp. II.
Using Formula (I_a)	$\frac{M}{H} =$	3949·4	4148·4
,, (I_b)	$\frac{M}{H} =$	3767·0	3768·7
,, (I_c)	$\frac{M}{H} =$	3786·0	3784·5

Taking the mean of the I_c results as the most probably accurate value, the percentage error from this mean was found for formulæ I_a and I_b—

Formula (I_a).—Exp. I. . .	Percentage of error		+4·3
Exp. II. . .	,,	,,	+9·6
Formula (I_b).—Exp. I. . .	,,	,,	−0·5
Exp. II. . .	,,	,,	−0·4

15. *Method of Sines.*—If the compass box and the scale were mounted so as to revolve about the centre of suspension of the needle, and if a graduated fixed external circle were provided in order to measure the amount of rotation, then the readings might be taken in another way. The needle being at zero to begin with, we might then place the magnet on the scale, and cause the apparatus to revolve until the needle is again at zero. By means of the graduated circle we might ascertain the exact amount of rotation which would represent α, or the angle through which the needle had been deflected from its position of rest. On the other hand, the angle between the magnet and the needle is no longer, as in the first method, $90° - \alpha$, but $90°$, inasmuch as the one is kept perpendicular to the other. It follows that in Art. 13, when we equate together the earth's and the magnet's magnetic couple, instead of $\cos \alpha$ we must substitute *unity*, and for $\frac{\sin \alpha}{\cos \alpha}$ or $\tan \alpha$ simply $\sin \alpha$. In other respects the expressions will remain the same as before.

We shall thus obtain the following modified formulæ—

Position A

$$\frac{M}{H}\left(1+\frac{2l^2-3\lambda^2}{d^2}\right)=\frac{d^3 \sin \alpha}{2} \quad . \quad . \quad . \quad (1)$$

approximately

$$\frac{M}{H}=\frac{d^3 \sin \alpha}{2} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Position B

$$\frac{M}{H}\left\{1-\frac{3}{2}\frac{l^2-4\lambda^2}{d^2}\right\}=d^3 \sin \alpha \quad . \quad . \quad . \quad (3)$$

approximately

$$\frac{M}{H}=d^3 \sin \alpha \quad . \quad . \quad . \quad . \quad . \quad (4)$$

16. *Determination of MH.*—It has been shown in Vol. I. p. 246 that the time of vibration (t) of a magnetic system oscillating under the action of the earth's magnetism may, if we dispense with torsion, be represented as follows:

$$t=\pi\sqrt{\frac{I}{MH}} \quad . \quad . \quad . \quad . \quad (1)$$

where I is the moment of inertia of the system, and MH the directive couple due to the mutual action of the magnet and the earth.

If t be experimentally determined, and if I be ascertained either experimentally or by calculation, then equation (1) will enable us to find MH. But we have already shown how $\frac{M}{H}$ may be obtained. Hence we have now sufficient data to enable us to obtain both M and H; that is to say, both the moment of the magnet and the horizontal intensity of the earth's magnetism.

It will at once be seen that it is quite essential to obtain *two* expressions in order that we may determine *each of the two* separate and independently varying quantities M and H.

Nor will it do to determine M once for all, for a magnet

generally becomes weaker through age, and a blow or the mere act of dropping our magnet on the floor may cause a very perceptible change in the value of M.

LESSON X.—Determination of H and M.

17. *Apparatus.*—A simple magnetometer. Fig. 16 shows a form which the student can readily construct for himself.[1] It consists of a base-board about 29 cm. long by 16 cm.

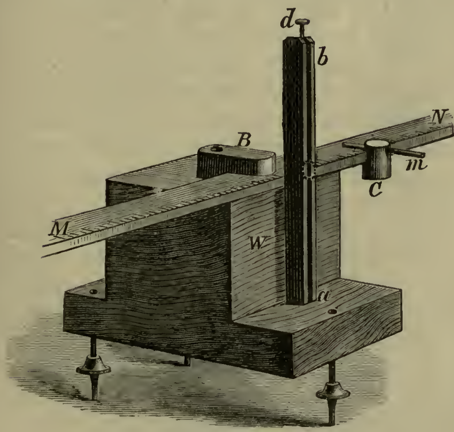


Fig. 16.—THE MAGNETOMETER.

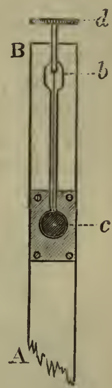


Fig. 17.

broad and 4 cm. thick; this is provided with three levelling screws. There is likewise an upright *ab* about 20 cm. high and $1\frac{1}{2}$ cm. square, and along the middle of its front a fine groove is cut. At *c* (Fig. 17) this groove opens out into a

[1] For the chief details of this lesson we are indebted to *Absolute Measurements in Electricity and Magnetism*, by Professor Andrew Gray (Macmillan and Co.) The magnetometer described is but little different from that devised by Mr. J. T. Bottomley.

small, very shallow chamber. A small mirror (such as is used for a Thomson's galvanometer), having a little magnet made of watch-spring attached to its back, is provided with a fibre of unspun silk, and suspended in the small chamber, the upper end of the fibre being fastened by wax to an adjustable brass pin, *d*. The chamber is shut in by a strip of glass, which is held in its place by means of two wooden side pieces, or by cement. The little chamber, if made of brass, as shown in the shaded portions of *c*, Fig. 17, will possess several advantages over a wooden chamber; for it will help to bring the mirror to rest by its damping effect, and it will be free from small splinters such as are apt to interfere with the motion of the needle in a wooden chamber. By making the chamber only just larger than the mirror, and very shallow, the instrument may be made to possess almost all the properties of a dead-beat arrangement; that is to say, after being disturbed, the needle will almost immediately return to its position of rest.

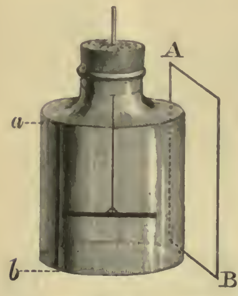


Fig. 18.

In addition to this magnetometer, several knitting needles about 1 mm. thick (or less) will be required. Also an apparatus for magnetising, consisting of a helix and battery, as well as a wire gauge and slide calliper for measuring the length and diameter of the wire. In order to determine the time of vibration of the magnet, the simple apparatus shown in Fig. 18 should be used. It consists of a wide-mouthed bottle about 16 cm. high and 8 cm. in diameter, provided with a cork, which has inserted into it a glass tube terminating in a hook. From this hook a fibre may be suspended, having the magnet attached to its lower end. A stirrup of paper or aluminium foil may be employed to support the magnet. A black vertical ink

line *ab* is ruled down the side of the bottle. On the opposite side is a sheet of white paper, AB.

It will likewise be necessary to have a block of wood, W, intended to support a wooden millimètre scale MN (Fig. 16), this being held in its place by the button B. The block is of such a size, that when placed on the base of the magnetometer, the scale and the needle may be nearly at the same horizontal level. We have said *nearly*, because the scale should be slightly lower, so that when the magnet *m* is placed on the scale, where it may be held in place by the end of a cork, *c*, its axis may be as nearly as possible in the same horizontal plane as the axis of the magnet attached to the mirror. Finally, a galvanometer scale will be necessary.

Method—Preparation of the Magnets.—Take about ten pieces of knitting needle, each about 7 cm. long, and bind them together by iron wire. Then heat them in the blowpipe flame or in the fire to redness, allowing them to cool slowly; they will thus be softened. Now file the ends so that they may be true, and straighten each of the wires. Again bind them together, heating them to bright redness, then very quickly remove them from the fire, and plunge them vertically into cold water. They will now be very hard. Next place the bundle in the centre of a helix of insulated wire, around which a strong current is passing. The wires should now be strongly magnetised. Each of these wires may be used as a separate magnet.

Setting up the Magnetometer.—Place the instrument on a firm slab or table, in the position where it is desired to determine H. The front of the instrument must lie along the magnetic meridian, and the whole must be levelled so that the mirror may swing freely. Set up the galvanometer scale also in the magnetic meridian, and at such a distance as shall be best adapted for obtaining a clear image on the galvanometer scale of the luminous slit as

reflected from the surface of the suspended mirror, or the scale may be set exactly a mètre away, and the image of a wire placed in the slit (see Art. 35) may be focused by a lens.

Taking Deflections.—Fix one of the magnets broadside on (the B position of Gauss) as regards the suspended needle, and take readings such as have been described in Lesson IX. Do this successively with several magnets. In order to determine the angle of deflection, it will be necessary to know the distance of the galvanometer scale from the mirror, and likewise the value in millimètres of one of its scale divisions.

Determining the Time of Vibration.—Place a magnet so that it hangs horizontally in the stirrup of Fig. 18. Then set it vibrating, but without any swinging motion, in such a manner that it makes equal oscillations on either side of the ink-mark. Then standing two or three yards away, in order that the error due to parallax may be small, note down the times of passages, using the method described in Vol. I. p. 188. In the same manner ascertain the time of vibration of the other magnets which were used in the deflection observations.[1]

Calculation of Moment of Inertia.—Determine the length and diameter of the wire by one of the methods of Vol. I. Then calculate the moment of Inertia I from the formula given in Vol. I. p. 244.

Example.—

Distance of mirror from scale $= 78 \cdot 5$ cm.

200 scale divisions $= 20 \cdot 35$ cm.

Thus distance of reading scale from deflected needle $= 771 \cdot 5$ scale divisions.

Half length of magnet $l = 2 \cdot 837$ cm.

---

[1] The above simple method has been greatly improved in its details, so as to increase largely its accuracy. See "Measurement of the Intensity of H at Glasgow," by T. Gray, B.Sc., *Phil. Mag.*, Dec. 1885, p. 484.

The three following sets of readings were taken :—

	Distance.	Deflection in Scale Divisions.	Tan 2α.	α.
(*a*)	13·2 cm.	68	$\frac{68}{771 \cdot 5}$	2° 31′
(*b*)	11·05 ,,	113	$\frac{113}{771 \cdot 5}$	4° 10′
(*c*)	10·10 ,,	146	$\frac{146}{771 \cdot 5}$	5° 21′

Hence

(*a*) $$\frac{M}{H} = (182 \cdot 29)^{\frac{3}{2}} \tan 2° 31' = 108 \cdot 18$$

(*b*) $$\frac{M}{H} = (130 \cdot 15)^{\frac{3}{2}} \tan 4° 10' = 108 \cdot 17$$

(*c*) $$\frac{M}{H} = (110 \cdot 06)^{\frac{3}{2}} \tan 5° 21' = 108 \cdot 13$$

$$\text{Mean} \quad 108 \cdot 16 \quad . \quad . \quad . \quad \text{(A)}$$

Weight (w) of magnet = ·467 grms.
Length (L) of magnet = 5·674 cm.
Radius (r) of magnet = ·05614 cm.

$$I = \left(\frac{L^2}{12} + \frac{r^2}{4}\right) w = (2 \cdot 6829 + \cdot 000788) \cdot 467 = 1 \cdot 2533.$$

T = time of oscillation = 1·77 seconds.

$$MH = \frac{\pi^2 I}{T^2} = 3 \cdot 948 \quad . \quad . \quad . \quad . \quad \text{(B)}$$

Hence

$$MH \times \frac{H}{M} = H^2 = \cdot 036504,$$

$$H = \cdot 191.$$

18. *Determination of M.*—The value of H for the locality having once been obtained, this may be used in subsequent experiments, provided that no change in the surrounding movable iron has been made. During a set of experiments it is well to determine H at the commencement and at the end of the series. If H has been thus determined, and if we assume it to be practically constant, or at least subject to very small variations, it is clear that a single observation of deflection will enable us to determine M; and if the

magnetometer has a fixed position, this observation will be of the simplest possible character. We have in fine, taking the A Position,

$$M = H\frac{(d^2-l^2)^2}{2d}\tan\alpha \quad . \quad . \quad . \quad . \quad (1)$$

and also

$$\tan 2\alpha = \frac{s}{L} \quad . \quad . \quad . \quad . \quad (2)$$

where s is the deflection in scale divisions, and L the distance of the mirror from the scale, also in scale divisions. From (2) $\tan\alpha$ may be ascertained, by the help of tables (see Vol. I. p. 55), and hence the value of M may be found from (1) with very little trouble.

If our object is to determine H for purposes connected with terrestrial magnetism we must conduct our experiment in a locality perfectly free from neighbouring iron. A small wooden house put together with copper nails is frequently used. But for each observation it is highly desirable to use the more complete instrument to be afterwards described.

19. The magnetometer may be employed for studying the variations of the moments of magnets under different conditions. This will be exemplified in the next lesson, in which we shall treat of the effect of temperature upon a magnet.

LESSON XI.—Effect of Temperature on Magnetism.

20. *Exercise.*—To plot a curve showing the variations of the magnetic moment of a magnet with changes of temperature, and to deduce the temperature coefficient.

Apparatus.—A bar magnet M (Fig. 19), about 10 cm. long and 1 cm. in diameter. Around this magnet is wound strong brass wire in order to enable it to be supported firmly from a wooden stand C. The magnet is thus fixed in a horizontal position in an evaporating basin of porcelain

V, which rests on two fire-bricks BB. Oil is placed in the basin so as just to cover the magnet. The oil is heated by a glass spirit-lamp L, the temperature being recorded by the thermometer T. The apparatus is placed at a fixed distance from the magnetometer, the magnet being in the

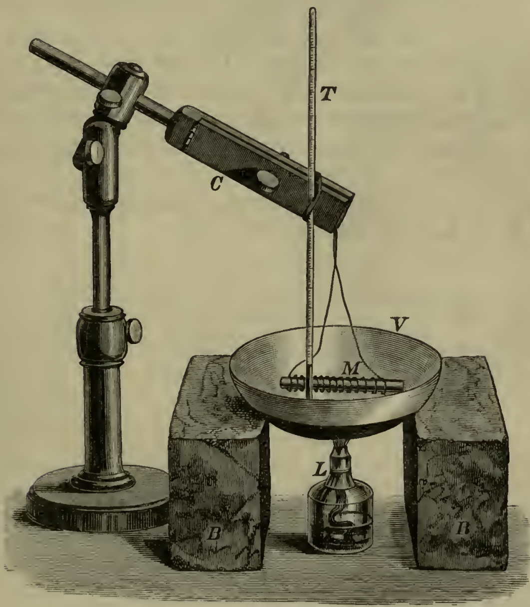


Fig. 19.

A tangent position of Gauss. For accurate work a second magnetometer will be necessary. It should be fixed some distance away from the other one.

Method.—All requisite adjustments having been made according to the preceding lessons, the oil is gradually heated, and the readings of the magnetometer and ther-

mometer are taken for a rise of say every 5° C. up to about 120° C. The lamp is then removed and the readings are repeated as the oil cools. It will be necessary to observe whether the zero point of the magnetometer has undergone any change during the progress of the experiment, such as that which would be produced by the solar diurnal variation of declination. The disturbance due to the declination change will have to be determined by means of a second similar magnetometer. Curves should be plotted exhibiting the simultaneous readings of the magnetometer and thermometer, from which the temperature coefficient of the magnet may be deduced, as shown in the following example.

Example.—A telephone magnet magnetised to saturation was heated and cooled twice. The results are tabulated :—

Experiment I.

Temperature.	Deflection. Heating.	Deflection. Cooling.
30	198	...
40	195	170·5
50	191	165
60	184·5	162·8
70·2	178	159
80	172	156·2
90	165·5	154
100	157	...

Experiment II.

Temperature.	Deflection. Heating.	Deflection. Cooling.
10	240	...
30	235	229
40	232	225·2
50	228	221
60	224	217
70	220	212
80	215·5	208
90	209·5	202·5
100	201	199
101	199	...

We perceive from these experiments—(1.) That the rate of loss for *rising* is greater than the rate of gain for *falling* temperatures. (2.) That we have a permanent loss of magnetism in the double operation which is less in Experiment II. than in Experiment I. To calculate the temperature coefficient between any two temperatures we make use of the formula :—

$$M_t = M_{t_0}(1 - q(t - t_0)),$$

where M_t is the moment at the temperature t, M_{t_0} is the moment at the standard temperature t_0, and q is the required coefficient.

Thus in Experiment I., with $t = 100$, $t_0 = 50$, we find, for rising temperature—

$$157 = 191(1 - 50q), \text{ or } q = \cdot 00356.$$

21. *Distribution of Magnetism.*—It is an interesting problem to determine the distribution of magnetism at different points of a bar magnet. The theoretical investigation as given by Biot, Green, Jamin, and Rowland is much beyond the scope of this treatise. A summary will be found in Mascart and Joubert's *Treatise on Electricity and Magnetism*, vol. i. pp. 393-398.

Many of the experimental methods that have been used for this purpose are open to grave objection; but we shall nevertheless give some of these methods, for not only are they of great historic interest, but they involve principles that should be known to the student, who may require some application of them in studying magnetic problems. The chief methods are

(1.) The Method of Vibration.
(2.) That of the Torsion Balance.
(3.) Test-Nail Method.
(4.) Rowland's Method.
(5.) The Deflection Method.

Of these the fourth is the only one that is really free from theoretical objection; it will be considered fully in the chapter on electro-magnetism.

The first two methods were used by Coulomb in his classical experiments.[1] The third method, depending upon

[1] See *Collection de Mémoires relatifs à la Physique publiés par la Société Française de Physique*, tome i.; *Mémoires de Coulomb*, Paris,

the force necessary to detach a magnetic particle from different parts of a magnet, has been the subject of several mechanical devices, and we may especially mention that due to Jamin.[1]

Lesson XII.—Distribution of Magnetism—Method of Vibrations.

22. *Exercise.*—To determine the distribution of magnetism along different points of a bar magnet and to compare the result with Biot's formula.

Apparatus.—(1.) A bar of hard steel, 730 mm. long by 10 mm. in breadth and 6 mm. thick, that has been graduated into centimètres, and magnetised by insertion into a helix of insulated wire, through which a strong current flows. (2.) A small massive cylindrical needle of glass-hard steel about 10 mm. long by 5 mm. in diameter,—a piece of a rat-tail file will do very well. This must be magnetised to saturation. (3.) A wooden case (see Fig. 20) with a glass window, in which the needle N fixed in a stirrup is suspended by a silk fibre from the roof, and prevented from swinging by a second fibre connecting the bottom of the needle with the floor of the case. At the back of the case a groove is cut, in which the magnet M may slide up or down at a constant small distance from the needle. The magnet may be clamped by a wooden button, so that any of the division marks may lie in the axis of the needle. The apparatus is clamped to the edge of the table by a brass hand clamp. (4.) For taking the time of vibration an American stop-clock is useful.

Gauthier Villiars. It would be difficult to find elsewhere examples of experimental work of such a thorough and ingenious kind as in this volume.

[1] See Jamin, *Cours de Physique,* tome iv. p. 342.

Theory.—According to Art. 16, if a magnet of moment M be set vibrating in a field of strength H, then

$$t=\pi\sqrt{\frac{\mathrm{I}}{\mathrm{MH}}} \quad . \quad . \quad . \quad . \qquad (1)$$

Fig. 20.—COULOMB'S APPARATUS.

where t is the time of vibration and I is the moment of inertia. Hence

$$\mathrm{MH}=\frac{\mathrm{K}}{t^2} \quad . \quad . \quad . \quad . \quad . \qquad (2)$$

where K is a constant for the same magnet. Now if this magnet be placed in a field of strength $H + F_1$, then from (2)

$$M(H+F_1)=\frac{K}{t_1^2}. \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where t_1 is the new time of vibration. Again, if the field be of strength $H + F_2$, then, as before,

$$M(H+F_2)=\frac{K}{t_2^2}. \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Hence

$$\frac{F_1}{F_2}=\frac{\frac{1}{t_1^2}-\frac{1}{t^2}}{\frac{1}{t_2^2}-\frac{1}{t^2}}=\frac{N_1^2-N^2}{N_2^2-N^2} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where N, N_1 and N_2 are the number of vibrations made in the same length of time in these cases.

Method of Experiment.—(1.) Place the apparatus in the magnetic meridian. (2.) Set the needle vibrating under the action of the earth alone, and determine its time of vibration by counting say 100 vibrations and observing the interval of time by the stop-watch. (3.) Place the magnet in position and observe now the time of vibration of the needle for different positions of the magnet. Care must be taken that the needle does not take up a swinging motion, which will interfere with the accuracy of the observation. (4.) Thus obtain $\frac{F_1}{F_2}$, by means of which you may graphically or otherwise compare the forces exerted on the needle at various positions of the magnet.

Now Coulomb supposed that the perpendicular component due to any small length of the magnet was represented by the force exerted on the small needle. But it is clear that the force which acts upon the needle will be not merely that due to that small length of the magnet which is nearly perpendicular to the needle, but will likewise embrace the resolved portions of the forces exerted by those magnetic regions to the right and to the left. Also the resolved portions to the right will, unless we are

observing the central point of the magnet, be different from those to the left. On the whole therefore, our observations will not accurately represent the force exerted by the various points of the magnet unless the needle be infinitely small and infinitely near the magnet. Inasmuch, however, as the perpendicular component of the force from any neighbouring region of the magnet rapidly decreases both on account of the distance (its value varying inversely as the square of the distance) and also on account of the obliquity, it seems probable that the observation will give us a fair approximation towards the truth, provided we are not too near the end of the magnet.

Biot, by comparing a magnet to a Volta's pile, has arrived at the formula—

$$y = A(\mu^x - \mu^{2l-x}),$$

where μ and A are constants, $2l$ is the length of the magnet, x is the distance of any point of the magnet from the end, while y is the intensity of free magnetism at the point x.[1]

Let us now apply this formula first to a set of observations taken by Coulomb, and secondly to a set made in the Owens College laboratory.

I. In Coulomb's experiment, when

$$x=1 \qquad y=90,$$
$$x=4{\cdot}5 \qquad y=9,$$

and the length of the magnet being 27, it followed that

$$90 = A(\mu - \mu^{26}) \quad . \quad . \quad . \quad . \quad . \quad (1)$$
$$9 = A(\mu^{4\cdot5} - \mu^{22\cdot5}) \quad . \quad . \quad . \quad . \quad (2)$$

By trial of different values of μ it is found that μ is nearly equal to $\frac{1}{2}$, hence μ^{26} and $\mu^{22\cdot5}$ may be neglected, (1) and (2) then become

$$90 = A\mu,$$
and
$$9 = A\mu^{4\cdot5}.$$

[1] From Biot's *Traité de Physique Expérimentale et Mathématique*, vol. iii.

From this we obtain $\mu = \cdot 5179$; also $A = 173 \cdot 76$.

Applying these values of μ and A, the following results were obtained by Biot from Coulomb's observations :—

Distance.	Intensity of free Magnetism.		Difference. Calculated – observed.
	Calculated.	Observed.	
0	173·76	165	+8·76
1	90	90	0·0
2	46·62	48	–1·38
3	24·14	23	+1·14
4·5	9·00	9·00	0·0
6	3·35	6	–2·65

II. (The Owens College physical laboratory.)

For earth alone $t = 1 \cdot 1075$

$$\frac{1}{t^2} = \cdot 815$$

Distance.	t_1 (Mean time of Vibration.)	$\frac{1}{t_1^2}$	$\frac{1}{t_1^2} - \frac{1}{t^2}$ (Observed)	$A(\mu^x - \mu^{2l-x})$ (Calculated)	Difference. Calculated – observed.
	sec.				
0	·29708	11·331	10·516	17·99	+7·474
1	·27750	12·986	12·171	16·82	+4·649
2	·26708	14·019	13·204	15·73	+2·526
3	·25750	15·0815	14·266	14·705	+0·439
4	·26416	14·331	13·516	13·748	+0·232
5	·27041	13·676	12·861	12·854	–0·007
6	·27562	13·164	12·349	12·019	–0·330
7	·27375	13·344	12·529	11·238	–1·291
8	·28791	12·064	11·249	10·507	–0·742
9	·29750	11·298	10·483	9·824	–0·659
10	·30500	10·750	9·935	9·186	–0·749
11	·31562	10·0385	9·223	8·588	–0·635
12	·32625	9·395	8·580	8·030	–0·450
13	·33375	8·9775	8·163	7·508	–0·555
14	·35000	8·163	7·348	7·020	–0·328
15	·37500	7·111	6·296	6·563	+0·267

$A = 17 \cdot 99$, $\mu = \cdot 935$, $l = 73$ cm.

It will be noticed that in both of these experiments there is a considerable difference between theory and observation near the end of the magnet.

Lesson XIII.—The Test-Nail Method.

23. *Exercise.*—To find the distribution of force along a short bar magnet.

Apparatus.—A spring balance in one of its modifications Fig. 21 shows one suited for the purpose. Here *ss′* is a spiral spring, having a silk cord attached to its upper end. The silk passes round a pulley mounted so as to rotate stiffly in a collar. At the end of the spiral spring is a small piece of soft iron. When the soft iron rests upon a magnet the force of attraction is measured by the amount of turning that must be given to the milled head *m* in order to detach the soft iron. This is indicated by means of the graduated disc *d* and the fixed index *i*. To ensure that the pull from the magnet is vertical the spiral spring works within the glass guard tube *g*. The apparatus is supported from an arm which may be raised or lowered at pleasure.

Theory.—If S denotes the strength of the magnet at any point, then the magnetism induced in the soft iron will be proportional to S, or equal, let us say, to KS, and hence the force necessary to detach the magnet must be proportional to S^2, or

$$F = \text{constant} \times S^2,$$

or

$$S = \text{constant}\sqrt{F},$$

that is to say, the strength at any point is proportional to the square root of the force required to detach the soft iron. The method is open to the objection that the amount of magnetism induced depends upon *the coefficient of induced magnetism* which may not, however, be strictly constant, but may vary to some extent with S. Again, the

presence of the soft iron is liable to cause a change of distribution of magnetism in the neighbourhood where it is placed.

Fig. 21.—SPRING BALANCE

Method.—Obtain the zero point of the balance by substituting for the magnet a piece of wood of the same size, turning the milled head until the soft iron just touches the wood. Now place the magnet in position and ascertain

the number of divisions through which the milled head must be turned until the soft iron leaves the magnet. The milled head must be turned slowly without any jerks, and a number of observations must be taken at each place, especially near the ends of the magnet, where such observations are most likely to vary.

Example.—Magnet divided into 174 equal parts.

Distance from middle of Magnet=D.	F.	$\sqrt{F}$.	$\frac{\sqrt{F}}{D}$.
13	9	3·0	·23
23	21	4·58	·20
33	39·5	6·28	·19
43	70	8·37	·19
53	125	11·18	·21
63	183	13·52	·21
73	308	17·55	·24

These results agree only approximately with Coulomb's conclusion that for short magnets, that is to say, for magnets whose length is less than fifty times their diameter, the magnetic strength (between the end and the centre) is directly proportional to the distance from the centre. If this had been quite true the value of $\frac{\sqrt{F}}{D}$ should have been a constant quantity.

CHAPTER III.

VOLTAIC ELECTRICITY—FUNDAMENTAL LAWS AND MEASUREMENTS.

24. We shall in this chapter, by a series of elementary experiments, introduce the student to the subject of voltaic electricity, and describe apparatus of simple construction which will be used in making electrical measurements.

Lesson XIV.—Fundamental Experiments.

25. *Apparatus.*—Two pint Bunsen's cells placed in a box arranged as shown in plan in Fig. 22. Each cell consists of

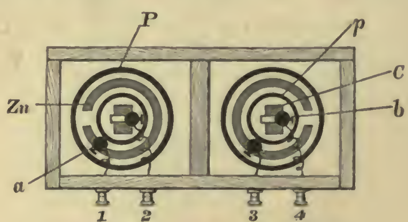


Fig. 22.—Plan of Two-Cell Battery.

a cylindrical glazed stoneware jar P, about 10 cm. in diameter and 15 cm. high. In this jar there is placed a cylinder of zinc *Zn*, made from a plate of zinc 14 cm. by 20 cm. that has been heated and then bent round until its edges

nearly meet. Within the zinc cylinder there is placed a porous pot *p* 14 cm. high and 5 cm. in diameter made of unglazed earthenware. This porous pot contains a rod of prepared carbon C 16·5 cm. in height and 3·5 cm. by 1·75 cm. in cross-section. The zinc and carbon have attached to them *screw clamps a* and *b*. The box for containing the battery is covered inside with pitch in order to prevent the fumes of the acid from acting upon the wood, and is provided with four binding screws numbered 1, 2, 3, 4, each of which has attached to it, inside the box, a thick copper wire covered with gutta-percha for making connections with the clamps.

The following additional apparatus and materials should be at the disposal of the student:—

Measuring vessels.
Glass funnel.
Glass tubing.
Stoneware jug with a spout.
No. 18 insulated copper wire.
No. 20 cotton-covered copper wire.
No. 30 pure iron wire.[1]
No. 30 copper wire.
Sulphuric acid.
Nitric acid.
Mercury.
Caustic soda.
File.
Emery paper.
Stiff nail brush.
3 Carbon rods, 6 to 12 in. long, about 2 in. thick.
India-rubber finger stalls.

Fitting up of the Battery.—This must be done in a draught cupboard or in the open air to prevent the fumes of the acid from affecting the operator.

Begin by removing all the clamps and clean the connecting surfaces and screws by means of a file and emery paper.

Proceed next to the making of mixtures and to the *amalgamation* of the zinc. Into one of the earthenware battery jars put a solution of caustic soda and water (1 of soda to 20 parts of water by weight), and into the other some sulphuric acid diluted with water (1 of acid to 12 of water by weight). In making this last mixture in the jug *pour* the water *first* into the jug, then gradually pour upon

[1] This should be kept in a bottle with quicklime.

it the acid, stirring meanwhile with a glass rod. If the acid be put in first and the water be added to it sufficient heat might be produced by the chemical union to crack the jar. Since all the sulphuric acid of commerce contains lead sulphate which is precipitated on dilution with water, the acid mixture will appear milky when first made. As the presence of lead is very injurious to the working of the battery it will be desirable to allow the mixture to settle and then decant off the clear liquid. A quantity of the mixture should thus be prepared and labelled "*Battery sulphuric acid.*"

The process of amalgamation is as follows:—

First. Dip the zinc into the solution of caustic soda in order to remove grease, and then wash it under a water-tap.

Secondly. Place the zinc in the dilute sulphuric acid until effervescence has commenced, then lift it out and lay it down in a flat dish.

Thirdly. Pour mercury that is free from lead and other injurious metals in a thin stream upon the inside of the cylinder, and also on the outside. Roll the cylinder about until nearly the whole surface of the zinc has a bright appearance.

Fourthly. Replace the zinc in the acid, and rub the surface with a stiff brush or with a rag, the fingers being protected in the operation by finger stalls. The whole of the zinc should be now well amalgamated. Remove it from the acid, wash it well with water, and allow it to drain.

Lastly. Collect any waste mercury and place it in a bottle labelled "*Amalgamation mixture.*"

Keep also the soda solution in a bottle appropriately labelled.

The process of amalgamation tends to make the zinc brittle and rotten if too much mercury be used. Napier (*Electro-Metallurgy*) allows $1\frac{1}{2}$ ounce of mercury for every effective square foot of zinc in the first operation, and half that weight for the second and all subsequent operations.

We find that 1 gramme of mercury will thoroughly amalgamate 100 square cm. of zinc surface. Three times this quantity of mercury may be used in the actual process, of which two-thirds will be recovered by draining off.

Examination and Preparation of the Porous Pots.—The porous pots being thoroughly clean and dry, subject them to the following test: Pour water into each pot, taking care not to wet the outside, and note the time by a watch. Then observe when first an indication of moisture appears on the outside surface of the pot, and again note the time. If the moisture appears immediately the pot is cracked, and should be rejected. A good pot, if made of red clay, should become moist all over in about two minutes; if made of white clay, in about double the time.[1] For low resistance cells the red-clay pots are to be preferred, but they possess the serious defect of being liable to disintegration, a fault possessed in a much less degree by the white pots.

Melt some paraffin wax, and plunge the open end of the porous pot vertically into the wax until this has soaked into it through a distance of about a quarter of an inch from the open end.

This will prevent the acids from creeping up the sides, and will likewise prove especially useful in preventing the sulphate of zinc formed when the battery is in action from becoming concentrated along the rim of the jar, and there crystallising, with the effect of disintegrating the porous material.

It is an excellent plan to put a flat india-rubber band round the top of the jar. This serves to protect the paraffin and to insulate the pot from the clamp at the top of the zinc, besides enabling the experimenter to handle the pot without staining his fingers with nitric acid.

Charging the Battery.—Fix the clamps upon the carbon and the zinc, and bring the parts of the battery together.

[1] A good pot should have a minimum leakage of 15 per cent in twenty-four hours, according to the French standard.

Now pour strong nitric acid through a funnel into the porous pot to within about an inch of the top. Next fill up the outer pot with battery sulphuric acid to a level *about an inch higher* than that of the acid in the inner pot, the reason for this difference of level being that the action of diffusion tends to empty the outer pot. Connect the zinc of one cell to the wire attached to the binding-screw No. 1, its carbon to that of No. 2, the zinc of the other cell to that of No. 3, and its carbon to the remaining screw.

Finally, tighten all the clamps, and then close and fasten the box, which may now be brought into the laboratory.

Necessary Precautions with the Battery.—The student must once for all be warned that nitric acid batteries may be the source of considerable danger to delicate instruments. Hence it is better that they should not be brought into the laboratory, being only used in a draught cupboard or placed outside a window. Since, however, this arrangement is not always convenient, we may employ a tightly-fitting box, such as we have described, *provided this box be not opened in the laboratory.*[1] The battery should be placed under the experimenter's table or bench in a position where it is not liable to be overturned.

Preliminary Connections. — Connect together binding screws Nos. 2 and 3 by means of a short piece of wire, and attach *main* or *leading wires* to Nos. 1 and 4. For this purpose No. 18 gutta-percha-covered copper wire will be found useful. The bared brightened ends of the wire are to be put round the binding-screws, or better still, we may employ a plate of copper (Fig. 23) provided with forked ends. This plan gives a better contact, and is therefore much to be preferred. Fig. 24 exhibits the manner in which the

[1] As an additional precaution it is sometimes advisable to place a wide-mouthed bottle containing lumps of carbonate of ammonia in the box with the battery, which will serve the purpose of neutralising the acid fumes. This, however, cannot be recommended as a rule, for a basic compound of copper will in this case form on the clamps, tending to destroy their good contact.

battery is usually connected. Here the thin vertical lines represent the carbons, the short thick wires the zincs, and the battery is said to be *in series.* In the diagram there are three cells, supposed to be connected together by wires going from the carbon of one cell to the zinc of the next, and so on. The end of the wire connected with the outer zinc is called the *negative pole* (written –), and that connected with the outer carbon is called the *positive pole* (written +). When these poles are connected together there will be a flow of electricity from the + to the – pole.

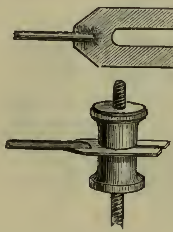


Fig. 23.
METHOD OF CONNECTION WITH BINDING SCREWS.

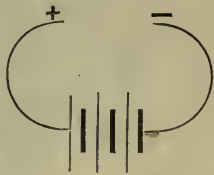


Fig. 24.
SCHEME OF BATTERY.

The battery now described should be used for the following groups of experiments :—

Group I.—(*a*) Bring the free ends of the leading wires together and then separate them ; a spark will be produced.

(*b*) Attach a file to one leading wire and rub the other pole along it ; the sparks will now be more brilliant.

(*c*) Attach a small piece of carbon rod to each leading wire, bring the carbons together and separate them, when a bright light will be produced. Observe that the carbon rods get very hot.

(*d*) Twist a piece of thin iron wire round one pole and then touch the free end of the iron wire with the other terminal of the battery ; it will be found that

several inches of the wire may thus be kept at a red heat, and if of short length it may even be fused.

(*e*) Use fine copper wire of the same diameter instead of the iron, and notice that it cannot be heated to redness.

Group II.—Additional Apparatus.—Pohl's commutator or instrument for changing the direction of the current (Fig. 25); a magnet suspended from a stand; a wire one

Fig. 25.—Pohl's Commutator.

The cups of the ebonite base are in connection each with its nearest binding screw. The switch hinged at A and B is moved by the insulating handle S.

When the terminals of the battery are connected at E and F, or C and D, then the ends of the main circuit are placed at A and B, and *vice versa.*

In the position shown, if a current entered at E it would ascend the left curved wire, descend the lateral wire to A, pass through the main circuit to B, ascend the right lateral wire and descend the curved wire to F. When the switch is pushed back the current will traverse the horizontal wires and be reversed.

mètre long stretched between the uprights (Fig. 26) and mounted on a board. With the aid of the suspended magnet set the wire in the magnetic meridian. Connect the ends of the wire with the commutator.

Make connections such as are exhibited in Fig. 26, on which the commutator is denoted by the cross. Trace out these and ascertain the position of the commutator switch that corresponds to a current from north to south along the wire. Call this Position I., and that which gives a current in the opposite direction Position II. Now break

the current, and then suspend by means of a fibre over or under the wire a short magnetic needle. The fibre should be attached to a stand so arranged that by means of a telescopic joint or otherwise the magnet can be easily raised or lowered. When the magnet is at rest turn the commutator into Position I. and note the direction in which the magnet is deflected. Then turn the commutator into Position II. and again note the direction of the deflection

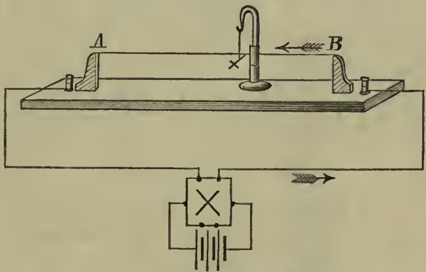


Fig. 26.—EXPERIMENT OF AMPÈRE.

which is produced. Proceed in this manner to verify the following table:—

WIRE HORIZONTAL.

Position of Magnet.	Position of Commutator.
	I.
Above wire	North end deflected to west.
Below wire	" " east.
Level of wire east side . .	South end dips.
" west side .	North end "
	II.
Above wire	North end deflected to east.
Below wire	" " west.
Level of wire east side .	North end dips.
" west side .	South "

WIRE VERTICAL.

Position of Magnet.	Position of Commutator. I. Current up the Wire.
North pole against the wire . .	North end deflected to east.
South pole " " . .	South end "
	II. Current down the Wire.
North pole " " .	North end deflected to west.
South pole " " . .	South end "

Now test your results against the following *memoria technica.* Imagine a man to be swimming against the current, which we may suppose to *enter* in at his *head* and leave at his feet, his *face* being turned *towards* the needle. Under these circumstances the *north-seeking* pole of the needle will be deflected towards his *right* hand.

Group III.—Apparatus.—Glass tubing $\frac{7}{8}$-inch in diameter, corks, soft-iron nails, iron filings. *Experiments.*—(*a.*) Take a piece of the glass tubing about $3\frac{1}{2}$ inches long, bore $\frac{7}{8}$-inch holes in two corks each about $1\frac{1}{4}$ inch in diameter. Into these holes the ends of the tubing have to be fitted to form reels.[1] Now make a small hole through one of the corks near the inside edge, and inserting through it the end of some No. 20 cotton-covered wire, proceed to wind this on the tube in the direction opposite to that of the hands of a watch, looking at the reel from above. About 6 inches of the wire should be passed through the hole before beginning to wind (see A, Fig. 27). Continue winding until four layers of wire are wrapped round the tube, and then bring the other end of the wire through a second hole in the cork (B, Fig. 27). Connect the ends of the wire with the battery. The helix will be found to behave as a magnet, and its polarity must be examined by means of the magnetoscope. Reverse the current and again examine the helix, which will now be found to have its polarity reversed.

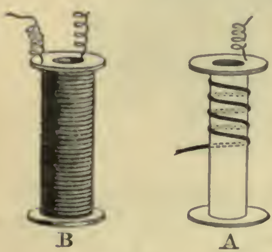


Fig. 27.
METHOD OF WINDING HELIX.

(*b*) Make a second helix, but wound in a direction the

[1] Turned wood reels can be used instead, as shown in the figure (see Appendix).

contrary to that of the hands of a watch, and then repeat the preceding experiments. The direction of magnetisation or polarity of this second helix will be found to be the opposite of that of the first.

(*c*) Notice that soft iron wires are readily drawn into the helices when the current is passing. Notice also that when the central hollow of the helix is filled up with such wires the magnetic power is greatly increased, the polarity being the same as that of the helix without the iron wires. See also if your results conform with the following rule: Look upon the helix from that end which makes the positive current appear to circulate in the direction of the hands of a watch. This end will be the S. pole and the other the N. pole of the helix. Hence if the helix could be swung freely it would point magnetic north and south, and the positive current would at the N. pole ascend on the west side and descend on the east.

(*d*) Place the helix conveying the current vertical, with a piece of cardboard across its end. Scatter filings over it, and obtain magnetic curves with and without a soft-iron core.

Group IV.—Dip the two ends of the battery wires into a small beaker containing dilute sulphuric acid, and leave them there several minutes, the terminals not being in contact with each other. It will be noticed that one terminal becomes covered with bubbles, which collect and escape to the surface, and that this is the one connected with the *negative pole.* The other terminal meanwhile becomes cleaner and brighter, as if the acid were dissolving it. That this is really the case will be seen by the liquid becoming *blue*, owing to the formation of copper sulphate. If the action be continued sufficiently long the negative terminal will become covered with a brown deposit, which on examination will prove to be pure copper.

The general explanation of these appearances is as follows: The current decomposes the liquid in which the terminals are placed, that is to say, it decomposes the molecule of water associated with sulphuric acid, the copper terminal connected with the positive pole taking the oxygen and sulphur, and producing sulphate of copper, while at the negative terminal the free hydrogen, which forms the remaining portion of the decomposed molecule, is allowed to escape.

When, however, besides free acid there is a sensible quantity of sulphate of copper dissolved in the liquid, the hydrogen at the negative pole, while yet nascent, that is to say, while in the act of assuming the gaseous state, instead of doing so, seizes upon the oxygen of the sulphate of copper molecule, thus displacing the copper which is deposited upon the negative terminal.

We shall see afterwards what advantage is taken of this action in plating operations.

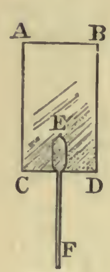


Fig. 28.
VOLTAMETER ELECTRODE.

Group V.—Proceed now to fit up a **Voltameter**, or instrument for decomposing water and collecting the products, as follows:—

(*a*) Cut off the shank of a 4-inch glass funnel to within half an inch from the top.

(*b*) Procure a piece of platinum foil, ABCD, of the size represented in Fig. 28, place it upon an iron plate, and direct the blowpipe flame upon it. Whilst the platinum foil is at a bright red heat lay upon one end of it a short piece of platinum wire, EF, and then by means of a few smart taps with a hammer weld the wire to the foil. Wind the end F of the platinum wire round one end of a piece of No. 20 copper wire about 6 inches long, sprinkle a little resin on the joint, and proceed to solder it by

means of a small soldering iron (see Appendix). The *electrode* will now be finished. Next make a second one of the same size.

(*c*) Fit a cork into the end of the funnel, and, piercing it with two holes by means of a knitting needle, pass the copper wires through these so that the platinum electrodes may be inside the funnel. Well warm the funnel all round, melt some paraffin wax and pour it in so as to fix the electrodes in position and cover the copper wire. The voltameter, which will be similar to that exhibited in Lesson XLVI., is now complete.

(*d*) Procure two test tubes of exactly the same size. Place the voltameter on a retort stand and pour into it some dilute sulphuric acid (say 1 part of acid to 50 of water). Fill the test tubes likewise with dilute acid and invert them over the platinum electrodes. Finally connect the terminals to the battery by means of *clamp screws* (see Appendix). The acidulated water will now begin to be decomposed, and the student will note the following particulars :—

(1) Gases are evolved from both electrodes.

(2) The gas in the tube connected with the negative electrode accumulates twice as rapidly as that connected with the positive.

(3) The gases respond to the tests for hydrogen and oxygen, the relative volumes being those in which these gases combine to form water.

(4) By collecting both gases in one tube an explosive mixture is obtained.

Here again we have evidence of the decomposing power of the electric current, and the student will observe how peculiar must be that action which gives us all the hydro-

gen at the one terminal and all the oxygen at the other. We may perhaps represent to ourselves what takes place by means of the following hypothesis, due to Grotthüss: First of all we may regard oxygen as an electro-negative element and hydrogen as electro-positive. Under these circumstances the oxygen ends of the various molecules will all point to the positive terminal, to which they will be attracted, while, on the other hand, the hydrogen ends will all point to the negative terminal, to which *they* will be attracted.

Now if the electric condition of these terminals be strongly enough developed, the positive terminal will attract the oxygen particle next it, and the negative terminal the hydrogen particles next *it*, and these will be given off at the respective terminals. This is the first operation.

The next will be a change of partners. The hydrogen of the molecule next the positive electrode having lost its partner, will attach itself to the oxygen of the molecule next but one to the electrode, the hydrogen of this to the oxygen of the molecule next but two, and so on until the whole line are once more properly paired. This is the second operation.

They are not, however, yet facing the proper electrodes, for the hydrogen will be facing the positive and the oxygen the negative. They will therefore have all to turn round about their centres through 180°. This is the third and final operation. After this the same round of operations is repeated.

Discharging the Battery.—When we have done with the battery it must be carried to the draught cupboard and there discharged. Remove the clamps, wash and dry them. Pour the nitric acid into a bottle labelled "*Old nitric acid for batteries;*" this may be used again, unless it be of a green colour. Thoroughly wash the porous pots and leave them to soak in water. Notice if any black spots appear

on the zincs, and if so reamalgamate such places; then wash the zincs and leave them likewise to soak in water. By soaking the porous pots and the zincs the zinc sulphate will be removed, which would otherwise tend to block up the pores of the pots and thus disintegrate them, and would likewise crystallise on the surface of the zincs. The sulphuric acid should be thrown away, for it is sure to contain nitric acid, which is very injurious to zinc.

26. The process of chemical decomposition effected by the electric current is called **electrolysis**. The experiments of Groups IV. and V. of the previous lesson are examples of electrolysis. A very important part of electrolysis relates to the deposition of metals, hence the next lesson will be devoted to the typical example of copper deposition.

Lesson XV.—The Daniell's Cell and Copper Plating.

27. *Apparatus.*—A Daniell's cell of the kind exhibited in Fig. 29, which forms a convenient arrangement. It consists of a glazed earthenware pot or outer vessel P, which is 13 cm. high by 9 cm. in diameter. In it stands a cylinder of zinc *Zn*, provided with three tags or tongues, *a*, *b*, *c*, and of these the last has a binding screw attached to it. These tags are formed by cutting away portions from the original sheet of zinc that has been employed to form the cylinder. The height of the cylinder is 10 cm., and its diameter 8 cm., so that when placed in the earthenware pot the zinc is supported by its tags, and the bottom of the zinc is more than 2 cm. from the bottom of the pot. Within the zinc cylinder there is a porous pot *p*, 13 cm. high and 5 cm. in diameter, and this contains a cylinder of copper C*u*, provided with a single tag, to which a binding screw is soldered. The porous pot has its mouth coated with paraffin after the manner already described (Art. 25).

A small flask *f*, containing crystals of copper sulphate each about the size of a small nut, is placed mouth downwards in the copper cylinder. At the bottom of the outer pot a few pieces of scrap zinc are placed.

Fig. 29.—DANIELL'S CELL.

The other materials required are as follows: Crystals of zinc sulphate and of copper sulphate; some telegraphic binding screws (see Appendix); india-rubber cork, ¾-inch diameter, with two holes; some plates of copper ·05 inch thick; a graduated measure; sulphuric acid, caustic soda, nitric acid; brass wire, No. 28; a glass rod, a beaker, a Bunsen's burner, and sundry materials for making solutions.

Charging the Battery.—Place a saturated solution of sulphate of copper in the porous pot. Into the flask already mentioned put crystals of sulphate of copper of about the size of small nuts, and fill it up with a saturated solution of this material, then invert it, and let it stand thus in the porous pot. The flask will now act as a supply reservoir to keep up the strength of the copper sulphate solution. Into the outer pot pour water in which some zinc sulphate has been dissolved (1 part of zinc sulphate by weight to about 20 parts of water). The battery will now be ready for use. Next connect the zinc and the copper by means of a short wire and leave the cell thus for some time with the current passing. In this condition it is said to be *short-circuited.*

Fitting up a Plating Bath.—Fig. 30 exhibits the

requisite arrangement. Here *ab* is an india-rubber cork, having two unconnected holes, an upper hole at the right, and a lower one at the left. Into the hole at *b* there is passed the shank of a telegraphic binding screw, which serves to support a copper plate A by means of the tag *d*, and to connect it with the wire from the positive terminal of the battery. This large plate is called the **Anode**. Into the hole at *a* passes the shank of a double binding screw, formed by uniting together two ordinary binding screws. This serves to support the plate C, which must be smaller than A, and which forms the main *cathode*, as well as a small **Test Cathode** T, and these are to be connected with the negative terminal of the battery. The whole arrangement is supported in a glass battery jar by means of brass wire, as shown in Fig. 30. This jar has to be filled with liquid, whose composition will be afterwards described.

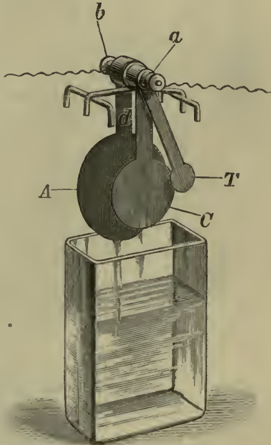


Fig. 30.—Plating Bath.

We may here mention that when in action the copper deposit goes from the anode to the cathode, and hence the propriety of these names.

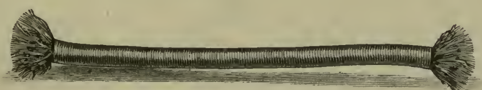


Fig. 31.—Scratch Brush.

Cleansing the Copper Plates.—In the *first* place a *scratch brush* (Fig. 31) should be made. This can be readily done

by driving into a board two long nails about 6 inches apart, and then winding fine brass wire continuously from the one nail to the other. Then bind the strands together by wire, and cut off the ends. The arrangement may now be thrust through a hole in a cork, in order that it may be provided with a handle, and we shall thus have a scratch-brush with two ends. *Secondly*, make a *lifting hook*, which is simply a rod of glass bent into the shape shown in Fig. 32, and provided at one end with a cork handle. *Thirdly*, prepare the following cleansing liquids, and label them as under:—

Fig. 32. LIFTING HOOK.

No. 1. *Alkaline Liquor for Cleansing Copper.*
1 part by weight of caustic soda.
10 parts by weight of water.

No. 2. *Sulphuric Acid Liquor for Cleansing Copper.*
1 part by volume of sulphuric acid.
10 parts by volume of water.

No. 3. *Dipping Liquor for Cleansing Copper.*
1 vol. of impure nitric acid (residue from battery).
1 vol. of water.

No. 4. *Brightening Liquor for Cleansing Copper.*
100 vols. of strong nitric acid.
1 vol. of strong hydrochloric acid.

Enough of these solutions should be prepared to cover the copper plates when they are placed therein. No. 1 should be contained in a porcelain evaporating basin, and the other solutions should be in glass beakers. *Fourthly*, the copper plates may now be cleansed as follows: (*a*) By means of the scratch-brush thoroughly clean both sides of the plates, going over the surfaces several times until the striæ run into each other; (*b*) wash each plate with water under the tap, rubbing it well with the fingers or with a rag; (*c*) boil the plate in the alkaline liquor No. 1. This will

cause a discoloration, owing to the formation of an oxide. Remove the plate by means of the *lifter*, which should be used throughout the subsequent operations. Wash the plate well under the tap, then carry it to liquor No. 2, in which it should remain sufficiently long to enable the acid to dissolve the dark-coloured oxide. Again wash it with water, and then place it in liquor No. 3 for about 15 seconds, after which it must be washed and placed for a few seconds in No. 4, and then quickly washed with *distilled* water. The plate should be now very bright and clean. If it is not so, the processes must be repeated. Let the plate now be preserved in a dilute solution of copper sulphate until required for use.

Deposition of the Copper.—The liquid with which the depositing bath must be charged is obtained by dissolving 100 grms. of copper sulphate in 500 cc. of water. Let it be boiled in a beaker until all is dissolved, and when cold let 25 grms. of sulphuric acid be added. The liquid should be bottled and labelled "*Copper depositing liquid.*"

Next place as much of this liquid in the depositing bath as will well cover the plates, and then connect the plates with the proper battery poles, attaching the negative terminal wire to the cathode or smaller plate, and the wire from the positive pole to the anode or larger plate. Now, place the apparatus in a place where it will not be disturbed and cover it up to keep out dust and prevent evaporation. The liquid should be stirred occasionally. The progress of the deposition may be ascertained by examination of the test plate.

In the course of a couple of days a bright copper deposit will be obtained on the cathode, whilst the anode will be found to be covered with a dark substance resembling mud.[1]

[1] This substance is of complicated composition. Besides containing disintegrated copper it may contain the impurities of commercial copper, such as tin, antimony, sulphur, nickel, silica, selenium, gold, cobalt, iron, and lead.

When a sufficient deposit has been obtained, remove the cathode, wash it well, and preserve it for future experiments.[1]

28. *The Galvanoscope.*—The existence of an electric current may be proved by reference to its (1) heating, (2) lighting, (3) chemical and (4) magnetic effects. An instrument arranged for the exhibition of any of these effects would, properly speaking, be a *current-indicator, detector*, or **Galvanoscope.** But as the magnetic effects produced by the direct action of a current on a freely suspended magnet are by far the most convenient for observation, galvanoscopes are almost invariably based upon the observation of the deflection of a magnetic needle. The methods of construction of galvanoscopes are extremely various. They may roughly be classified into *Vertical Galvanoscopes* and *Horizontal Galvanoscopes.* Fig. 33 shows a vertical galvanoscope of the kind largely used by telegraphic engineers, and called by them a **Detector.** The instrument consists of a vertical coil wound at right angles to the plane of the paper, within which is a pivoted magnetic needle. The needle is loaded so as to rest in a vertical position. Fastened to the same axis as the needle is a pointer, which moves over a circle

Fig. 33.
THE VERTICAL DETECTOR.

[1] For further information the student should consult the various treatises on electro-plating, such as:—*Practical Guide for the Gold and Silver Electroplater, and the Galvanoplastic Operator*, by Dr. Wahl. London: Sampson Low, 1883. *Art of Electro-Metallurgy*, by Dr. Gore. London: Longman and Co. Muspratt's *Chemistry*, new ed., p. 792, Article "Electro-Metallurgy," etc.

placed between the pointer and coil, graduated into degrees. When a current passes, this needle, with its pointer, tends to place itself in a horizontal position.

It may be asked how far an instrument such as a detector may be used as a current measurer or **Galvanometer.** If the angle of deflection of the needle were strictly proportional to the current passing through the coil, then the instrument would be of great value in comparative measurements. But this is by no means the case, nor can the indications be valued by the help of any simple rule. In order, therefore, to render the instrument of service, it must be submitted to the process of **Calibration.** We shall later on describe the necessary process, and meanwhile confine ourselves to the assumption that the greater the deflection the greater must be the current circulating in the coils. This assumption will be made in the next lesson, which deals with some further fundamental experiments made with a horizontal galvanoscope.

LESSON XVI.—The Galvanoscope.

29. *Apparatus.*—A simple galvanoscope, or the following materials for fitting one up, will be required: A tooth-powder box about 3 inches in diameter, four binding screws, No. 28 silk or cotton-covered wire, 6 inches of $\frac{1}{2}$-inch copper strip, wood for making a simple reel, namely, a strip 9 inches by $\frac{1}{4}$ inch by $\frac{1}{2}$ inch, a magnetic needle 2 inches long provided with an agate cap, a sewing needle for pivot, galvanometer card or card-board for making it, a piece of common window glass, thin board ($\frac{1}{8}$ inch thick) on which to mount the card.

Making, Winding, and Fitting the Reel.—Divide the strip of wood into three equal oblong pieces, and fit them together in order to form a reel. Trim the ends so as to make the reel fit somewhat tightly into the box. Make a small

hole at one end of the reel and pass through it 3 inches of wire. Then wind continuously until the reel is nearly filled with wire. Finally, pass the other end of the wire through a second hole in the reel, then fit the reel into the box. Pass the ends of the wire through holes in the lower part of the box, and connect them with binding screws screwed into the box. The bright ends of the wire may be put round the ends of the binding screws, and then firmly held in their place by screwing the binding screws well into the wood. It is better still to make a soldered contact, but if the binding screws are firm this will not be necessary.

Mounting the Card.—Gum or glue upon a thin board a card graduated into degrees. Cut the board into a circular shape so as just to fit inside the box. At its centre fix the point of a needle so as to project upwards above the board for about quarter of an inch or less. Upon this point the agate cap of the magnetic needle is supposed to rest.

Fitting the Lid.—Mark off a circle 2 inches in diameter on the lid by means of compasses, and then cut out a hole having the circle marked as its boundary. Take off the rough edges by means of a file and sand-paper.

Next place the board which has the scale attached to it on a sheet of glass, and cut the glass round its edge by means of a diamond or substitute for a diamond. Snip off the glass with pincers; the glass ought now just to fit inside the lid.

Putting the Pieces together.—In the first place adjust the card in the box so that the zero line of the graduation shall lie along the direction of the strands of the wire. Put the needle on its pivot, and cover the whole with the box-lid. The instrument is now complete. Before being used it must be placed in such a position that the needle points to zero, in other words, the strands of the wire as well as the needle must lie in the magnetic meridian. G. of Fig. 34 shows the completed galvanoscope.

Use of Copper Strip.—Where strong currents have to be observed, it will be necessary to make use of a copper strip, through which, and not through the wire of the galvanoscope, the current must be passed. In this case the galvanoscope as above described, being properly pointed, ought to be placed on the wooden block to which the copper strip is fastened. In the arrangement sketched in Fig. 34 the strip of copper is bent so as to form the three sides of a square. It is pivoted to the wooden block so as to move stiffly.

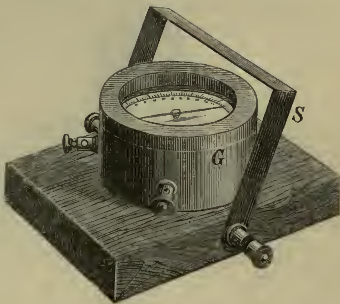


Fig. 34.
GALVANOSCOPE WITH COPPER STRAP.

This is done by screwing the binding screws through holes in the copper into the wood. According to the strength of the current the copper strip must be turned round its bearings into a plane more or less oblique to that of the block, this obliquity being greatest when the current is strongest, and least when the current is weakest. Or we may, by means of a sliding arrangement, place the galvanoscope at a greater or less distance from the copper strip. In Fig. 35 the copper strap is mounted on a wooden hoop, and the galvanoscope is mounted so as to slide on a graduated platform. By either of these arrangements, or by a union of both, we can bring the most powerful currents within the range of the scale of the galvanoscope.[1]

Fitting up the Apparatus.—The copper strip must be

[1] The former of these arrangements exhibits the principle of Obach's galvanometer, the latter the principle of Thomson's current meter, instruments which are employed in measuring currents of different strengths.

connected with the battery by means of the appropriate binding screws. When in action the copper strip must lie in the plane of the magnetic meridian. The arrange-

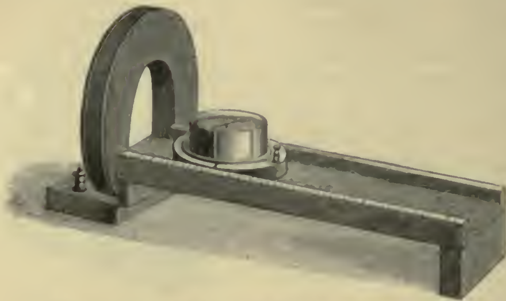


Fig. 35.—SLIDING METHOD OF CHANGING SENSIBILITY.

ment may, if necessary, be firmly fixed to the table by a wooden clamp. The battery and commutator must be east or west of the galvanoscope (see Fig. 36), and the leading wires should remain in a fixed position during the performance of the experiments. Of these the following

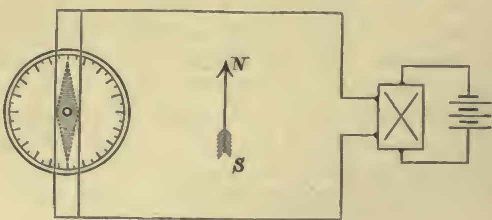


Fig. 36.

are examples, which were all made by means of the copper strip:—

Experiment I.—One cell was found to give a deflection of 48°, whilst two cells in series gave a deflection of 49°, or very nearly the same as before.

Experiment II.—The two zinc terminals were connected together, and the two carbon terminals were likewise connected together, so as to form one large cell (see Fig. 37). This method of connection is known as that of *multiple arc* (where many cells have to be arranged in this manner,

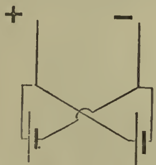


Fig. 37.—Two Cells in Multiple Arc.

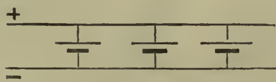


Fig. 38.—Cells in Multiple Arc.

it is best to place them as shown in Fig. 38). It was found that the two cells in multiple arc gave a deflection of 62°.

Experiment III.—A piece of carbon rod, 8 inches long, placed in the circuit reduced the strength, so that one cell now gave only 16°, while two cells in series gave 28°, thereby showing that, when there is a resistance external to the battery, the current is increased by adding to the number of the cells.

Experiment IV.—It was shown that the greater the length of carbon rod in circuit, the less was the deflection.

Experiment V.—Two carbon rods of the same length placed alongside each other gave a greater deflection than one rod alone.

Experiment VI.—A piece of iron wire was coiled in a spiral and placed in the circuit. The deflection was noted, and then the iron was heated by means of a spirit lamp. Thereupon the deflection became less, but when the wire was allowed to cool the needle returned to its previous position.

30. *Theory of the Battery.*—It may here be desirable to

give a short account of the principles of action of the voltaic battery, premising that in all probability these are not yet fully understood, so that any statement we make must only be regarded as a working hypothesis. If a zinc rod or wire be soldered or closely united to a similar copper rod or wire, an electric separation is produced at and over the joining surfaces, in virtue of which the zinc becomes positively and the copper negatively electrified. This electrical difference is not, however, great, and its existence can only be experimentally verified by means of a delicate electrometer. Imagine now (Fig. 39) a circuit of the following nature, namely a thick semicircular zinc rod soldered or united at two junctions to a similar copper rod. Shall we have a current from this arrangement? Unquestionably not. At the upper junction there is no doubt a source of electric irritation, in virtue of which positive electricity is driven to the zinc or left-hand side, and negative electricity to the copper or right-hand side, and if these two electricities could be allowed freely to unite in the remainder of the circuit, we should certainly have a current as long as the electric irritation was kept up. But this is not the case, for the lower junction is a similar source of electrical irritation, and will prevent the union of the two electricities, so that what we shall finally have will be, not a current, but a distribution of statical electricity, in virtue of which the zinc will remain positively and the copper negatively electrified. Before we can get a current we must be able to retain the irritation at the one junction and neutralise it at the other.

Fig. 39.

It is this which is accomplished by means of the battery liquid. Suppose that we dispense with the lower junction and allow the rods to swell out into two plates or terminals of their own material, which are to be immersed in

a vessel containing dilute sulphuric acid (Fig. 40). A molecule of this dilute acid may be regarded as being composed of two members or parts, one of these containing the oxygen, which we may regard as negatively electric, and the other the remainder of the molecule, including the hydrogen, which we may regard as positively electric. The first effect of the immersion of the electrodes in dilute acid may be regarded as a polarisation or pointing of these liquid molecules after the manner which we have previously described, namely, the ends containing oxygen pointing to the zinc, and the ends containing hydrogen to the copper terminal. Now, if the positive electricity of the zinc terminal be more intense than that of the hydrogen portion of the dilute acid molecule, the oxygen portion will leave this hydrogen portion and will unite with the zinc, which will thus be oxidised, and, in like manner, at the other end the hydrogen portion of the dilute acid molecule will go to the copper terminal, carrying its positive electricity with it. By this means negative electricity will constantly be carried to the zinc and positive electricity to the copper terminal, so that the electric difference of these terminals will be neutralised. Meanwhile we may imagine that at the upper junction, the source of electric irritation continuing to exist, a constant supply of positive electricity is carried down the zinc side and a similar supply of negative electricity down the copper side, both of which are, as fast as they descend, neutralised after the manner we have now described. But a current of negative electricity flowing down the right-hand side is equivalent to a current of positive electricity flowing up, so that, taking both sides together, we have virtually a current of positive electricity flowing round the circuit in a direction the opposite to that of the hands of

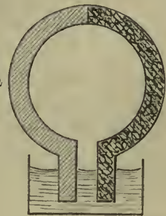


Fig. 40.

a watch, and passing in the liquid from the zinc to the copper.

The combination of the zinc with the oxygen, and the solution of the oxide in the liquid, involves of course the gradual wasting away of the zinc, which may be said to be slowly burned in a liquid manner. This burning is the source of energy in the arrangement, to which the zinc serves as fuel, while the peculiar construction of the circuit is adapted to convert this energy into the form of a current of electricity. We have in fact to look for the *mechanical equivalent* of the energy displayed to the burning of the zinc, and for the *peculiar form* which this energy takes to the arrangement of the circuit. If we did not amalgamate the zinc there would probably be a difference in hardness and chemical composition, between different parts of the same plate. These differences would give rise to local currents, so that the energy due to the combustion of the zinc would be partly spent on these local currents, to the weakening of the main current of the battery, which it is our object to strengthen as much as possible. Thus amalgamation of the zinc, by equalising the chemical composition all over the plate, prevents the formation of these local currents, so that the whole energy of the combustion is directed towards the main current. But it will be asked, What becomes of the hydrogen which is set free on the copper plate? It cannot, of course, combine with the copper, and will ultimately no doubt form bubbles and escape to the surface. Meanwhile, however, it may envelop the copper terminal, and, by means of the tendency to send a current in the opposite direction or **Polarisation** thus produced, act detrimentally upon the production of the current, which will become quickly enfeebled from this cause.

It becomes therefore a matter of importance to prevent this deposition of hydrogen and consequent polarisation, so as to obtain a constant current from our battery. This

is done in Bunsen's battery, which we have just been describing. Here, under ordinary circumstances, while the amalgamated zinc would be gradually oxidised by the dilute sulphuric acid, the hydrogen would be deposited on the carbon plate, which plays the part of the copper, and thus polarise it; but this deposition is prevented by immersing the carbon plate in strong nitric acid enclosed in a porous cell. By this means the nascent hydrogen is immediately oxidised by the oxygen of the acid, and its deposition upon the carbon plate is effectually prevented. The nitric acid will of course, owing to the loss of oxygen, become gradually changed in its composition, and useless for the purpose.

31. *Electromotive Force.*—We have here spoken of the electric difference which is continuously kept up at the junction of dissimilar metals; this may be termed (for the present purpose) the **Electromotive Force** of the arrangement, and is generally denoted by the letter E. This electromotive force may be regarded as chiefly, at all events, depending upon the electro-chemical difference between the two plates, so that zinc and copper would give one value of E, zinc and carbon a second, zinc and platinum a third, and so on. Suppose we confine ourselves to zinc and carbon, then, if we have a single cell, its electromotive force will be E. If, however, we have two cells in series, that is to say, the zinc of the one cell being connected with the carbon of the next, we shall have a total electromotive force equal to 2E,—if three cells in series, 3E, and so on.

32. *Ohm's Law.*—It must not, however, be imagined that if two circuits have the same electromotive force the current will necessarily be the same in each. This leads us to discuss the law which regulates the rate of flow, intensity, or strength of the current produced, known as Ohm's law, because it was discovered by Ohm, a German physician.

In order to explain this law, imagine that we have a thick cylindrical metallic rod (Fig. 41), of which the upper cross-section A is kept at an electric potential or level different from that of the lower cross-section B. This difference of electrical level we shall call E, and indeed, it is merely a manner of expressing the cause of electromotive force. In consequence of this electrical difference between the top and bottom being kept up at these places, there will be a continued flow of electricity from the top to the bottom, the strength of which will depend amongst other things upon the value of E; double E and you double the flow, make E three times as great and you increase the flow in the same proportion, and so on. Thus the strength of the current or C is *proportional to the electromotive force or E.*

Fig. 41.

In the next place the flow of electricity will be *proportional to the cross-section* of the rod at A, so that if we double the cross-section we shall double the flow. The double cross-section virtually makes the single rod into two rods, and this law hardly requires further explanation. The next point is that if we double the length of the rod we halve the flow, other things being the same—in other words, the flow is *inversely proportional to the length* of the rod. To prove this, let us suppose that the rod in the above diagram is cut by an imaginary cross-section half-way between the top and the bottom. The electrical difference between this section and the bottom will only be one half of that between the top and the bottom, or it will be $\frac{E}{2}$, and yet, since we have not altered the state of things, we shall have the same current C as before in the lower half of the rod. In other words, we may either regard the current C as produced by an electrical difference E between the top and bottom of the rod, or by an electrical difference equal to $\frac{E}{2}$ between the middle and bottom of the

rod. Now had there been an electric difference = E between the middle and the bottom, we should obviously have had a double current—in other words, for the same electrical difference the current is inversely proportional to the length of the rod.

Finally, the amount of current will depend *upon the nature* of the rod—if it be of copper there will be a large flow for a small electrical difference, if it be of wood the flow will be much smaller, and if of ebonite there will be scarcely any flow whatever.

All that we have now stated is conveniently expressed by Ohm's law and the other laws associated with it. The following is a statement of Ohm's law: Let C represent the strength of the current in a circuit, E the electromotive force, and R the resistance this current experiences from the materials of the circuit, then

$$C = \frac{E}{R}.$$

To define the resistance, or that which impedes the flow of electricity, we must bear in mind what we have already indicated above (1.) that the conductivity is directly, and hence the resistance is inversely, proportional to the cross-section of a rod or wire; (2.) that the conductivity is inversely, and hence the resistance is directly, proportional to the length; (3.) that the conductivity depends on the substance of which the rod or wire is composed, each substance having its own specific conductivity; hence the resistance also depends on a specific resistance, which will vary inversely with the specific conductivity In fine, resistance may be regarded as the reciprocal of conductivity, so that we may either assert that the current is jointly and directly proportional to the electromotive force and the conductivity of the circuit, or directly proportional to the electromotive force and inversely proportional to the resistance.

The resistance of a circuit is usually divided into two parts—the internal or essential resistance of the battery, consisting chiefly of that of the liquid into which the plates are immersed, and the external resistance, which may be varied according to circumstances. The laws now given apply equally to the internal and to the external resistance. If we denote the former by R and the latter by r, then Ohm's law will stand as follows:—

$$C = \frac{E}{R+r}.$$

We may now apply Ohm's law to give an explanation of the experiments of pages 76 and 77.

In the first place, for one cell, without any external resistance except the copper strip, we shall have $C = \frac{E}{R}$, while for two such cells in series we shall have $C = \frac{2E}{2R} = \frac{E}{R}$. Thus both theory and experiment agree in demonstrating that the current is the same in these two cases—as a matter of fact the galvanoscope indications were 48° and 49°.

Again, when the two cells are connected together in multiple arc, we have virtually one large cell of a double cross-section. Here the current will be, since the resistance is halved owing to the cross-section being doubled, $C = \frac{E}{\frac{1}{2}R} = \frac{2E}{R}$. Accordingly we ought to have a double current, and, as a matter of fact, the galvanoscope indications increased from 48° to 62°. We must not, however, in the meantime attempt to use these numbers to give us an accurate measurement of the comparative intensity of the current in the two cases; this will come afterwards, when we describe the galvanometer. Suffice it, however, that both by theory and experiment the current is much larger when we have the two cells connected in multiple arc than when we have a single cell or two cells in series. Thirdly, when we interpose a considerable external resistance, such as a piece of carbon rod, not only is the current

greatly reduced in strength (from 48° to 16°), but the two cells in series give us decidedly more than a single cell, the numbers being 16° for a single cell, and 28° for the two cells in series. This follows at once from Ohm's law, which will give us for a single cell under these circumstances $C_1 = \frac{E}{R+r}$, and for two cells $C_2 = \frac{2E}{2R+r}$. Now if r is considerable, C_2 will be decidedly greater than C_1, and if it be very great compared to R, C_2 will be nearly double of C_1.

Finally, we see from Experiment IV. that a rise of temperature increases the resistance of an iron wire, and the same law will hold for other metals.

When the external part of a circuit is composed of varying resistances, we must remember, in applying Ohm's law to it, that the same quantity of electricity passes in one second through every cross-section of the circuit. For if this were not the case, more positive electricity might be carried into some region than was carried out of it, so that positive electricity would there accumulate, or less might be carried in than was carried out, so that the region would become more and more negative. But both of these suppositions are inadmissible, inasmuch as when a current is established we have a constant state of things. We must therefore suppose that the quantity of electricity passing any cross-section in unit of time, or, in other words, the current, is *constant throughout the circuit.* Now under these circumstances, what we have already said will lead the reader to infer that the difference of potential (which we shall take to be the cause of the electromotive force) between any two points in a circuit must so dispose itself as to be proportional to the resistance between these points, so that the greater the resistance so much the greater is the electromotive force. In other words, we have in the whole circuit a given electromotive force E to dispose of, and this must be distributed along the circuit, so that the force between two points shall always be *proportional to the resistance* between these points.

33. *The Units of Theory and Practice.*—Ohm's law may be written in three ways:—

$$C=\frac{E}{R} \quad . \quad . \quad . \quad . \quad (1)$$

$$E=CR \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$R=\frac{E}{C} \quad . \quad . \quad . \quad . \quad (3)$$

If in (1) $E=1$ and $R=1$, we define

Unit current as the current in a circuit with unit E. M. F. and unit resistance . . . . (4)

If in (2) $C=1$ and $R=1$, then we define

Unit E. M. F. as the E. M. F. in a circuit with unit current and unit resistance . . . . (5)

If in (3) $E=1$ and $C=1$, we define

Unit resistance as the resistance in a circuit with unit E. M. F. and unit current . . . . (6)

Having then fixed upon independent values for any two of the units, the third will be determined by one of the definitions (4), (5), or (6). We are at liberty to select any units we please. Thus, for instance, the unit current might be that produced by a Daniell's cell of a certain construction and size when its poles were connected by a specified wire; and the unit of resistance might be that between the ends of a cylinder of pure silver of specified diameter and length. But every one is now agreed that it is desirable that the units should be derived from the fundamental units of length, mass, and time adopted in this work (see Vol. I. Appendix), namely the centimètre, the gramme, and the second. Accordingly methods have been devised of defining the electrical units with reference to these three fundamental units. A method of deriving these units will be given in Appendix B, and tables of the various systems of units in Appendix C.

The units so obtained are of very inconvenient mag-

nitude for practical purposes, and hence *practical units* have been chosen by taking a submultiple of the unit of current and multiples of the units of E. M. F. and resistance. Thus are obtained :—

The ampère	$= 10^{-1}$	of the	C. G. S.	unit of current.
The volt	$= 10^{8}$	,,	,,	E. M. F.
The ohm	$= 10^{9}$	,,	,,	resistance.

The units which we are here discussing are called **Electromagnetic Units**, to distinguish them from units of different nature called **Electrostatic Units**, which are derived from the effects of electrostatic repulsion and attraction.

It will be seen from the numerical values above given that if we have a circuit in which the resistance is one ohm and the E. M. F. one volt, then the strength of the current will be one ampère; for $\frac{10^8}{10^9} = 10^{-1}$.

Ohm.—A Committee of the British Association found that the resistance of the ohm is represented nearly by the resistance of a column of pure mercury 105 cm. long and 1 sq. mm. in section at 0° C. They caused coils of an alloy of silver and platinum to be issued as standards. Resistance coils based on these standards are called B. A. ohms. Recent experiments of Lord Rayleigh and others prove beyond doubt that the B. A. ohm is more than one per cent too small. The B. A. ohm therefore can only really be regarded as an empirical unit, just as is the case with the standard mètre. An attempt is, however, being made to substitute for the old standards new ones of correct value. These are called *True Ohms*, sometimes *Rayleigh Ohms*. In accordance with the recommendations of a Congress held at Paris in 1884 a *legal ohm* is defined to be the resistance of a column of pure mercury about one centimètre longer than that defining the B. A. ohm, or 106 cm. More exactly the relation between the units is

1 Congress ohm = 1·0112 B. A. ohm.
1 B. A. ohm = ·9889 Congress ohm.

In Germany the Siemens unit or S. U. is largely used. It is supposed to denote the resistance of a column of pure mercury 1 sq. mm. in section and 1 mètre long at 0° C.

$$1 \text{ S. U. unit} = \cdot 9540 \text{ B. A. ohm.}$$

A megohm is one million ohms. A microhm is one millionth of an ohm.

Volt.—

$$1 \text{ Congress volt} = 1 \cdot 0112 \text{ B. A. volt.}$$

A Daniell's cell has approximately an E. M. F. of one volt.

Ampère.—The ampère in common use being dependent on the ratio of the volt to the ohm, is left unchanged, and has the same value as the Congress ampère.

A milliampère is one thousandth of an ampère.

34. *The Mirror Galvanometer.*—To take advantage of Ohm's law for electrical measurement the student must be provided with a galvanometer. The best form of galvanometer will be one in which *currents are simply proportional to the deflections.* This is the case, as we shall immediately prove, with the mirror galvanometer, an instrument of extreme value to the electrician.

When a small magnet is suspended at the centre of a coil, and is deflected through an angle α by a current C circulating in the coil, the following relation may be considered to be true:—

$$C = K \tan \alpha \qquad (1)$$

where K is a constant. This will be formally proved in a later chapter. Meanwhile we shall assume it to be true for the case of our mirror galvanometer. In Vol. I. p. 55 we have considered the measurement of small angles

by the aid of a mirror and a beam of light, and it was there shown that

$$\tan 2\alpha = \frac{n}{L} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where n is the number of millimètre divisions that the reflection traverses on the scale, and L is the distance of the scale from the mirror in millimètres. Now for small values of α we may assume

$$\tfrac{1}{2} \tan 2\,\alpha = \tan \alpha \quad . \quad . \quad . \quad . \quad (3)$$

Hence from (1), (2), and (3)

$$C = \frac{K}{2L} \times n = \text{constant} \times \text{deflection} \quad . \quad . \quad . \quad (4)$$

or *the current varies directly as the observed deflection.* For all the work of this chapter we shall consider this to be true.

In the following lessons it will be necessary for the student to have a mirror galvanometer of simple construction. The student may easily learn how to put together such an instrument, and it is desirable that this should be attempted by all students.

Lesson XVII.—Construction of Mirror Galvanometer.

35. *Materials.*—A wooden base B (Figs. 42 and 43) 8 inches in diameter and 1 inch thick. A pillar P, 3 inches in diameter and 4 inches high, bored with a small hole passing along its axis. A reel R, 3 inches in diameter and $1\frac{1}{2}$ inch thick, with flanges of half an inch and a central hole $1\frac{1}{4}$ inch in diameter, with a small recess on one face. A plug to fit the hole of the reel. The reel with plug is seen in the two upper figures of Fig. 44. The above may be prepared by any wood-turner. A round piece of glass for window, to fit the recess in the reel. Bobbins

of No. 28 silk-covered, and No. 20 cotton-covered, wire. Three binding screws (No. 3 telegraph binding screws.) A brass rod r to support the *directing magnet* M, which may be of crinoline steel, and which is fixed to a cork C. The cork slides up or down the rod. The *magnetic needle for*

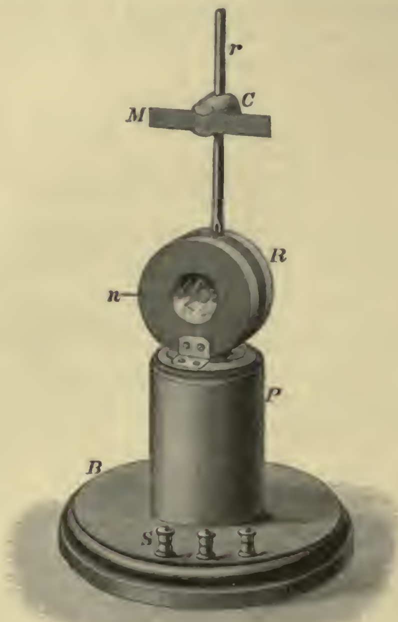


Fig. 42.—SIMPLE MIRROR GALVANOMETER.

suspension requires to be attached to the back of a small mirror of microscopic glass, silvered. It has aluminium foil for a damper and a cocoon fibre for suspension. The magnetic needle is made of watch spring.

Scale, Lamp, and Lens (Fig. 45).—The scale requires three

pieces of wood. The base B—16 inches × 6 inches × 1 inch thick. The front A—16 inches × 9 inches × $\frac{1}{2}$ inch thick. The shade S—16 inches × 4 inches × $\frac{1}{2}$ inch thick. The front has a $\frac{3}{4}$-inch hole *h* $7\frac{1}{2}$ inches from the bottom. A paper scale *ab* 16 inches long is divided into millimètres. The lamp is a small paraffin lamp P that may be hooked on to the scale, which is provided with two staples for the

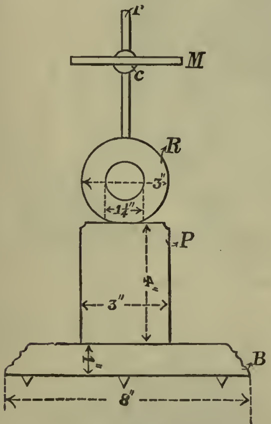


Fig. 43.
DIMENSIONS OF MIRROR GALVANOMETER.

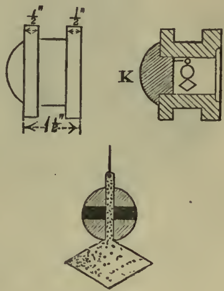


Fig. 44.
PARTS OF MIRROR GALVANOMETER.

purpose. A lens of 5-inch focus is fitted on a cork supported by a bottle L laden with shot. The lens is used for focusing (see Fig. 45).

Construction I.—Winding the Reel.—This may be done by hand, but it is far more expeditious to employ the simple machine of Fig. 46. The reel R is slipped upon the somewhat conical axis, where it is wedged firmly. A few turns of wire are then wound round the axis, and the wire is then wound regularly on the reel with a moderate tension from the bobbin B, which may be mounted on a metal axis supported by two uprights so as to revolve.

The winding machine and bobbin holder should be clamped or screwed down to the table. First wind one layer of

Fig. 45.—SCALE, LAMP, AND LENS.

No. 20 wire, then give it a coat of melted paraffin applied

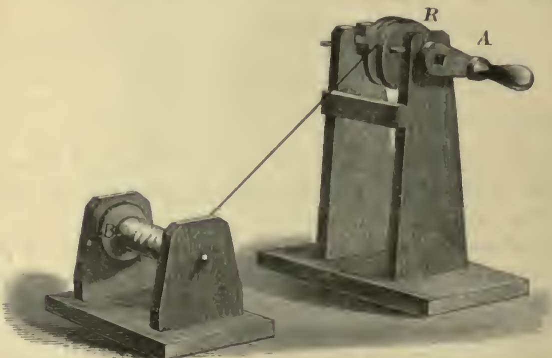


Fig. 46.—WINDING THE REEL.

with a brush, then wind a second and third layer, applying paraffin to each. Leave about 12 inches of wire for

making connections; this may be wound round the axis. Secondly, replace on the winding machine the bobbin of No. 20 wire by the bobbin of No. 28 silk-covered, and wind on about 300 turns. Paraffin will not be required with this wire, the insulation of the silk being sufficiently good if the wire be not roughly handled. Should any bare places appear they should be covered with tissue paper that has been steeped in paraffin.

The free ends of the wires should be dipped in paraffin, and they should be run together so as to leave the reel at the same place. A piece of ribbon is wound round the reel to keep the wire in its place and as a protection from dust.

II.—Fit together the woodwork, screw the pillar to the base, fasten the reel on the top of the pillar by brackets of zinc or brass, run the wires down through the hole in the pillar and through that in the base, solder the end of the No. 20 wire, and the beginning of the No. 28 to the same binding screw, the other ends going to separate binding screws placed one on each side of the common binding screw; thus the three screws will serve for the two coils, and by using the extreme screws, the two coils may be used in conjunction. It is best to first solder short lengths of wire to the shanks of the binding screws before passing them through the holes in the base, and then solder the ends of these wires to the free ends of the coils, for it is difficult to solder the latter directly to the short shanks of the binding screws. The base may either be supported by levelling screws (three window-fasteners do very well) or raised until it is horizontal by means of three small wooden feet.

III.—The next thing will be to make the needle. Harden and magnetise $\frac{1}{4}$ inch of watch spring, and fix it to the back of a small mirror by wax. Cut out a piece of aluminium foil in diamond shape, leaving a tag to which the mirror must be fixed. The completed needle

is seen in Fig. 44, where the circular glass mirror, the horizontal magnet, and the diamond-shaped aluminium damper—all these being in the same plane—will be recognised. A hole must be pierced in the end of the tag with a small needle for the reception of the suspending fibre.

IV.—Fix a small piece of wire to the inner portion of the plug K (Fig. 44), and suspend the needle from it by means of a single fibre of cocoon silk. This operation is one requiring considerable skill and care; it does not, however, require special description.

V.—Next arrange the cork with the directing magnet on the rod. Put in the window with a little putty.

VI.—Fasten the scale together, stretch a wire across the hole, and glue the paper scale upon the cross-piece.

Setting up of Galvanometer and Scale.—Place the instrument in the magnetic meridian, and set the scale about 2 feet away, the centre of the scale being opposite to the mirror and parallel to it. Raise the galvanometer or scale, and bend the aluminium support of the needle slightly, if necessary, until the reflection from the mirror falls on the scale. Focus by means of the lens until a distinct image of the wire is obtained in the middle of the image of the hole upon the scale. Bring this image to the middle of the scale by turning the directing magnet. The instrument will now be ready for use.

36. *Use of Box of Coils.*—It is necessary in many experiments to have the means of varying by degrees the amount of resistance in a galvanic circuit. A box of coils arranged in series is generally used for this purpose whenever the requisite variation is capable of being made by steps, none of which are less than a unit of resistance. Fig. 47 exhibits the interior of such a box of coils as usually arranged, so as to serve the double purpose of a resistance box and a Wheatstone's bridge. In the present lesson it is only required for the former purpose. A plan of

the arrangement of the coils is seen in Fig. 48. On a block of ebonite *abcd* there are mounted a good many thick brass connecting pieces distributed in three rows,

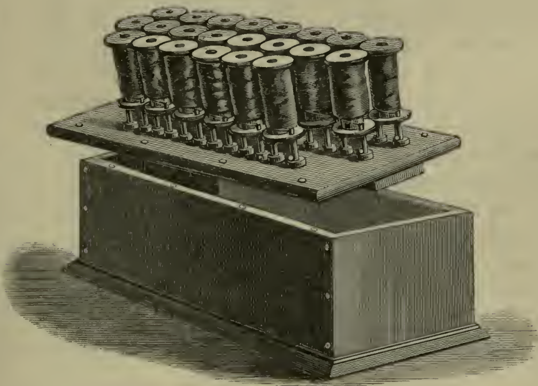


Fig. 47.—Interior of Box of Coils.

somewhat in the shape of the letter S. The parts AB, BC are known as the *Proportional Arms*. These are connected with the *Rheostat Arms* DEF by means of a

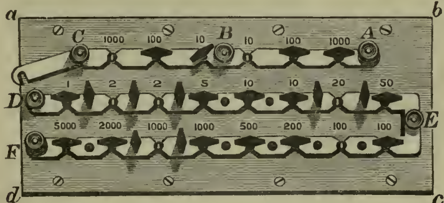


Fig. 48.—Plan of Box of Coils.

brass piece CD, movable at pleasure by unscrewing its clamp screws at C and D. At A, B, C, D, E and F are binding screws. Between each of the brass pieces there is

a space into which a well-fitting brass plug or stopper may be placed so as to make perfect metallic contact from the one piece to the other. The plugs may be inserted or removed at pleasure, being provided with an ebonite handle for the purpose. The holes and plugs are all exactly similar, so that any plug would fit any hole. On the inside of the ebonite lid there are bobbins for holding the wire. Each bobbin consists of a brass tube covered with a layer of paper. It is provided at each end with an ebonite disc. The bobbin is kept in position by two screws which pass through the lower ebonite disc. These screws are each in connection with the wire of the bobbin below and with the corresponding brass segment on the upper side of the ebonite lid; but they may be separated from the latter if necessary by unfastening two small screws. Usually, however, the screw joints at these points are soldered in order to render the contact more secure.

The wire employed for these resistances is of German silver,[1] selected both on account of its high resistance and the small variation of this due to change of temperature. The wire is covered with one or two layers of white silk. In winding the wire is doubled upon itself, and then wound so doubled. This method is adopted in order to avoid *self-induction*, and also to avoid any *electro-magnetic* effect which might vitiate the galvanometer readings. These are terms which will be afterwards explained. For the lower resistances thick wires are employed, in order that a great length of wire may be obtained, and thus a more exact adjustment secured. Furthermore, the lower resistances may be subjected to greater heat than the higher resistances. The actual sizes of the wires used in the rheostat arm are exhibited in the following table :—

[1] German silver is an alloy of 50 to 60 parts of copper, 25 to 30 parts of zinc, and 15 to 20 parts of nickel.

Table E.

Series of Wires suitable for Resistance Coils.

Ohms.	Diameter of Wire in Inches.	Ohms.	Diameter of Wire in Inches.
1	·05	100	·020
2	·05	200	·013
5	·04	500	·013
10	·031	1000	·008
20	·031	2000	·008
50	·022	5000	·005

The resistances of the various proportional arms are 10, 100, 1000, the sizes of wire as given above being used for these resistances.

The student will understand that a set of standard resistances plays in the measurements of resistance the same part that a set of standard masses plays in the measurements of mass.

37. *Care and Use of the Box of Coils.*—The success of some of the subsequent measurements will depend largely upon the observance of the following precautions: (1.) The ebonite should be free from dust, etc., especially in the intervals between the brass pieces. A little paraffin oil should be rubbed over the surface when it is cleaned. (2.) The plugs should be bright and free from grease. They must be made so as to fit well into their places, and they should be tightened by means of a screw motion. Occasionally they may be just touched with the finest emery paper, but this should be done as seldom as possible, for otherwise the plugs may become loose in their holes. (3.) The connecting pieces and the surfaces of the connecting screws should be bright and clean, and the screws should be firmly screwed in their places. It is hardly necessary to remind the student that when a plug is inserted into its hole between two brass segments, the result is that the current virtually passes through these segments and through the plug, which present very small resistance, and not sensibly through the bobbin

which is underneath. When, however, the plug is withdrawn, the current must all pass through the bobbin.

38. *Figure of Merit.*—In order to express the sensibility of a galvanometer in measurable terms, it is usual to determine the current in **ampères** which will be required to produce a deflection on the scale of one division.

The current required is called the *Figure of Merit.* This current, and therefore the figure of merit, will depend upon the position of the directing magnet, and also upon the distance of the scale from the galvanometer. It is desirable that the scale should be kept at a fixed distance from the galvanometer, so that it is the position of the directing magnet that will have to be raised or lowered in order to obtain the required sensibility.

LESSON XVIII.—Figure of Merit of Galvanometer.

39. *Apparatus.*—A mirror galvanometer and its accessories, a box of coils, a Daniell's cell, a plug key (Fig. 49).

Fig. 49.—PLUG KEY.

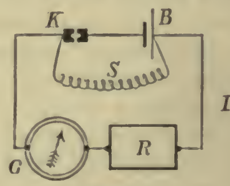


Fig. 50.

Method.—For the purpose of this lesson it will be necessary to obtain approximate values of the resistance of the battery and of the galvanometer.

Resistance of the Galvanometer.—Make connections as in Fig. 50, where B is the battery, K a plug key, G the galvanometer, R the box of coils, and S a shunt, or in other words, a short piece of wire placed so as to *short-circuit* the

battery at pleasure. When the shunt is of sufficiently small resistance, the deflection of the galvanometer may be reduced to a readable amount. Furthermore, the *combined resistance* of the battery and shunt will be so small, that in comparison with that of the rest of the circuit it may be neglected. If now resistance be introduced into the lower part of the circuit by taking plugs out of the resistance box until the original deflection is halved, we shall know that the total resistance has been doubled, so that the added resistance must be equal to that of the galvanometer. For it will be seen at once that the great body of the current will go by the short-circuit, and that this will not be appreciably affected by increasing the resistance by the withdrawal of plugs from the box of coils. Thus the difference of electromotive force at the extremities of the shunt wire may be regarded as constant, and hence the current in the main circuit will be inversely proportional to the resistance.

Resistance of the Battery.—To determine this the same principle is applied, only the shunt is now transferred (Fig. 51) to the galvanometer. Here the resistance of the galvanometer being very great compared to that of the shunt, the great body of the current will go through the circuit and shunt, and only a very small portion of it through the galvanometer. The intensity of the current will therefore be virtually regulated by the resistance of the circuit and shunt, and this intensity will of course be recorded by the galvanometer. Thus by this arrangement the galvanometer records the strength of the current, but does not sensibly interfere with it. Now let us introduce, by means of the box of coils, resistance until the deflection of the galvanometer is halved; this means that the current is halved, and that the resistance of the whole circuit is doubled. Hence the additional resistance introduced must be equal to that of the battery, as the joint resistance of shunt and galvanometer is negligible.

In making these tests, if the battery should vary, the results will be affected. It is therefore important to make the tests quickly, and the battery circuit should only remain closed while the tests are being made. More especially is this true when the battery is short-circuited, for it is a rule that the smaller the resistance of the circuit the more liable is the battery to be inconstant.

Figure of Merit.—Make connections as in Fig. 52. Employ resistances so as to give successively deflections of about 150, 100, and 50 divisions.

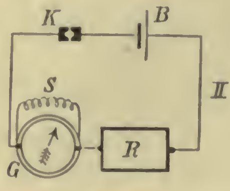


Fig. 51.

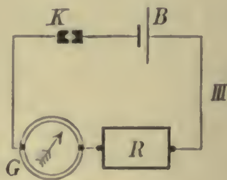


Fig. 52.

Divide the electromotive power of the battery in volts (a Daniell's cell with E. M. F. = 1·08 volt approximately) by the total resistance in ohms of the circuit. This will give the current in ampères, which, when divided by the deflection, will give the figure of merit required.

Example—Resistance of Galvanometer.—To reduce deflection from 240 to 120 divisions, 9 ohms were required, which is the resistance of the galvanometer.

Resistance of Battery.—To reduce from 220 to 110, 3 ohms were required.

Figure of Merit.—

R = Total Resistance.	D = Deflection.	Figure of Merit = $\frac{1·08}{DR}$.
2962	150	·00000243
4342	100	·00000248
8652	50	·00000249

Mean ·00000247 ampères,
or 2·47 microampères.

The directing magnet had its north pole to north, and was placed at the top of the support.

40. *Determination of E. M. F.*—Unless we are provided with standards of E. M. F., it will be difficult to determine the E. M. F. of a cell in volts. No official standard has been yet issued. The best available is a cell of Latimer Clark's construction, as will be explained in a later chapter.

LESSON XIX.—Comparison of Electromotive Forces by the High Resistance Method.

41. *Exercise.*—To compare together the electromotive force of various cells.

Apparatus.—A coil of high resistance—at least 5000 ohms, a mirror galvanometer and its accessories.

Method.—This consists simply in observing the deflections produced when the high resistance is in circuit.

Theory of the Method.—Let E_1 be the electromotive force of one of the cells (say a Daniell's cell), and E_2 that of another cell (say one of Bunsen's). Also let B_1 and B_2 be the respective resistances of these cells, while R is the resistance of the external circuit, including the galvanometer. When the Daniell's cell is in circuit we shall have, by Ohm's law,

$$C_1 = \frac{E_1}{B_1 + R},$$

and when the Bunsen's cell is in circuit we shall have

$$C_2 = \frac{E_2}{B_2 + R};$$

hence

$$C_1 : C_2 : : \frac{E_1}{B_1 + R} : \frac{E_2}{B_2 + R} \quad . \quad . \quad . \quad . \quad (1)$$

Now if R be very great compared to B_1 or B_2, this proportion will virtually become (since $B_1 + R$ is sensibly the same as $B_2 + R$)

$$C_1 : C_2 : : E_1 : E_2 . \quad . \quad . \quad . \quad . \quad (2)$$

In other words, the electromotive forces are to one another in the same proportion as the currents, that is to say in the same proportion as the deflections produced.

The higher the resistance R the more accurate will be this method. To obtain an idea of the error produced let us imagine that the observed deflections are 100 and 200 divisions. This will, according to formula (2), also be the ratios between the electromotive forces. But if the resistance of the batteries be respectively 5 ohms and 0·5 ohm, and R be = 5000 ohms, we shall have by formula (1)

$$E_1 : E_2 :: \frac{100}{5005} : \frac{200}{5000 \cdot 5} :: 500050 : 1001000 :: 1 : 2 \cdot 002,$$

or almost exactly the same as before.

Example.—Standard cell (E. M. F. = 1·46 volt) gave 190 divisions of deflection through 20,000 ohms. With the same resistance the results with other cells were

Daniell. 148 divisions, hence E. M. F. $= \dfrac{1 \cdot 46 \times 148}{190} = 1 \cdot 14$ volts

Bichromate, 260 ,, ,, ,, $= \dfrac{1 \cdot 46 \times 260}{190} = 2 \cdot 00$,,

Lesson XX.—Proof of Ohm's Law.

42. *Apparatus.*—A mirror galvanometer; a coil of No. 40 German-silver wire at least 5000 ohms in resistance; two small plates of copper having binding screws soldered to them, and fixed to a board so that their inner edges are just a mètre apart; a German-silver wire (No. 36) which is stretched tight along the board and soldered to the copper plates; a key or commutator; a few cells of a constant battery will likewise be required.

Method.—Make connections as shown in Fig. 53, in which B is the battery, K the key, G the galvanometer, R the high resistance, PQ the German-silver wire. The unconnected end of the wire leading from the high

resistance should be filed into a wedge shape, and then thrust through a cork, which is intended to serve as a handle and prevent the temperature of the observer's hand from affecting the wire. Observations are made in the following order: place the free end at different points along the wire, and read the deflection produced at each point; reverse the poles of the battery, and then repeat the observations in the opposite order. The two results may be somewhat different, owing to possible variation in the strength of the current. The mean should therefore be taken.

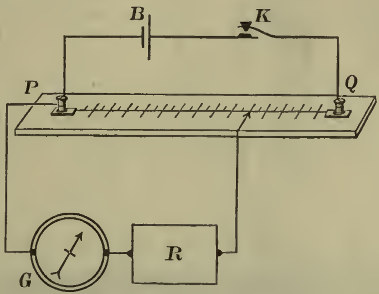


Fig. 53.—Proof of Ohm's Law.

Now if the differences of potential or electromotive force along the wire are, as Ohm's law would indicate, proportional to the resistances, that is to say, to the length of wire between the two points at which the potential is taken or tapped, it follows that the number expressing this resistance should have a constant ratio to that expressing the difference of potential. But the difference of potential will be expressed by the current of the galvanometer which it produces, so that ultimately this current will be proportional to the distance between P and the free end of the galvanometer wire.

That this proportion holds fairly well will be seen from the following series of experiments. But before exhibiting

this series we would remark that the galvanometer circuit is to be here regarded as one which taps the main circuit above the wire and indicates the difference in potential by means of the deflection produced, without sensibly interfering with this main current.

Example.—

Reading on PQ.	Deflection.			
(1)	I.	II.	Mean (2).	$\frac{(2)}{(1)}$
10	11	12	11·5	1·150
20	24	24	24·0	1·200
30	37	35·5	36·3	1·210
40	48	47·5	47·8	1·195
50	60	59·5	59·8	1·196
60	70	70·5	70·3	1·171
70	82	82	82·0	1·171
80	94	94	94·0	1·174
90	106	106	106·0	1·178
100	118	118	118·0	1·180

The results of this lesson may best be represented

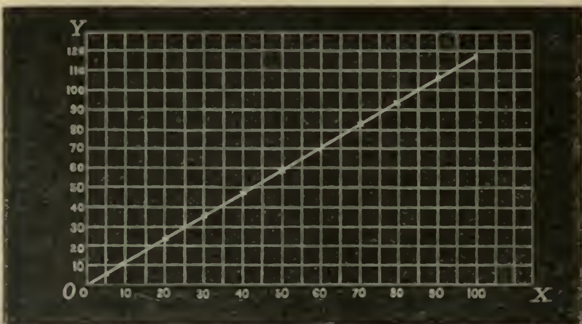


Fig. 54.

graphically. Let the line of abscissæ or horizontal line represent distances on the wire, and the line of ordinates

the observed potentials at these points. On plotting the observations (see Vol. I. p. 275) we obtain a nearly straight line (Fig. 54). This shows at once that the fall of potential between two points is proportional to the resistance between these points. This method of recording results will enable the student to understand the principle of Wheatstone's Bridge.

43. *Wheatstone's Bridge.*—Suppose that OAC and O′A′C′ (Fig. 55) are two wires whose resistances are represented by

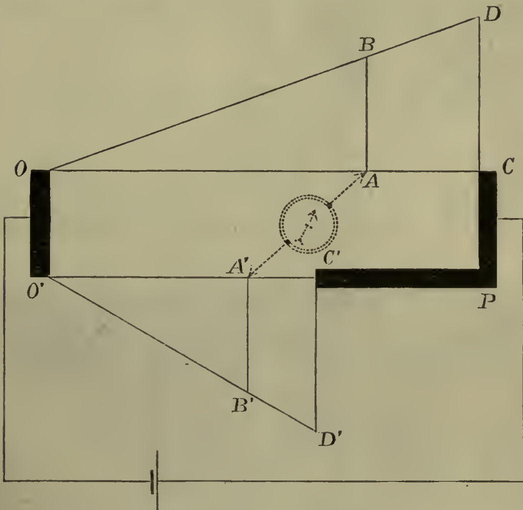


Fig. 55.—THEORY OF WHEATSTONE'S BRIDGE.

their lengths. Let their ends be connected by thick copper pieces, OO′ and CPC′, of which the resistances may be neglected. We shall suppose that a battery is connected with OO′ and CPC′, whereby these parts are kept at a constant difference of potential, represented by the equal

lines CD and C′D′ (the potential at OO′ being supposed for convenience equal to zero). The fall of potential along the wires will be given by OBD and O′B′D′. Take any point A in OC and find the potential at this point by erecting an ordinate AB. Now a corresponding point A′ can be formed along O′C′, such that the potential A′B′ at this point shall be equal to AB. In this case a galvanometer connecting A and A′ would not indicate any current, since these points being at equal potentials no current would pass from the one to the other through the galvanometer. But under these circumstances what must be the relation between the resistances OA, AC, O′A′, A′C′? We have

$$\frac{\mathrm{OA}}{\mathrm{OC}}=\frac{\mathrm{AB}}{\mathrm{CD}}=\frac{\mathrm{A'B'}}{\mathrm{C'D'}}=\frac{\mathrm{O'A'}}{\mathrm{O'C'}},\text{ or }\frac{\mathrm{OA}}{\mathrm{OA+AC}}=\frac{\mathrm{O'A'}}{\mathrm{O'A'+A'C'}};$$

hence

$$\frac{\mathrm{OA}}{\mathrm{AC}}=\frac{\mathrm{O'A'}}{\mathrm{A'C'}}.$$

This is the principle of Wheatstone's Bridge, which is

Fig. 56.

usually arranged in the form shown in Fig. 56, where P, Q, R, S are four resistances. If these be in the ratio

$\frac{P}{Q} = \frac{R}{S}$, then the galvanometer will not be affected. When this adjustment is made any one of the four resistances may be determined if the other three are known.

Lesson XXI.—The Wheatstone's Bridge.

44. *Apparatus.*—A mirror galvanometer with its accessories. A slide half-mètre Wheatstone's bridge, or the following materials for its manufacture: (1.) Board of varnished wood 2 feet long, 4 inches broad, $\frac{3}{4}$ inch thick. (2.) Some sheet copper $\frac{1}{16}$ inch thick. (3.) Seven telegraphic binding screws. (4.) Uncovered German-silver wire, 2 feet long, No. 28 B. W. G. (5.) A half-mètre boxwood scale $\frac{1}{2}$ inch broad and $\frac{1}{8}$ inch thick. This should be divided along one edge into half centimètres. (6.) Sixteen $\frac{1}{4}$ inch brass screws. (7.) Two small pieces of copper, $\frac{1}{2}$ inch square, $\frac{1}{16}$ inch thick.

Method of Manufacture.—The completed bridge is shown in Fig. 57. At CDE and FGH are L-shaped pieces of

Fig. 57.—Simple Wheatstone Bridge.

sheet copper, each provided with two binding screws. AB is a straight piece of copper with three binding screws. Between E and H is the boxwood scale, having a German-silver wire stretched along its upper surface and soldered to the copper at E and H. In making the bridge—(1.) The copper strips should be cut out. Fig. 58 shows their shapes and dimensions. They will require to be drilled with holes just large enough to receive the shank of a binding screw at the places marked with large circles; also with smaller holes at the places shown, in

order to receive the screws for fastening the coppers to the board. (2.) The boxwood scale must be screwed down in the position shown in Fig. 57, the heads of the screws being screwed below the surface of the scale. (3.) The binding screws must be soldered to the copper straps. This operation may be performed without a soldering iron, the strips being heated in a Bunsen's flame. Also at the shaded portion of the L-shaped pieces a square piece of copper must be soldered, in order that the level of the copper may be brought up to the level of the scale. (4.) Bore holes in the wood to admit the shanks of the binding screws, then fasten the copper strips down by means of the screws. (5.) One end of about 2 feet of the German-silver wire, which must not have an insulating covering,[1] is to be soldered to E, then, the other end being held by an assistant, the wire must be stretched straight along the scale and then soldered at H. The soldering should be performed by means of a small iron, and care should be taken that the wire is in metallic communication with the coppers at the ends of the scale.

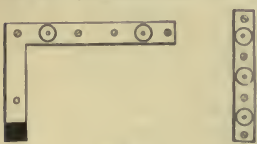


Fig. 58.

The Bridge Connections.—Fig. 59 shows the necessary arrangement for comparisons of resistance. Here the letters correspond with those of Fig. 56, which should be first drawn by the student, in order to help him in making his connections. It will be noticed that in the arrangement used in practice R and S are varied at will by sliding the battery terminal along the German-silver wire. To do this more conveniently the battery terminal is thrust through a cork at C, and the end of the wire is filed into a wedge-shaped form. A portion of the cork may be cut away if necessary, so that it may be held

[1] To remove silk from a wire without injury to the wire, it should be boiled in a strong solution of caustic soda.

against the edge of the base board as it is moved along. The galvanometer (G) should be provided with a simple shunt (Sh) for lessening its sensibility at will.

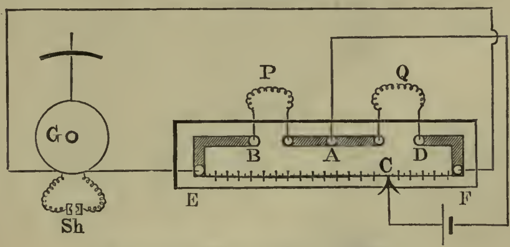


Fig. 59.—Scheme of Bridge Connections.

Operations with the Bridge.—(1.) Measure 8 feet (*i.e.* four times the length of the base board) of No. 36 silk-covered copper wire. Make it into a doubly wound spiral, and connect its bared ends across the gap P of the bridge. Make a second spiral in exactly the same manner, and connect it across the gap Q. Shunt the galvanometer, and touch the end F of the bridge with the free battery terminal, the galvanometer will be deflected, say, to the right; now touch the end E and the deflection should be to the left. This will show that the connections are correct, that the contacts at P and Q are good, and that neither of the spirals is broken. Find roughly the position at which the galvanometer shows no deflection, then remove the shunt and obtain the position of equilibrium more accurately. Call the position of equilibrium a, then, since the bridge may be considered to be divided into 1000 parts, we shall have

$$\frac{P}{Q}=\frac{a}{1000-a}.$$

Example.—The bridge reading is 499, hence

$$\frac{P}{Q}=\frac{499}{1000-499}=\frac{499}{501}=\frac{1}{1{\cdot}004}$$

or P is very nearly equal to Q. (2.) Make a third spiral of 8 feet of No. 36, place it in the same screw holes as the spiral at P, so that the two spirals are in *multiple arc*. Now compare the two resistances.

Example.—

$$a=331 \qquad 1000-a=669$$

$$\frac{P}{Q}=\frac{331}{669}=\frac{1}{2\cdot02}$$

or the wires in multiple arc have only half the resistance of the single wire. (3.) Place the two spirals at P in *series* by connecting their ends by small clamps, and again compare the resistances.

Example.—

$$a=668 \qquad 1000-a=332$$

$$\frac{P}{Q}=\frac{668}{332}=2\cdot01$$

showing that the effect of doubling the length of the wire is to double the resistance. (4.) Take 8 feet of No. 28 copper wire for Q and balance it against 8 feet of No. 36 for P. Find the average diameter of both wires by the micrometer-calliper or the microscope (the latter preferred).

Example.—

$$a=819 \qquad 1000-a=181$$

$$\frac{P}{Q}=\frac{819}{181}=4\cdot52$$

Diameters—No. 28 = ·015 inch ; No. 36 = ·007 inch.

$$\frac{\text{quare of diameter, No. 28}}{\text{Square of diameter, No. 36}}=\frac{\cdot000225}{\cdot000049}=4\cdot59,$$

from which we see that the resistance varies inversely as the square of the diameter.

Lesson XXII.—Manufacture of a One-Ohm Coil

45. *Apparatus.*—The same as before, with the addi-

tion of some silk-covered German-silver wire, No. 28 B. W. G.; materials for mounting the coil, and a standard ohm.

Method.—Cut off one mètre length of the wire and measure its resistance, then calculate what should be the length to give one ohm resistance. From the total length cut off a piece rather greater than the calculated length, and proceed to mount it with attached terminals for future use. The methods of mounting that might be adopted are very various. In Fig. 60 we have one of the

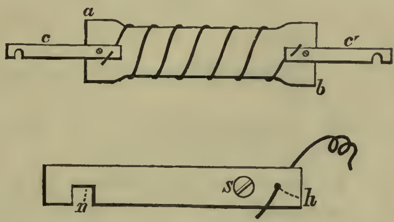


Fig. 60.—The One-Ohm Coil.

simplest of these. Here *ab* is a flat piece of wood, notched along its two edges. The wire is wound between the notches, and the ends are soldered to two copper strips *c* and *c*′. Each copper strip must have a small hole *h* through which the German-silver wire may be passed before it is ultimately soldered to the copper; also a hole for a screw *s* for securing the copper to the wood, and a notch *n* which fits the binding screw of the bridge. Ribbon or tape is wound outside the wire for protection, its ends being secured by small tacks. The wire having been mounted must have its resistance tested; if this should be found to be rather too great the wire should be unsoldered at one end, drawn through the small hole in the copper, and again soldered. This adjustment must be repeated if necessary. It is not, however, desirable to spend much time in making an exact

adjustment, but when this is sufficiently good the resistance should be measured as exactly as possible, and its value recorded on the wood.

Example.—One mètre of the wire gave against the standard ohm a balance on the bridge at 52, hence its resistance was $\frac{520}{1000-520} = 1{\cdot}083$ ohm. Hence the length of 1 ohm in millimètres will be $\frac{1000 \times 1}{1{\cdot}083} = 923$ millimètres. A piece 925 mm. was cut off and mounted, as described; its resistance was found rather too great. On reducing the length about 1·5 mm. it was found to be almost an ohm.

LESSON XXIIA.—Calibration of Galvanoscope.

45*a*. *Apparatus.*—That of Lessons XVIII., XIX., and XXI.; also a Bunsen's or a Grove's cell.

Method.—(1.) Charge the cell and compare its E. M. F. with that of a standard cell by Lesson XIX. (2.) Measure the resistance of the galvanoscope by Lesson XXI. (3.) Measure the internal resistance of the cell by Lesson XVIII. (4.) Connect the cell, box of coils, and galvanoscope in series, and take readings of the latter with different resistances in the circuit. (5.) Calculate the current in ampères producing the different deflections. Draw a table up for use with the instrument; also plot a curve showing the relation between the currents and deflections.

Example.—A vertical galvanoscope was calibrated.

E. M. F. of cell	= 1·87 volt.
Resistance of cell	= ·25 ohm, nearly.
Resistance of galvanoscope	= 9·75 ohms.

If R be the resistance from box of coils, then the current C in ampères will be $C = \frac{1{\cdot}87}{R + {\cdot}25 + 9{\cdot}75}$, from which was calculated the following table:—

R.	C.	Deflection.
0	·187	75
1	·170	73
3	·144	69
5	·125	65
10	·0935	58
15	·0748	51
20	·0623	45
30	·0467	37
50	·0311	25
100	·0170	13
200	·0089	7

The curve plotted from these numbers was regular.

CHAPTER IV.

THE MEASUREMENT OF RESISTANCE.

46. *Definitions.*—The problem of measuring resistance consists practically in ascertaining its value in terms of the ohm or other unit between any two points of a material at any stated temperature. If the material be of uniform section, such as a wire, its **Specific Resistance** or the resistance at 0° C. between the opposite faces of a cube whose side is of unit length may be readily found, for since the resistance R is directly proportional to the length l, and inversely proportional to the area of cross-section a, and otherwise depends only on a constant ρ, which expresses the specific resistance, we must have :—

$$\mathrm{R}=\rho\frac{l}{a}$$

or

$$\rho=\frac{a\mathrm{R}}{l} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

For example, if the resistance of 3 mètres of copper wire of No. 28 S. W. G. be ·435 ohm at 0° C., and if the cross-section be ·00111 sq. cm., we have in C. G. S. units (see Appendix),

$$\rho=\frac{\cdot 435\times 10^{9}\times \cdot 00111}{300}=1609{\cdot}5.$$

If the equation giving ρ were expressed in ohms instead of absolute units, the value would be $1609{\cdot}5\times 10^{-9}$, an extremely small value. It is customary to express it

instead in *microhms* or millionths of an ohm. The value in microhms will be

$$1609{\cdot}5 \times 10^{-9} \times 10^{6} = 1{\cdot}6095.$$

The specific resistance of a substance may be defined with reference to weight in the following terms: The specific resistance is that of a unit length of the substance formed into a wire of such a thickness that it is of unit weight. The value so obtained would differ from ρ. This method of definition has the advantage that the weighing of a wire is a more accurate and less troublesome process than the determination of its diameter, which in any case ought to be determined from the weight. The information it supplies is also useful in selecting a material whose resistance for unit length and unit weight shall be a minimum. Nevertheless in the following lessons the usual definition of specific resistance will be adopted.

Instead of expressing the resistance of thick wires in terms of ohms, Sir Wm. Thomson prefers to express the *conductivity* in terms of the **MHO** (ohm written backwards), which he defines as the unit of conductivity, being the reciprocal of the unit of resistance. Thus *a resistance* of n ohms would have a *conductivity* of $\frac{1}{n}$ mhos. For example, a mètre of No. 0, S. W. G. copper wire, with a resistance of ·000322 ohm, would have a conductivity of 3107 mhos. This device thus avoids the use of small numbers when low resistances are being measured.

Frequently a certain substance is selected as a standard one, and the resistances or conductivities of other materials are expressed in terms of this standard. The ratio is known as the **Relative Resistance**, or **Relative Conductivity**. The standard substance is usually silver or mercury in the state of purity, the latter being much preferable, since different specimens of pure silver may vary considerably.

The following table gives the specific and relative con-

ductivities of a number of commonly occurring metals and alloys :—

Table F.

Specific and Relative Conductivity of Metals and Alloys at 0° C.

	Resistance in Microhms between opposite faces of centimètre cube.	Resistance in Ohms of wire 1 mètre long and weighing 1 gramme.	Approximate Relative Conductivity, Mercury unity. Mean values.
Silver, annealed .	1·521	·1544	63
Copper ,, . .	1·616	·1440	59
Gold ,, . .	2·081	·4080	44
Aluminium ,, . .	2·945	·0757	31
Zinc, pressed . .	5·689	·4067	16
Platinum, annealed .	9·158	1·96	10
Iron ,, . .	9·825	·7654	9·2
Tin, pressed . .	13·36	·9738	8·2
Lead ,, . .	19·85	2·257	4·6
Mercury, liquid .	99·74	13·06	1·0
Platinum-silver, hard or annealed . .	24·66	2·959	3·7
German-silver, hard or annealed . .	21·17	1·850	4·2
Gold-silver, hard or annealed . .	10·99	1·668	8·6
Brass . . .	5·8	...	17·2

47. *Divisions of the Chapter.*—The work of this chapter will fall under the following heads :—

(1.) Description of the use of the Box of Coils as a Wheatstone Bridge.

(2.) Use of the Differential Galvanometer.

(3.) Use of Wheatstone Bridges with Slide Adjustment.

(4.) Determination of very low Resistances.

(5.) Determination of very high Resistances.

(6.) Resistance of Batteries and Electrolytes.

48. *Different kinds of Galvanometers.*—In each of these

sections a galvanometer, usually of high sensibility, will be required. Speaking generally, when the resistance to be measured is high, the galvanometer should be wound with a large number of turns of fine wire, and therefore must have a high resistance; and when the resistance to be measured is low, the galvanometer should have a low resistance, and consequently must be wound with only a limited number of turns of thick wire. According to the purpose desired galvanometers are classed as :—

(1.) High Resistance, or Long Coil Galvanometers.

(2.) Low Resistance, or Short Coil Galvanometers.

An instrument of either type may be wound *differentially*, that is to say, be provided with two coils of exactly similar construction, and placed symmetrically with reference to the suspended magnetic system, so that this system will be uninfluenced by equal currents flowing in opposite directions in the two coils.

By providing a galvanometer with many coils it may be employed either as a high or a low resistance galvanometer. Usually, however, instrument makers prefer to make the two types of instrument of somewhat different construction. The next lesson will describe a useful form of high resistance galvanometer adapted for the greater portion of the work of this chapter.

Part I.—Use of Box of Coils as a Wheatstone's Bridge.

Lesson XXIII.—The High Resistance Differential Astatic Galvanometer.

49. *Exercise.*—To adjust and test the galvanometer.

Apparatus.—The galvanometer. Fig. 61 shows a front view of the complete instrument, while Fig. 62 shows the back and side views of the instrument without its outer case.

(a) *The Base.*—A round ebonite base AB is provided with three levelling screws, a circular level *l*, and four binding screws (1, 2, 3, 4).

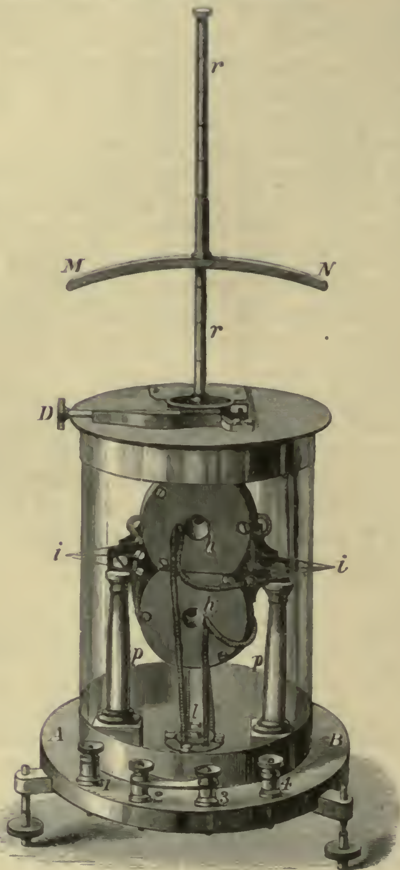


Fig. 61.—Reflecting Galvanometer—Front View.

(β) *The Coils and their Supports.*—From the base rise two brass columns *p*, *p*, supporting a brass plate *a*, *a* (Fig. 62). Against the back face and also against the front face of the plate are screwed four coils *c*, *c*, *c*, *c*. Three of these are of the same construction, but the upper back coil is provided with a subsidiary

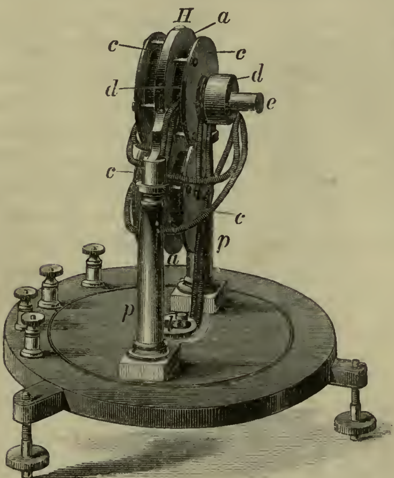


Fig. 62.—Reflecting Galvanometer—Side and Back Views.

coil *d*, within which slides a small adjusting coil *e*.[1] The coils are held in their places by brass plates screwed on to the brass framework. The wire is wound on suitable bobbins, so as to be heaped up towards the centre. This is done with the view of obtaining the maximum effect on the needle.[2]

[1] The purpose of *d* and *e* is to make the upper and lower coils have the same magnetic effect when the instrument is used differentially (see p. 144).

[2] See Jenkin's *Electricity and Magnetism*, 3d edition, p. 196.

Each coil has a central aperture, and the edges of the wound coils are protected with shellac.

(γ) *The Connections.*—One end of each of the coils is brought into connection with one of the binding screws on the base of the instrument by means of the spirally coiled wires seen in the figures. The other ends of the coils are put into connection with the screws at *i*, *i*, which are insulated from the main framework, and where also the two ends of the secondary coil and those of the small adjusting coil are brought. The connections are so made that when a battery is joined to the main terminals 1 and 4, 2 and 3 being connected together, the coils are in series, and all unite to deflect the needle of the galvanometer in the same direction. This is exhibited in the upper diagram of Fig. 63,

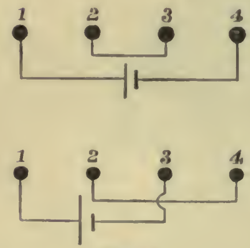


Fig. 63.
METHODS OF MAKING THE CONNECTION.

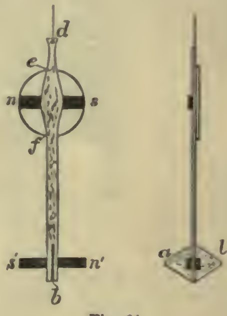


Fig. 64.
GALVANOMETER NEEDLE.

where, of the two galvanometer coils, one is supposed to extend between 1 and 2, and the other between 3 and 4. But when the battery is at 1 and 3, and 2 and 4 are in connection with each other, as shown in the lower diagram of the same figure, where we may suppose the same current to

go in a right-handed direction between 1 and 2, and in a left-handed direction between 3 and 4, the currents in the coils will oppose one another, and may be made to have no effect on the needle by help of the adjusting coil at the back. For the purpose of readily making these connections, two brass connecting pieces of suitable length are usually provided with the galvanometer.

(δ) *The Needle.*—Fig. 64 gives us the front and side views of the needle. An aluminium wire *db* is flattened at the top and pierced with a small hole for the admission of a silk fibre. It is also flattened at the part *ef*, where a light concave mirror is attached to it. Across the middle of the mirror there is fastened a magnet, *ns*. A second magnet *s′n′*, nearly identical in size, shape, and magnetic strength with the former, is placed at the lower end of the wire, with its poles in the opposite direction to those of *ns*. This is called an astatic system, since it possesses very little directive force as far as the earth's magnetism is concerned. It is usual to adjust the system so that when freely suspended the face of the mirror should look towards the west. At right angles to the lower needle is an aluminium vane *al*, the object of which is to help by its resistance to the air in bringing the suspended system more rapidly to rest. The arrangement now described is suspended by a short fibre, consisting of a single thread of cocoon silk, from the end of a brass pin, the milled head of which is seen at H (Fig. 62). The suspension is so adjusted that the mirror is seen through the centre hole *h* (Fig. 61) in the upper coil, while the aluminium damper and lower magnet swing within the opening *h* of the lower coil.

(ϵ) *The Case and Directing Magnet.*—The instrument is

covered with a cylindrical glass case, having a brass top. This supports a vertical brass rod *rr*, intended to hold the curved directing magnet MN. The latter is arranged so as to slide up and down the rod, and it may be rotated either by the hand directly or by the help of the slow motion screw D. It is convenient to have the brass rod *rr* graduated.

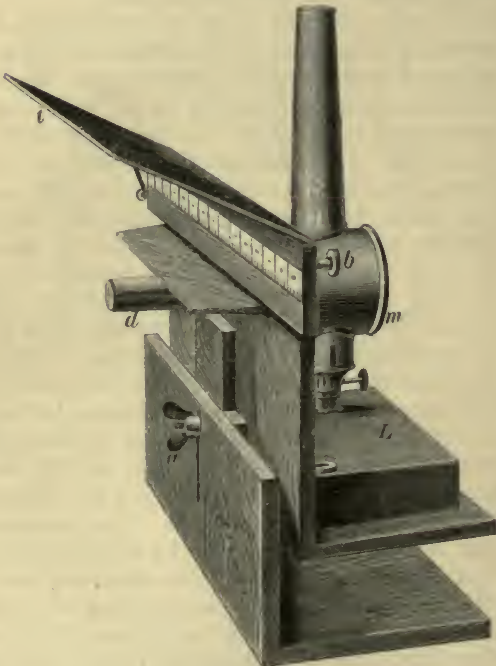


Fig. 65.—Galvanometer Lamp and Scale.

(ζ) *The Scale.*—Fig. 65 exhibits a convenient form of scale. It has the following adjustments: (1.) it

may be raised or lowered and clamped in the proper position by means of the screw seen at *a*; (2.) it may be moved horizontally through a range of about two inches by the screw *b*, working with a rack and pinion movement; (3.) the scale may be shadowed more or less by placing the hinged top *t* at any desired angle, at which it may be clamped by a screw. At *d* there is a hollow tube in which a second tube slides, having a convex lens of 5 inches focal length at one end, and a fine wire stretched across the tube near the other end. The intention of this is to bring the image of the fine wire to a focus in the middle of the illuminated circle reflected from the mirror upon the scale. The scale, which is usually of well varnished paper, glued to the wood, had better be graduated in half millimètres. Many scales, however, are graduated in fortieths of an inch. The zero of the scale is generally at the middle.

(η) *The Lamp.*—This is an ordinary paraffin lamp L provided with a copper chimney *m*, having a plain glass window in front and a concave mirror behind. It must be placed on the ledge behind the opening, with the edge of the flame towards the hole. Its height may be adjusted by sliding the wick-holder in or out of the body of the lamp. The exact placing of the lamp is a matter of much consequence if it be essential to obtain a well illuminated image of the wire. It may be desirable to add that when lighting the lamp, after raising the wick by the screw provided for the purpose, and after trimming and lighting the wick, the latter must be screwed down so as to be below the metal lips of the wick-holder. The chimney should then be placed on, and the light turned up until the flame is of the desired height. This will

ensure the proper burning of the vapour of the paraffin.

(In setting up the galvanometer it will be necessary to have a magnetic compass (a prismatic one being the best), a mètre scale, a T square, a piece of plain mirror, some microscope covering glasses, elastic bands, and chalk.)

Position of the Galvanometer and Scale.—The galvanometer must have a support unaffected by vibrations; the scale must likewise have a stable support, which need not, however, be of such a rigid character as that provided for the galvanometer.

The arrangement most frequently adopted is to place the galvanometer and scale on a slate slab 1 inch thick, and, say, 55 inches long by 26 inches wide, fixed to the wall by stone brackets. The height of the slab from the floor should be about 32 inches. The slab should be fixed with its length in the magnetic east and west. A better plan is to have separate slabs for the galvanometer and the scale. These should be fixed by stone brackets to the wall, with their centres 40 inches apart. They may be 14 inches wide, and should project from the wall to a distance of 27 inches. They should be so high that an ordinary table may be placed under them, leaving sufficient space for the reception of the working apparatus. Should it be undesirable to make use of the laboratory wall for supporting the slabs, brick or stone piers with suitable foundations may be built up directly from the earth, each pier being surmounted with a slab of slate or stone.

It will be necessary to partially darken the place selected for the galvanometer, either by shutters or curtains. All iron which has not a fixed position should be removed.

Enumeration of the Galvanometer and Scale Adjustments.—

The chief adjustments that require to be made are the following:—

(1.) The galvanometer must be levelled.

(2.) The needle must be made to swing freely.

(3.) The plane of the coils must be in the magnetic meridian.

(4.) The scale must be in the magnetic meridian.

(5.) The scale must be horizontal and one mètre from the mirror.

(6.) The zero point of the scale, the centre of the mirror, the flame of the paraffin lamp, and the wire of the galvanometer scale must be in the same vertical plane.

(7.) The reflection from the mirror of the wire of the scale must be focused on the scale clearly and distinctly, so that its position with regard to the small divisions of the scale may be easily read off.

(8.) The suspending thread of the needle must be free from torsion.

We shall describe these adjustments and others of minor importance in the order in which they should be performed.

(*a*) *Preliminary Adjustments.*—The galvanometer and scale having been placed in their respective positions, the former should be levelled, after which the case should be removed to some distance off, so that the directing magnet may not influence the needle. If now the openings of the coils be not protected by glass fronts, it will be necessary to cover them with circles of microscopic glass, which may be held in place by elastic bands. A small square piece of mirror glass should also be tightly fastened by an elastic band against the top of the front upper plate (that holds the front upper coil in place), in a symmetrical position with respect

to the hole in the coil. This mirror is intended to assist in placing the coils and the galvanometer mirror in the same plane, and we shall assume that the plane of this mirror is parallel to the plane of the coils, this assumption being justifiable if the workmanship of the galvanometer be good. Next, the needle must be made to swing freely. The head of the suspension pin should be raised *slowly and without a screw motion* until the centre of the galvanometer mirror is brought into the centre of the hole. The needle should now be free to move.

(β) *Adjustment to Meridian, etc.*—If the suspending thread were altogether free from torsion, the needle of the instrument might be used for the direct setting of the galvanometer coil in the magnetic meridian. But the thread being short it may have received sufficient torsion from an accidental twist to prevent the adjustment being accurately made by this means. We shall therefore perform the setting by the help of an independent magnet, such as an ordinary or, better still, a prismatic compass. By the help of the compass rule two chalk lines on the slab, one in the magnetic meridian (near the position which the galvanometer will occupy), and one at right angles to the meridian. Place a mètre scale against the plane of the coils, and then turn the instrument until the mètre scale is judged to be parallel to the chalk line drawn in the meridian. A second chalk line drawn in the meridian near the place of the scale will enable the latter to be set correctly also in the meridian, and at a distance of one mètre from the galvanometer mirror.

(γ) *Final Adjustments.*—The lamp having been lighted, the scale should be raised or lowered until the reflection from the galvanometer mirror falls so as to be

neither too high nor too low. By moving the lamp horizontally a little, and raising or lowering the wick-holder, the position will be found in which the image is brightest.

Two reflections should now be seen on the scale, one due to the square mirror fastened on the plate, and the other due to the galvanometer mirror. The former should appear, provided the instrument has been properly made, as a square patch of light over the zero point of the scale; while the latter, if the suspending fibre possesses torsion, may be some distance away. In this case the head of the suspending pin should be turned until the reflected spot of light from the galvanometer mirror falls so as to coincide with the reflection from the fixed mirror.

The scale should now be screwed or clamped down, and the galvanometer should have its position fixed by Sir William Thomson's method of the hole, the slot, and the plane as follows:—Mark the position of the levelling screws on the slab, then remove the galvanometer. Let us suppose that *a*, *b*, *c*, Fig. 66, are the positions occupied by the points of the levelling screws. At *a* make a conical hole about $\frac{1}{8}$ inch deep, and just sufficiently wide to admit the end of the screw, and then at *b* make a V-shaped groove of the same depth as the hole at *a*. Next replace the galvanometer, whose three levelling screws will now rest, one on a plane, a second in a hole, and the third in a slot. The galvanometer will therefore have a defined position without the possibility of strain being caused by expansion or contraction of its base.

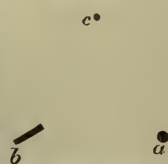


Fig. 66.—The Hole, Slot, and Plane.

The galvanometer should now be levelled and finally adjusted, and then the case of the instrument replaced. The ebonite base should be dusted, in order that the insulation may be as perfect as possible.

TESTING THE INSTRUMENT.

The Sensibility.—This will depend upon the position of the directing magnet. To clearly understand the precise influence of this magnet let us suppose that the vertical plane passing through the magnetic meridian is represented by the plane of this page, the north being to the *left hand* of the reader. In the nearly astatic combination (Fig. 67 I.), the upper magnet being generally made somewhat stronger than the lower, will place its north-seeking pole towards the magnetic north. The magnetic field produced by the earth may be looked upon as caused by a very large magnet, whose S. pole lies to the left, while its N. pole lies to the right of the page. Let us denote by H the strength of this field which tends to keep the nearly astatic combination in the meridian.

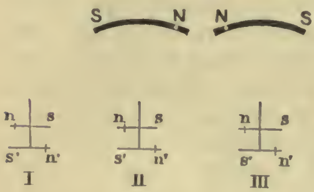


Fig. 67.

Now, when the directing magnet is placed over the needle, with its poles lying in the same direction as those of the imaginary magnet which produces the earth's magnetism (Fig. 67 II.), it will follow that the field in which the needles swing will be strengthened, and may be expressed by $H + M$, where M is the strength of the field due to the directing magnet above. Also, in this case, the nearer the directing magnet is to the needles the greater will be the force required to produce a given deflection, or, in other words, the less will be the sensibility of the galvanometer.

Should, however, the directing magnet have its poles placed in the reverse position (Fig. 67 III.), the strength of the field will be $H - M$. In this case the approach of the magnet will first of all diminish the strength of the residual field, and thus increase the sensibility of the instrument. If a position is reached, as may be the case when the directing magnet is of sufficient strength, where $H - M = 0$, then the system will be under the control of no directing force, and will be unstable; while, if the directing needle is pushed still nearer, the position of the astatic system will be reversed, and the needles will swing round through 180°.

This reasoning has been tested experimentally. The directing magnet was first placed at the top of the standard supporting it, with its S. pole to the magnetic north. The time of vibration of the needle was then taken by means of a stop-watch. The magnet was then reversed and the time again determined. This proceeding was repeated several times as the directing magnet was gradually brought nearer the needle. The following table gives the results of the experiment :—

Position of Directing Magnet.	Time of Vibration. S. Pole to North.	N. Pole to North.
Top . .	2·9 seconds.	3·3 seconds.
Position 1 .	2·7 ,,	3·5 ,,
,, 2 .	2·5 ,,	4·4 ,,
,, 3 .	2·0 ,,	Point of instability.
,, 4 .	1·6 ,,	Could not be brought to zero.
Bottom . .	1·1 ,,	,, ,, ,,

Here, when the S. pole of the directing magnet is to the north, the strength of the field is seen to increase as the magnet is lowered, the vibrations becoming more rapid. But when the magnet is reversed the opposite is the case, the strength of the field gradually diminishing until a zero, or point of instability, is reached, after which it no doubt increases in an opposite direction; but this,

of course, cannot be tested, as the needle cannot then be brought to zero.

The most sensitive position lies evidently between the positions 2 and 3. The positions 2·5 and 2·75 were found to be stable positions, but it was noticed that the resting point frequently altered. This may have been caused by the directing power being insufficient to control variations in the torsion of the fibre, or it may have been due to solar diurnal changes in the magnetism of the earth, to which such an arrangement would be peculiarly sensitive. It was therefore decided that position 2 should be regarded as the most sensitive working position.

Experiments showing the Sensitiveness of the Galvanometer.—(1.) Touch the brass terminals, one with the finger and the other with the thumb, a deflection will be produced which will be increased when the fingers are moistened. The current is due to the slight variation in the nature of the brass terminals and in the moisture of the fingers.

(2.) Obtain two copper wires of the same kind, connect them with the galvanometer, and place their free ends in distilled water. A deflection will be produced. Take one of the wires out of the water, touch it with the finger, and then replace the wire in the water. A far greater deflection will now be obtained, which may, however, be in the opposite direction to the previous deflection.

(3.) Connect one terminal with the gas-pipe, now touch the other terminal, when a deflection will be produced, due to the battery formed by the observer, the wooden floor on which he stands, and the brass terminal. Here the gas-pipe acts probably as a conducting link between the floor and the other terminal.

(4.) Leaving the one terminal connected with the gas-pipe, give a small spark to the other terminal

by means of an electrophorus. At the instant when the spark is given a deflection will be observed. A greater effect may be produced by disconnecting the wire leading from one of the terminals to the gas-pipe, thus leaving both terminals insulated, and then giving a charge of several sparks to one of the terminals. If we now touch the galvanometer with the finger a deflection, due to the sudden discharge, will be noticed.

(5.) Observe the influence on the galvanometer needle of a wire conveying a current, of a small magnet, of the observer's watch, of the buttons on his coat, etc. etc.

50. *Theory and Use of Shunts.*—To reduce at will the sensibility of the galvanometer, shunts, the use of which will be already familiar to the student, are employed. Figs. 68 and 69 show two such arrangements in frequent use, the

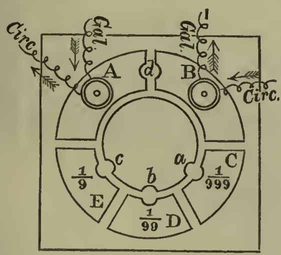


Fig. 68.
Plan of Circular Shunt Box.

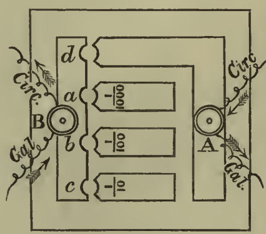


Fig. 69.
Plan of Square Shunt Box.

corresponding parts in each being similarly lettered. Fig. 70 exhibits the general plan of the shunt connections. When a plug is inserted at *d* the galvanometer is short-circuited through the thick metal portions between A and B; but when the plug is removed from *d* and inserted at *a*, *b*, or *c*, the galvanometer is shunted through one or other

of the resistance coils of the shunt. The resistance that a shunt must have in order to diminish the current in any ratio may be readily ascertained. Let G be the resistance of the galvanometer and S the resistance of any shunt, while C denotes the main current which we wish to shunt. This current will divide between the galvanometer and shunt in the inverse ratio of the resistances in each;

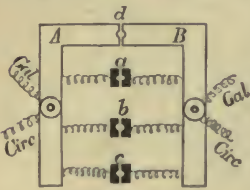


Fig. 70.—SCHEME OF SHUNTS.

that is to say, in the ratio of S to G, and hence the current C_1 going through the galvanometer will be

$$C_1 = \frac{S}{G+S} C \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Suppose now that we wish to allow only the $\frac{1}{n}$th part of the current to go through the galvanometer, or, in other words, let $C_1 = \frac{C}{n}$. In this case we shall have from (1)

$$\frac{C}{n} = \frac{S}{G+S} C, \qquad \text{or } S = \frac{G}{n-1}. \quad . \quad . \quad (2)$$

from which we find as follows for the values of n in common use :—

If $n=10$ $\quad S = \frac{1}{9}$ of G.
$n=100$ $\quad S = \frac{1}{99}$ of G.
$n=1000$ $\quad S = \frac{1}{999}$ of G.

The positions a, b, and c are marked either with the numbers $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$, implying the fractions of the whole current which they pass through the galvanometer, or with the numbers $\frac{1}{9}$, $\frac{1}{99}$, $\frac{1}{999}$, implying the ratio between their resistances and that of the galvanometer.

When the shunts are made of the same metals as the galvanometer coils, their shunting power will not be altered with change of temperature. Since, however, the galvanometer coils are usually made of copper and the shunts of

German silver, a correction will be necessary in calculations involving the resistance of the shunts.

The student must not fail to notice that the effect of shunting the galvanometer is to diminish the total resistance of the circuit, and hence to increase the main current, so that if the main current is desired to be constant, additional resistance must be introduced A set of resistances called **Compensating Resistances** are accordingly sometimes used for this purpose.

Lesson XXIV.—The Box of Coils used as a Bridge.

51. *Exercise.*—To learn the use of a box of coils for measuring resistance by Wheatstone's method.

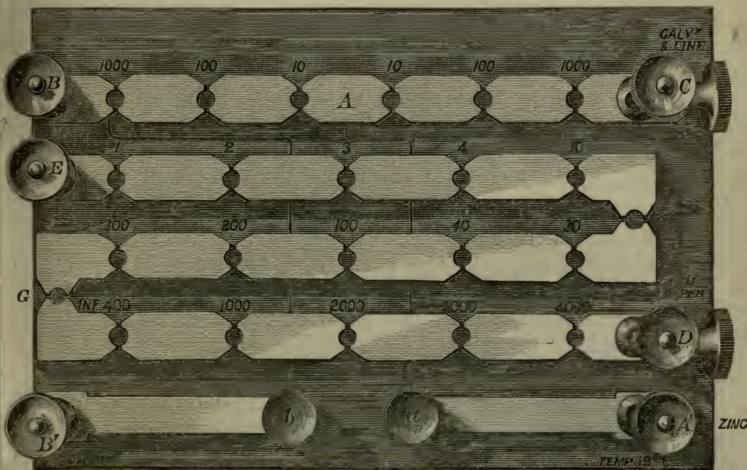


Fig. 71.—The Post Office Resistance Box.

Apparatus.—(1.) *The Box of Coils.*—One of the best-known arrangements is the *Post Office Resistance Box.* A plan

of this box will be seen in Fig. 71, in which AC and AB are the proportional arms, and EFGD the rheostat arm. The bridge will best be understood by comparing it with the typical diagram (Fig. 72), in which the parts are lettered in the same way as in the figure of the box. At A (Fig. 71) no binding screw is provided, but a wire passes under the ebonite top of the box to a stud at *a*, so that on pressing the key *a*A′ the binding screw at A′ is in connection with A. In like manner the terminal at B′ may be put in contact with the point B by pressing the key B′*b*. The rheostat arm is connected with the proportional arms by a brass connecting piece (not shown), which ought to be strongly clamped by the binding screws at B and E. At C and D are double binding screws—one for the wire of the unknown resistance, or line wire, besides which there will be the galvanometer wire at C and the battery wire at D. The order in which the various resistances occur will be seen in the diagram. At the place marked INF is the plug called the "*infinity plug*." Should this plug be removed the connection between the parts of the rheostat arm on either side of it will be completely broken.

Fig. 72.

(2.) *The Leclanché Battery.*—This form of battery is chosen for measurements of resistance, since it deteriorates but little on standing, so that it is always ready for use. On the other hand it runs down very rapidly when short-circuited. But when the circuit resistance is small, as is the case when our object is rather to find the direction of deflection than to measure its amount, the current is only required for a few seconds at a time, and in this case any variation

in the strength of the current is of little consequence. Again, when the current is required for a longer period, as it is when accurate determinations are being made, the resistance in the circuit will necessarily be so high that the constancy of the battery will be unaffected, inasmuch as it is doing little work. It is convenient to have four cells of this battery fitted with a switch, so that 1, 2, 3, or 4 cells may be thrown into circuit as required (see Appendix).

(3.) *The Connecting Wires.*—These should be of gutta-percha covered copper wire. The wires leading to the galvanometer and battery may be No. 20 B. W. G. Those leading to the unknown resistance should, however, be thicker, and be provided with copper strips soldered at the ends, as we have shown in Fig. 23. The use of these strips ensures a greater surface of contact.

Method of making the Connections.—In Fig. 73 we have a plan of the connections where G is the galvanometer, S the shunt, X the unknown resistance, L the Leclanché battery, and CBB′A′ the Post Office bridge. The wires going to the same parts should be brought together as much as possible. When *a* and *b* are pressed down, the contacts indicated in the figure are made. The galvanometer should be at least a mètre away from the measuring apparatus, and if the resistance to be measured consists of many turns of wire, it is necessary that it should be so far distant that it cannot directly affect the galvanometer.

Method of Measuring Resistances—The resistance of the connecting wires that go to the unknown resistance should first be measured. (We shall suppose that these wires are each about two yards long: for the purpose of distinction they will be spoken of as the *resistance connectors.*) In order to do this, take the wires out of the binding screws at *m* and *n*, and clamp their extremities together.

Place the $\frac{1}{999}$ shunt in the galvanometer, and put on one cell of the battery. Make P = 10, Q = 10, and R = 0, that is to say, keep all the plugs of the rheostat arm in. Press the battery key *first*, in order that the momentary current due to self-induction may have ceased before bringing the galvanometer into circuit. Then, whilst it is down, press the galvanometer key for a few seconds.

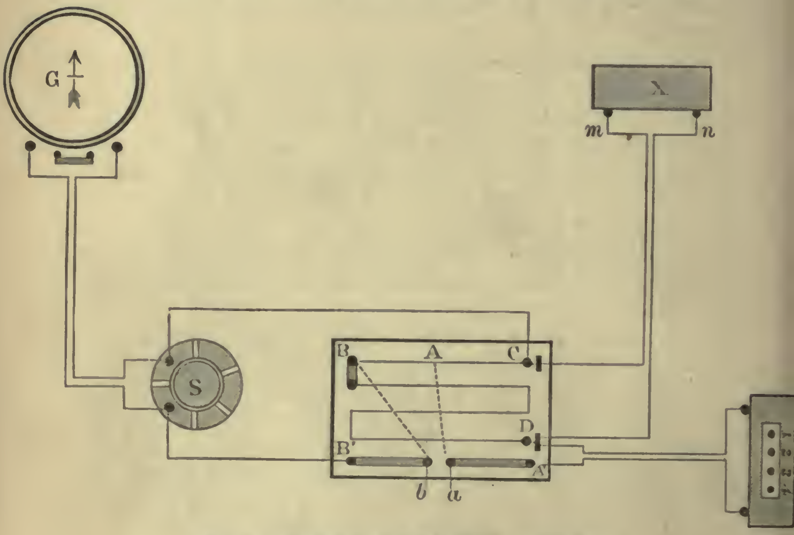


Fig. 73.—CONNECTIONS FOR MEASURING RESISTANCES.

The galvanometer will now be deflected, say to the *right*. If no deflection is obtained there must be a faulty connection at some place. In this case examine the battery and galvanometer connections, and especially ascertain whether any of the leading wires are broken. With gutta-percha covered wires such an accident may easily be overlooked, for the wire frequently becomes broken while the covering remains complete.

Next, P and Q remaining the same as before, make R = ∞. On pressing the keys momentarily as before, the deflection should now be to the *left*, *i.e.* in the opposite direction, for the leading wires should have a resistance between that of R = 0 and R = ∞. If the deflections are not in opposite directions the connections are probably wrong, and should be examined. Various resistances must now be tried, until a balance is obtained. The order of procedure will best be seen by studying the following table, which gives the result of an actual measurement. The student is advised to arrange his results in this form until he is quite familiar with the use of the bridge.

No. of Cells.	Shunt.	P.	Q.	R.	Value of X which would balance = $\left(\frac{QR}{P}\right)$	Deflection.
1	$\frac{1}{999}$	10	10	1	1	To right.
1	$\frac{1}{999}$	100	10	1	·1	To right.
1	$\frac{1}{99}$	1000	10	1	·01	To left.
4	No shunt	1000	10	6	·06	To right.
4	No shunt	1000	10	5	·05	To left.

From this we see that the resistance of the leading wires is between ·05 and ·06 ohm. To obtain this resistance more accurately the extent of the deflections must be noted in the two last cases, and the true value of X found by *interpolation*, as shown below:—

Value of $\frac{QR}{P}$.	Deflection.
·06 ohm	36 divisions to right.
·05 ,, ...	37 ,, left.

Hence ·01 causes a difference of seventy-three divisions, and hence the value of $\frac{QR}{P}$, which would correspond to no

deflection of the galvanometer, will be $\frac{QR}{P} = X = \cdot05 + \frac{\cdot01 \times 37}{73} = \cdot05507$ ohms approximately.

The above result will require a further correction, due to the fact that the rheostat arm, when all the plugs are in, has a certain finite though small resistance. To obtain its value, substitute for the resistance connectors a piece of copper wire, No. 18 B. W. G., say about a foot long; likewise clean all the plugs of the rheostat arm and press them well into their places. Make $P = Q = 10$. On pressing the keys a deflection will be obtained, but by shortening or lengthening the wire it will be possible to obtain a length of wire which will just balance the rheostat arm. Measure the length of the wire, obtain its diameter, and calculate its resistance by the aid of tables. It was found, for example, by this means that the resistance of this balancing wire was ·008 ohm. Hence the corrected resistance of the leading wires is more nearly $\cdot05507 + \cdot00008 = \cdot05515$ ohm. (More accurate methods of finding the resistance of the plugged rheostat arm will be found in Part IV.)

The resistance of the leading wires being known, the measurement of the resistance of several coils should be proceeded with, as exhibited in the following examples:—

Examples.—I. Galvanometer coil of copper—

$P=100$, $Q=10$, $R=9896$, $\frac{QR}{P}=989\cdot6$. No deflection.

,, ,, ,, 9897, ,, 989·7. Slight deflection.

$X=989\cdot6-\cdot055=989\cdot545$ ohms. Temp. 15° C.

II. Galvanometer coil of copper—

$P=1000$, $Q=10$, $R=1020$, $\frac{QR}{P}=10\cdot20$. Deflection of -4.

,, ,, 1019, 10·19. ,, $+21$.

$$X=10\cdot19+\frac{\cdot01\times21}{25}-\cdot055=10\cdot143.$$

In these examples the resistance of the rheostat arm is neglected. Temperature 15° C.

52. *Correction for Temperature.*—According to the experiments of Dr. Matthiessen, the resistance of metals increases with the temperature, according to the empirical formula :—

$$R_t = R_o\{1 + \alpha t + \beta t^2\}$$

where R_t is the resistance at $t°$ C., R_o that at 0° C., and α, β are numerical constants. The values of α and β for a few useful metals are exhibited in the following table :—

TABLE G.

TEMPERATURE COEFFICIENTS FOR RESISTANCE.

Metal.	Temperature Coefficients.	
	α.	β.
Most pure metals .	·003824	+ ·00000126
Mercury . . .	·0007485	− ·000000398
German silver .	·0004433	+ ·000000152
Platinum silver .	·00031	...

In the previous experiments the resistances in the box were made of platinum silver, and the coils had been adjusted to the standard at 19° C. Since our measurements were made at 15° C., the actual resistance used to balance X would be less than that marked on the box, for the temperature at the time of measurement was 4° less than the standard temperature. Referring to the formula used in the Wheatstone's bridge, namely,

$$X = \frac{Q}{P}R,$$

we see at once that, since Q and P are of the same metal, any rise in temperature will not affect the ratio of Q to P. The resistance R is therefore the only one requiring correction. For our purpose it will be sufficient to suppose that an alteration of 4°, when the resistance R is at 0°, would cause exactly the same proportionate change as when R is at 19°; hence our resistance becomes

$$R_{15} = R_{19}\{1 - (4 \times ·00031)\}.$$

Applying this formula to the preceding examples, we find

$$\text{I.}\quad X_{15} = 989{\cdot}545(1 - 4 \times {\cdot}00031) = 988{\cdot}318,$$
$$\text{II.}\quad X_{15} = 10{\cdot}143(1 - 4 \times {\cdot}00031) = 10{\cdot}130.$$

53. *Resistance of the working Galvanometer.*—When a second galvanometer is available, the resistance of the working galvanometer may be obtained by means of Wheatstone's bridge in the ordinary manner, the working galvanometer being the unknown resistance. The necessity of an additional galvanometer may, however, be avoided by a method due to Sir W. Thomson, which will be now described.[1]

LESSON XXV.—Measurement of Galvanometer Resistance (Thomson's Method).

54. *Apparatus.*—Wheatstone's bridge, etc.; galvanometer, whose resistance is to be measured; a Daniell's cell of high resistance; resistance boxes for regulating the battery.

Theory of the Method.—Let the student draw the theoretical diagram (Fig. 72), and consider what, when $\frac{P}{Q} = \frac{R}{X}$, will be the effect of joining through a galvanometer the points B and C in the two branches of the current. There will evidently be no effect upon the galvanometer, since B and C are at the same potential, and this is the ordinary way of using Wheatstone's bridge.

Suppose now that instead of the galvanometer being placed between B and C it is placed between C and D, as shown in Fig. 74, and that a key is placed between B and C. What will happen in this case if $\frac{P}{Q} = \frac{R}{X}$? It is clear

[1] See a note by Sir W. Thomson added to Mr. Mance's paper on a method of measuring the resistance of a conductor or battery, *Pro. Roy. Soc.*, vol. xix. (1871), p. 247.

that here we shall have a current passing through the galvanometer, but since, when connection is made by the spring key between B and C, no current will pass between these points, it is manifest that no diversion of current is produced, and hence that the value of the current passing through the galvanometer will not be altered by making such connection. Hence no alteration will take place in the current of the galvanometer in making this connection, provided that P : Q :: R : X.

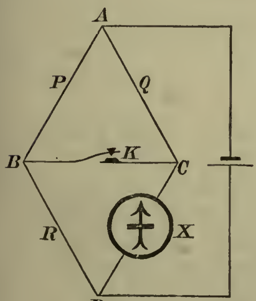


Fig. 74.

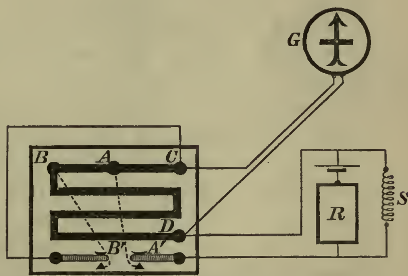


Fig. 75.
MEASUREMENT OF GALVANOMETER RESISTANCE.

Practice of the Method.—We shall suppose that the resistance of the galvanometer is approximately known to begin with.[1] The requisite connections with the Post Office bridge are seen in Fig. 75. In this figure the keys are at B′ and A′. Here S is a shunt placed across the battery circuit (in which a resistance R is included to keep the battery steady), this being necessary, since otherwise the current will be too great to make the test practicable. In order still to reduce the current as much as possible it is

[1] The resistance of a galvanometer is usually supplied by the maker of the instrument. If this is not the case the resistance may be approximately obtained by measuring the resistance of one of the shunts supplied with the instrument, or by the method of Lesson XVIII.

desirable to make the resistance in the arms of the bridge very high.

Let us suppose that P = 1000, Q = 1000, R = 4000. Press the battery key, when a deflection of the galvanometer will be observed, probably so great as not to be readable. Now place the directing magnet low, with its S. pole to the north, so as to strengthen the earth's magnetic field, and then turn the magnet until the spot of light is brought to the centre of the scale. Next press the key B′. If a deflection results, it will be necessary to alter the rheostat arm of the bridge, and to repeat the operation of adjusting the magnet and pressing B′ until the effect of opening or closing the circuit CBB′ has no influence on the current in the galvanometer.

Example.—We measured the resistance of the $\frac{1}{9}$ shunt, which was about 617 ohms; hence the resistance of the galvanometer should be $617 \times 9 = 5553$ ohms. Using the above method with $P = Q = 1110$, we found galvanometer resistance = 5550, but it was not possible to tell within 5 ohms. This was owing to the necessity of bringing the directing magnet near the needle to balance the current in the galvanometer, which reduced the sensibility of the instrument. The galvanometer under measurement being a differential one, it was decided to measure one of the coils at a time, and send a weak current from an accessory battery through the other in such a direction as to balance the current in the coil under measurement. This method, by rendering it unnecessary to lower the magnet, gave much better results, since the adjustment could be made within 1 ohm. Each coil was found to have a resistance of 2775 ohms.

PART II.—USE OF A DIFFERENTIAL GALVANOMETER.

55. The differential method of measuring resistances has, in common with the bridge method, the merit of being

independent of the constancy of the working battery, and also of being a zero method, but owing to the additional skill and labour requisite for the construction of a differential instrument, the former method (at one time used extensively for measurements connected with telegraphy) is now almost entirely superseded by the bridge method. The differential method should nevertheless be known to the student, for not only is it of service in physical investigations, but it is useful in checking results obtained by using the bridge.

Lesson XXVI.—The Differential Method.

56. *Exercise.*—To compare the higher resistances in two boxes of coils.

Apparatus.—A differential galvanometer (see Lesson XXIII.), a battery and connecting wires.

Testing and adjusting the Galvanometer.—A differential

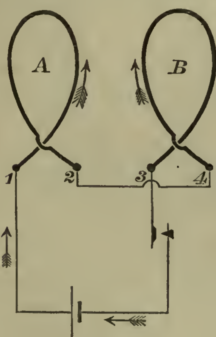


Fig. 76.

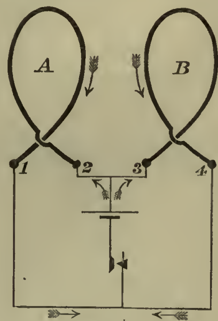


Fig. 77.

galvanometer, in order to be a perfect instrument, should satisfy two conditions. (1.) Equal and opposite currents in the two coils should be without influence on the needle. (2.) The resistance of the two coils should be equal.

Fig. 76 exhibits the connections necessary to test the first of these two conditions. Here the two coils A and B have sent through them *the same current*, but in opposite directions. If a deflection be produced, the small adjusting coil mentioned on p. 119 should be pushed in or out until no deflection results. In order to test the second condition, the connections are altered to those of Fig. 77, which shows the battery current divided between the two coils, and this division will be in equal proportion to each, provided that the resistances of the coils be equal. Should

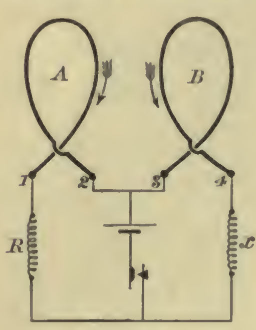


Fig. 78.

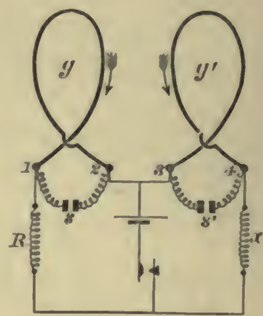


Fig. 79.

this not be the case it will be necessary to add an external resistance to one of the two coils until no deflection is produced.

Direct Comparison of Resistance.—Let the connections remain as in Fig. 77, but place the resistance coils R and x, which are to be compared together, as shown in Fig. 78. Make x a high resistance, and adjust R until no deflection is obtained, then $R = x$, provided that the galvanometer is rightly adjusted. We may make the test quite independent of any such assumption, if we substitute for x a third box of coils, out of which a resistance R_1 is taken to balance R, for then must $x = R_1$. This is the best

way of using the galvanometer. The student will see that this is analogous to Borda's method of double weighing.

Comparison by means of Shunts.—The range of utility of the differential galvanometer may be considerably extended by the use of shunts. Fig. 79 shows the connections with the shunts. Let g and g' be the resistances of the galvanometer coils, s and s' that of the corresponding shunts. Let us denote by B the current in the main branch in which the battery is placed; also let C denote the current that flows through the coil g, and C_1 the current that flows through the coil g'. Now, in conformity with the law of the division of currents,

$$C = B\,\frac{\text{Equivalent resistance of circuit } 34x3}{\text{Total resistance of } 34x3 \text{ and } 21R2} \times \frac{1}{\text{shunting power of } s}$$

$$= B\,\frac{x + \dfrac{s'g'}{s'+g'}}{x + R + \dfrac{sg}{s+g} + \dfrac{s'g'}{s'+g'}} \times \frac{s}{s+g} \quad . \quad . \quad . \quad (1)$$

In like manner the current C_1 through the coil g' is as follows—

$$C_1 = B\,\frac{R + \dfrac{sg}{s+g}}{x + R + \dfrac{sg}{s+g} + \dfrac{s'g'}{s'+g'}} \times \frac{s'}{s'+g'} \quad . \quad . \quad . \quad (2)$$

If the galvanometer is properly adjusted, $g = g'$, and by altering s, s', and R until there is no deflection, we get $C = C_1$, and

$$x = \frac{1 + \dfrac{g}{s}}{1 + \dfrac{g}{s'}}\,R.$$

To make use of this formula with convenience, two sets of shunts must be arranged, one to shunt each galvanometer coil, of $\frac{1}{9}$ and $\frac{1}{99}$ of the resistance of the coil.

Remembering that we also have the option of using no shunt at all, or, in other words, of using a shunt of in-

finite resistance, the above formula will give a range of resistance measurement from ·01 ohm to 1,000,000 if we are provided with a box of coils ranging from 1 to 10,000 ohms, for if $s=\frac{g}{99}$ and $s'=\infty$, then, by the above formula,

$$x=100\text{R},$$

and if

$$\text{R}=10{,}000$$
$$x=1{,}000{,}000.$$

Again, if

$$s'=\frac{g}{99} \text{ and } s=\infty$$
$$x=\cdot01\text{R},$$

and if

$$\text{R}=1$$
$$x=\cdot01.$$

We are thus provided with a method of comparing resistances which has a range equal to that of the Post Office box of coils.

PART III.—USE OF THE SLIDE BRIDGE.

57. When the resistance to be measured does not amount to more than 2 or 3 ohms, or when we wish to compare as accurately as possible a resistance which is approximately an ohm with a standard unit, the slide bridge should be employed. The student having been already made acquainted (Lesson XXI.) with the general principles involved in the slide bridge, will now be prepared for making measurements in an accurate manner.

58. *Low Resistance Galvanometer.*—It will be necessary in this section to make use of some pattern of low resistance reflecting galvanometer with a low resistance coil of from ·25 to 3 ohms. Fig. 80 shows an instrument of the tripod pattern. It has a single coil placed within a brass box having a glass front, within which is suspended the mirror and needle seen through the central opening *m*. A lower needle, with an aluminium damper, and having reversed polarity, swings below the coil in the position *d*.

On the top of the instrument is a brass cap supporting the directing magnet, the latter being turned by the tangent screw *b*. By liberating the screws *c* and *c′* the cap with

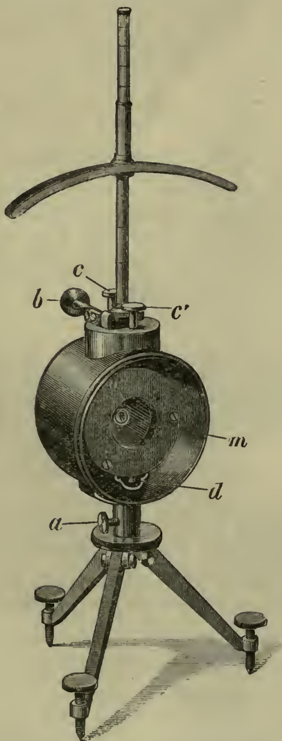


Fig. 80.—THE TRIPOD GALVANOMETER.

the directing magnet may be removed, when the head of the pin that supports the needle will be exposed. The body of the instrument may be turned and clamped in any position by the screw *a*.

The adjustments are very similar to those of the high resistance galvanometer.

LESSON XXVII.—Use of the Slide Mètre Bridge—Method I.

59. *Exercise.*—To compare the resistances of two coils.

Apparatus.—A slide mètre bridge, Fig. 81. Here are seen thick bars of copper, of which the bars MB, KL, EF, GH, and CN are fixed permanently to the base-board *a*, *b*, *c*, *d*, while the alternate bars BK, LE, FG, and HC are movable at will, being simply clamped by binding screws at B, K, L, E, F, G, H, and C.

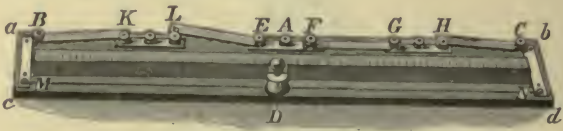


Fig. 81.—SLIDE MÈTRE BRIDGE (Elliott's Pattern).

Between the ends M and N a thick German silver wire is soldered, so that the distance between the portions of the ends of the wire just clear from solder is equal as nearly as possible to one mètre. A millimètre scale is fixed along the middle of the base-board mounted on the top of a block of wood, which raises the scale some 45 mm. above the bridge wire. The slider D may be placed at any position along the scale, and this position read off to $\frac{1}{10}$ of a millimètre by observing the index line drawn on the upper plate of the slider. When the knob of the slider is pressed, metallic contact is made between the slider and the bridge wire by means of a small notched plate of platinum, which is pushed from underneath the slider into contact with the wire. On releasing the

knob a spring within the slider ensures the breaking of the contact.

Other Apparatus.—Two Leclanché cells, a spring key, a low-resistance galvanometer.

Theory of the Method.—The chief error in the use of the bridge, as we have used it in Lesson XXI., lies in assuming that the resistance of the thick metallic end portions of the bridge and the solderings there may be neglected. The middle portions need not be taken into account. But however well the bridge may be constructed, these resistances cannot be got rid of; we must therefore arrange our method of observation so that they may either be entirely eliminated or, if retained, allowed for. The first method which we shall give is not a perfect one, but it is sufficiently good for many measurements, and does not assume a previous knowledge of the constants of the bridge. Let the copper rods BK and HC be removed from the two end spaces, and the resistances P and Q inserted in their place (Fig. 82). Suppose that P and

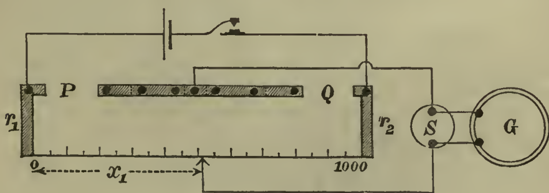


Fig. 82.—CONNECTIONS FOR METHOD I.

Q are balanced against each other in the ordinary way, the bridge reading being x_1. Now it may not happen that the index line on the slide is exactly over the part of the platinum plate that makes contact with the bridge wire. Let λ be this error, so that the true bridge reading is $x_1 + \lambda$. Now let r_1 be the resistance of the thick end of the bridge at which P is placed, and r_2 that of the

end where Q is placed, all in terms of a division of the bridge wire. Then

$$\frac{P}{Q}=\frac{r_1+x_1+\lambda}{r_2+1000-(x_1+\lambda)} \quad . \quad . \quad . \quad . \quad (1)$$

If the position of P and Q be reversed, a new relation between these resistances will be obtained, as under—

$$\frac{P}{Q}=\frac{r_2+1000-(x_2+\lambda)}{r_1+x_2+\lambda} \quad . \quad . \quad . \quad . \quad (2)$$

where x_2 is the new bridge reading. Either of these expressions would give $\frac{P}{Q}$ if r_1, r_2 and λ were known.

By adding the numerators together and the denominators together we obtain

$$\frac{P}{Q}=\frac{1000+r_1+r_2+(x_1-x_2)}{1000+r_1+r_2-(x_1-x_2)} \quad . \quad . \quad . \quad (3)$$

An expression which is free from λ, and which contains both in the numerator and denominator, the (not large) quantities r_1 and r_2 added to a large number.

In the absence of definite information regarding r_1 and r_2, we may safely disregard them in expression (3), substituting for it the following formula, employed by Siemens:[1]—

$$\frac{P}{Q}=\frac{1000+(x_1-x_2}{1000-(x_1-x_2)} \quad . \quad . \quad . \quad . \quad (4)$$

It will be found from (3), by assuming values for r_1, r_2 and x_1, x_2, that the greater x_1-x_2 is the greater will be the error in using the formula, so that the ratio of P to Q should not be great (see Appendix A.)

Practice of the Method.—Open the gaps at the ends of the bridge, and at P and Q insert the resistances, which we shall suppose to be provided with copper straps by which they may be clamped to the binding screws, the copper straps and the binding screws having been pre-

[1] See Sabine, *The Electric Telegraph*, p. 312.

viously cleaned with a file and emery paper. Make the connections as in Fig. 82, and test them by moving the slider first to 0 and then to 1000, making battery and galvanometer contacts at each place, when, if the connections are good, the deflections should in the one case be to the right and in the other to the left. (These tests should be made with the galvanometer shunted.) Next observe whether the contact of the platinum of the slider with the German silver wire gives rise to a thermo-current sufficiently strong to cause a deflection of the unshunted galvanometer. This may simply be done by pressing the knob of the slider—the battery key being raised, when a deflection will certainly be obtained if the galvanometer is at all delicate. This thermo-current becomes a source of trouble when the adjustment of the balance is approaching completion, hence we must consider some method by which the effects arising from it may be avoided. There are three such methods which we shall now discuss.

(1.) The position of the galvanometer and battery exhibited in Fig. 82 is that usually adopted with the slide bridge. Now let these positions be interchanged so that the battery wire is connected with the movable contact. No doubt it has been thought that when the galvanometer and battery are thus interchanged, the surface of the bridge wire may become injured by the passage of the current. But since the currents employed are weak, very little injury can, we think, be caused in this way, and if the interchange be made, the thermo-current at the movable junction is simply united with that from the battery either acting with it or against it. We shall not, however, even now be free from thermo-currents, for they may still be produced at the ends of the bridge where the German silver wire comes in contact with the

copper. Such currents may, however, be to a great extent avoided by packing the ends of the bridge with cotton wool so as to keep these places at a constant temperature.

(2.) A second method (the connections being as in Fig. 82) is to place in the battery circuit a commutator, by which the current may be reversed, and to obtain a balance for the two directions of the current. In the one case the thermo-current will aid, while in the other it will oppose the battery-current, and hence the mean of the two will give a correct result. It is here assumed that the thermo-current remains constant during the time of application of the test—hence it will be necessary to protect the slider from the heat of the hand as much as possible by wrapping cotton wadding around it.

(3.) The second method involves a double reading of the bridge, but this may be avoided by proceeding as follows:—Making the ordinary connections (Fig. 82), obtain a balance as nearly as possible in the usual way by *first* pressing the battery key and *then* the movable contact. Now keep the latter pressed down, the former being raised, and allow the galvanometer to come to rest under the action of the thermo-current. Let the battery key be now pressed, if a *different* deflection is produced, *which is permanent*, it shows that the balance is not perfect, and that further adjustment is necessary. The student must be warned that a slight *transient* deflection in the opposite direction may be obtained, owing to the self-induction of the coils in the circuit, but this must be disregarded. Taking these precautions, this last method is to be specially recommended.[1]

[1] See "Results of a Comparison of the BA units of Electrical Resistance," by G. Chrystal and S. A. Saunder. *Report B.A.*, 1876, p. 13.

Example.—Determination of resistance of coil marked "R_2" compared with standard ohm P.

Left.		Right.
P	$x_1 = 493{\cdot}4$	R_2
R_2	$x_2 = 509{\cdot}9$	P

$$\frac{R_2}{P} = \frac{1000 - (x_1 - x_2)}{1000 + (x_1 - x_2)}, \text{ hence } R_2 = 1{\cdot}0336 \text{ ohm.}$$

Lesson XXVIII.—Use of Slide Mètre Bridge—(*continued*) Method II.[1]

60. *Apparatus.*—The same as before, with the addition of mercury cups and copper straps for making connections. Also a number of resistance coils provided with stout copper terminals.

Theory of the Method.—Let the central gaps of the bridge be opened, and the resistances P and Q under comparison placed there, also in the end gaps place two other coils R_1 and R_2, and complete the connections as figured (Fig.

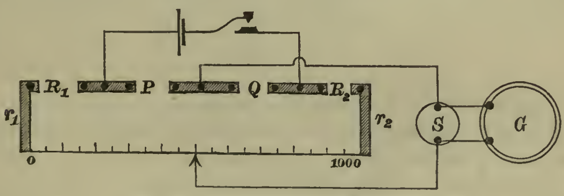


Fig. 83.—Connections for Method II.

83). When a balance is obtained, leaving λ out of consideration, we have

$$\frac{P}{Q} = \frac{r_1 + R_1 + x_1}{r_2 + R_2 + L - x_1} \quad . \quad . \quad . \quad . \quad (1)$$

[1] Adapted from Matthiessen and Hockin, *The Laboratory*, pp. 343 and 391; and Clerk Maxwell, vol. i. p. 402.

where r_1, r_2, and x_1 have the same meanings as in the previous method, and L is the length of the bridge wire, it being supposed that r_1, r_2, R_1 and R_2 are expressed in terms of a division of this bridge wire. On reversing P and Q we obtain

$$\frac{P}{Q}=\frac{r_2+R_2+L-x_2}{r_1+R_1+x_2} \quad . \quad . \quad . \quad . \quad (2)$$

Treating (1) and (2) as in the last lesson, we obtain

$$\frac{P}{Q}=\frac{r_1+r_2+R_1+R_2+L+(x_1-x_2)}{r_1+r_2+R_1+R_2+L-(x_1-x_2)} \quad . \quad . \quad . \quad (3)$$

For the sake of brevity let $r_1+r_2+R_1+R_2+L=\epsilon$, and $x_1-x_2=\delta$ (being the difference of the bridge readings), then

$$\frac{P}{Q}=\frac{\epsilon+\delta}{\epsilon-\delta} \quad . \quad . \quad . \quad . \quad (4)$$

This will be the working formula. It may be simplified when δ is small compared with ϵ by dividing the numerator by the denominator, and neglecting small quantities of the second order; we obtain then

$$\frac{P}{Q}=1+\frac{2\delta}{\epsilon} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The effect of introducing R_1 and R_2 is virtually to increase the length of the bridge wire and so increase the delicacy of the method. This will be seen from the following examples. The greatest possible value of δ is 1000 mm., *i.e.* the total length of the bridge, and the least value, say 0·2 mm. Now let $\epsilon=5000$, then from (4) we obtain

$$\frac{P}{Q}=\frac{5000+1000}{5000-1000}=\frac{3}{2}=1{\cdot}5,$$

which gives us the maximum ratio of P to Q for which this method is applicable.

Again from (5) we obtain

$$\frac{P}{Q}=1+\frac{\cdot 4}{5000}=1\cdot 00008,$$

which gives us the minimum ratio of P to Q for which the method is applicable.

Let $\epsilon=10000$.	In this case the greatest ratio is	$1\cdot 22$
	the least	$1\cdot 00004$.
Let $\epsilon=200000$.	In this case the greatest ratio is	$1\cdot 01005$
	the least	$1\cdot 000002$.

We thus see that the effect of increasing ϵ is to decrease the range of applicability of the method, but at the same time to increase the accuracy of the comparison, inasmuch as under these circumstances the extreme bridge readings correspond to a continually decreasing range of proportional difference between the resistances.

Before the preceding formulæ can be applied it will be necessary to determine r_1, r_2, R_1 and R_2.

Determination of r_1 and r_2.—Let us suppose the connections of Fig. 82 to be made, P and Q being two coils of known resistance, whose ratio is about 100 to 1, then

$$\frac{P}{Q}=\frac{r_1+x_1}{r_2+1000-x_1} \quad . \quad . \quad . \quad . \quad (1)$$

and, on reversing,

$$\frac{P}{Q}=\frac{r_2+1000-x_2}{r_1+x_2} \quad . \quad . \quad . \quad . \quad (2)$$

From (1) and (2) we obtain

$$r_1=\frac{Qx_1-Px_2}{P-Q} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$r_2=\frac{Q(1000-x_2)-P(1000-x_1)}{P-Q} \quad . \quad . \quad . \quad (4)$$

From (1) and (2) we may also obtain the value of r_1+r_2,

thus, by the addition of the numerators and denominators, we have

$$\frac{P}{Q}=\frac{r_1+r_2+1000+(x_1-x_2)}{r_1+r_2+1000-(x_1-x_2)} \quad . \quad . \quad . \quad (5)$$

From which we have, by compounding,

$$\frac{P+Q}{P-Q}=\frac{r_1+r_2+1000}{x_1-x_2} \quad . \quad . \quad . \quad . \quad (6)$$

or

$$r_1+r_2=\frac{P+Q}{P-Q}(x_1-x_2)-1000 \quad . \quad . \quad . \quad (7)$$

Determination of R_1 and R_2.—Make the connections of Fig. 83, but remove R_2 and substitute for it the copper rod that is provided for bridging the gap. Taking readings with P and Q, that is, with coils of known resistance, we obtain

$$\frac{P}{Q}=\frac{R_1+r_1+x_1}{r_2+(1000-x_1)},$$

whence

$$R_1=\frac{P}{Q}(1000-x_1+r_2)-x_1-r_1.$$

In like manner we may determine R_2.

Practice of the Method.—The only additional practical information necessary to give is that regarding the use of mercury cups as connectors. This method of making connections is found to give contacts whose resistance is both very small and very constant, provided that the following precautions are observed: (1.) Each mercury cup must be provided with a disc of copper fastened to its bottom. This disc must be well amalgamated. (2.) The ends of the copper rods with which the resistances are provided must likewise be well amalgamated, and arranged so as to press firmly against the plates at the bottom of the cups by means of clamps or elastic bands. The mercury cups may be made simply by boring holes in a piece of varnished wood, connection from the binding screws of the bridge being

made by thick copper bars having one end clamped under the screw, and the other dipping into the cup.

Example.—Determinations of r_1 *and* r_2.

$$P=101, \quad Q=1.$$

Left.		Right.
P	$x_1=992{\cdot}4$	Q,
Q	$x_2=\ \ 7{\cdot}8$	P,

$$r_1=\frac{992{\cdot}4-787{\cdot}8}{100}=2{\cdot}046,$$

$$r_2=\frac{992{\cdot}2-767{\cdot}6}{100}=2{\cdot}246, \quad r_1+r_2=4{\cdot}29.$$

This calculation was checked by formula (7) as follows :—

$$r_1+r_2=\frac{101}{99}\ 984{\cdot}4-1000=4{\cdot}29.$$

Determination of R_1 *and* R_2.

$$\frac{P}{Q}=10, \quad x_1=508{\cdot}5,$$

$$R_1=10(1000-x_1)-x_1+10r_2-r_1=4427.$$

This result was checked by measuring the resistance of the whole length of the bridge wire by a second Wheatstone bridge. The total resistance of the bridge, including the ends, was 0·234 ohm, hence the value of a division, taking the value of the ends as 4·3, is

$$\frac{{\cdot}234}{1004{\cdot}3}={\cdot}000233 \text{ ohm per division.}$$

The resistance of R_1 was found by the bridge to be 1·0335 ohm, which is equivalent to $\frac{1{\cdot}0335}{{\cdot}000233}=4435$ divisions.

R_1 was taken as the mean of 4427 and 4435 = 4431 divisions.

R_2 was similarly found to be 4360 divisions.

Value of ϵ.—We have

$$\epsilon=R_1+R_2+r_1+r_2+1000=4431+4360+4{\cdot}3+1000=9795{\cdot}3.$$

(For convenience of calculation R_2 was increased until ϵ became 10,000 nearly.)

Comparison of Resistances.

Resistance P compared with a standard ohm.

$$x_1 - x_2 = \delta = 10 \cdot 8,$$
$$P = 1 + \frac{2\delta}{\epsilon} = 1 + \frac{21 \cdot 6}{10,000} = 1 \cdot 00216 \text{ ohm.}$$

Lesson XXIX.—Use of Slide Bridge—Carey Foster's Method.

61. *Exercise.*—To measure the resistance of a coil that is nearly equal to a standard ohm, by Carey Foster's method, and to determine its temperature coefficient.

Apparatus—The experience gained by the student in the foregoing lessons will have shown him that the bridge arrangements already described require some very desireable improvements, especially, (1.) A better slider, permitting a more accurate adjustment, and provided with a vernier for reading off its position. (2.) A bridge wire of non-oxidisable material, drawn with care, so as to be of the same diameter throughout, and of sufficient hardness not to become injured by the pressing upon it of the contact-piece. (3.) Some convenient method of interchanging the coils without displacing them from the mercury cups. (4.) Suitable water baths, for keeping the coils under measurement at a constant temperature.

(I.) A bridge is exhibited in Fig. 84 which will satisfy the first two conditions. A rod *mn*, supported at the two ends of the bridge so as to be capable of being moved longitudinally by means of the screw *s*, passes through an opening at the back of the slider (Fig. 85), and may be clamped to the slider by means of the screw *c*. When the clamp screw *c* is unfastened, the slider with the connecting wire *d* may be moved from one end of the bridge to the other quite readily, but when *c* is clamped the slider can only

be moved by turning the screw s, thus allowing a fine adjustment movement of about an inch. The contact-piece, when pressed by the knob k against the wire, may be held

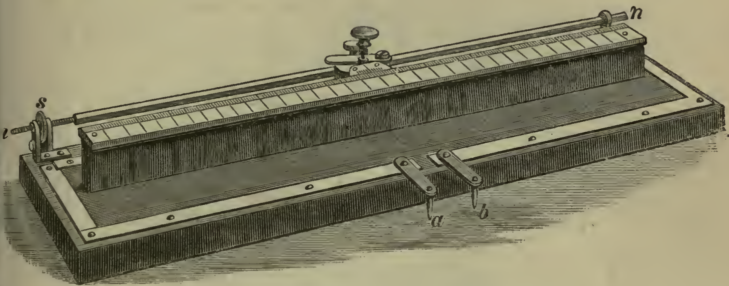


Fig. 84.—Improved Slide Bridge.

in position by means of the lever l. The wire, of 1·5 mm. diameter, is made of platinum-iridium, which, in addition to its sufficient hardness and non-liability to oxidation, has a low temperature coefficient. It will be seen that the bridge has only a central gap terminating in two copper rods a, b, which are intended to fit into two mercury cups.

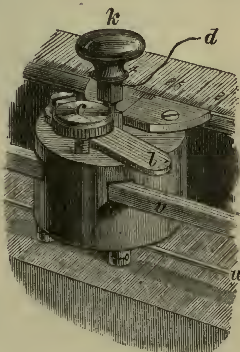


Fig. 85.—The Slider.

(II.) In order to facilitate the making of the connections and the interchange of the coils a *switch board*, such as is shown in plan (Fig. 86), will be required. A B C D is a base-board of ebonite or well varnished mahogany, with four large mercury cups, m, n, m', and n', and eight smaller cups, a, b, c, d, a', b', c', d', the latter fixed at the end of four bars of copper, with which they are in metallic connection. Bars of copper connect the two lower

pairs of cups either to two mercury cups, into which the ends of the bridge are brought, or the connection may be made by means of the clamp screws shown. The upper bars are

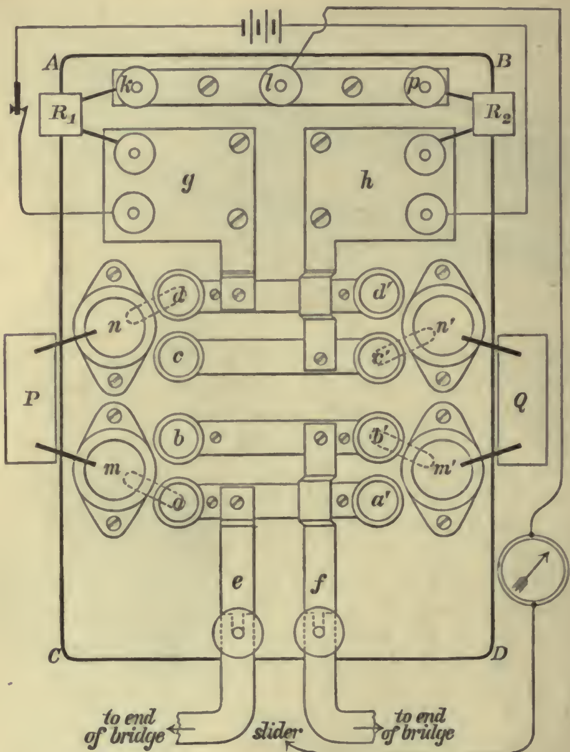


Fig. 86.—The Switch Board (S. P. Thompson's Pattern).

in connection with plates of copper, g and h, each provided with two binding screws. Finally, a bar of copper with three binding screws, k, l, p, is fixed at the top of the

board. Short bars of copper with bent ends are provided for making connections between the large and the small mercury cups.

(III.) For maintaining the coils at a constant temperature during the measurement they must be kept in zinc cans, through which a current of water is flowing. It is found that the temperature of the water from the town mains is sufficiently constant for the purpose. The cans should be provided with stirrers and thermometers.

(IV.) A copy of a standard ohm will be required. The coil, which is of platinum silver, thoroughly insulated by

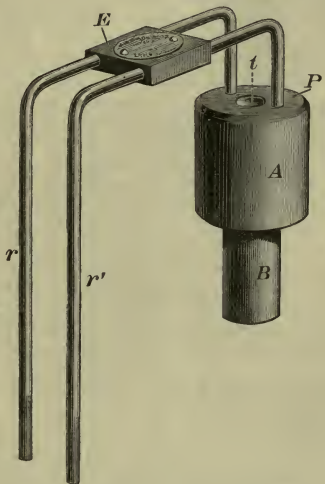


Fig. 87.—Standard Resistance Coil.

silk and paraffin, is mounted within a brass case AB (Fig. 87). Its ends are soldered to thick copper rods r and r' for the purpose of making connections with mercury cups. The coil is at the lower portion of the case B, the upper portion A being filled in with paraffin up to the top P.

A hole *t*, for the reception of a thermometer, runs through the case. The standard is accompanied by a certificate, giving the temperature at which it is correct, the temperature coefficient, and instructions to be observed in using the ohm. The following is a copy of such a certificate relating to a standard BA coil:—

No. 38. Right at 16°·5.

The resistance of this coil at the temperature of 16°·5 C. is equal to

ONE BA UNIT,

representing one $\frac{\text{Mètre}}{\text{Second}} \times 10^7$ (electro-magnetic absolute measure).

The coil is made of an alloy of platinum and silver, and within 5° on either side of the above temperature the resistance varies ·031 per cent for each degree centigrade, increasing with the increase of temperature. The temperature of the coil should be observed by noting the temperature of water in which the lower portion (up to the shoulder) is to be placed. The water must be stirred from time to time. The coil is heated by every observation, and no two accurate observations can be made within ten minutes of each other. No current should be allowed to pass through the coil from a powerful battery, nor from a weak battery except for very short periods.

Mercury cups are supplied for connecting the coils with other conductors. Before using the coil the ends of the copper wire and the copper plates in the mercury cups should be reamalgamated. To do this, dip the ends of the wire and the plates into a solution made by dissolving mercury in nitric acid (taking care that a little metallic mercury be always present in the solution). Then wipe the wires and plates with blotting paper, when they will be found covered with brilliant metallic mercury. The mercury in the cups should be clean.

The following is the form of the certificate relating to the new standards:—

This is to certify that the resistance coil X has been tested by the Electrical Standard Committee and that its value at a temperature of 4° C. is P Legal Ohms.

It has been assumed for the purpose of this comparison that 1 Legal Ohm is equal to 1·0112 BA units.

(V.) *Other Apparatus Required.*—Supply of hot water, galvanometer, battery, and key.

Theory of the Method.—Make the connections figured (Fig. 88), where P and Q are the coils which it is wished to

compare, while R_1 and R_2 are two resistances approximately equal to P or Q. The reading will thus be brought to the centre of the bridge. When a balance is established we have

$$\frac{R_1}{R_2}=\frac{P+r_1+x_1\rho}{Q+r_2+(L-x_1)\rho} \quad . \quad . \quad . \quad . \quad (1)$$

where x_1 is the bridge reading, ρ is the resistance of a

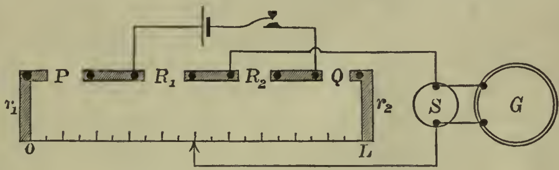


Fig. 88.—CONNECTIONS OF METHOD III.

division of the wire, and L is the total number of divisions. Reverse the position of P and Q, then we have

$$\frac{R_1}{R_2}=\frac{Q+r_1+x_2\rho}{P+r_2+(L-x_2)\rho} \quad . \quad . \quad . \quad . \quad (2)$$

x_2 being the new bridge reading.

From (1) we have

$$\frac{R_1}{R_1+R_2}=\frac{P+r_1+x_1\rho}{P+Q+r_1+r_2+L\rho} \quad . \quad . \quad . \quad (3)$$

and from (2) we have

$$\frac{R_1}{R_1+R_2}=\frac{Q+r_1+x_2\rho}{P+Q+r_1+r_2+L\rho} \quad . \quad . \quad . \quad (4)$$

From (3) and (4) we get

$$Q+r_1+x_2\rho=P+r_1+x_1\rho,$$

hence

$$Q-P=\rho(x_1-x_2) \quad . \quad . \quad . \quad . \quad (5)$$

an expression which is independent both of the length of the bridge and of the resistance of its ends, and which gives us the difference between the two resistances in terms of the difference of the bridge readings and of the

resistance of unit of length of the bridge wire. This formula is hence one of extreme simplicity and convenience. When P is of unit resistance, we obtain

$$Q = 1 + \rho(x_1 - x_2) \qquad . \quad . \quad . \quad (6)$$

Practice of the Method.—The appropriate connection must first be made by help of Figs. 86 and 88. The coils R_1 and R_2, each of about an ohm, will not have to be displaced, and must have their ends clamped under the binding screws. It is important that these coils be kept at a uniform temperature, hence it is better that they should be wound together and placed in the same water bath. P and Q, the coils under comparison, must have the ends of the copper rods forming their terminals well amalgamated, and they are then placed in the large mercury cups. Clamps or elastic bands should be used to press the rods firmly to the bottoms of the mercury cups. A stream of water from the town mains should be passed through the water baths, and readings taken of the bridge and the thermometers. Next, the position of the copper connectors must be changed and a new observation taken. We must now, as advised by Fleming, raise the temperature of the bath containing the coil Q to about 20° C. above the temperature of P, and allow the temperature to fall slowly some 5 degrees, by which time the temperature of the coil will probably be that of the bath. Readings are then taken in the two positions of the switch as before. The value of the observations will depend largely upon the attention that has been given to the regulation of the temperature; the wire being embedded in paraffin, its temperature will not be known within 0°·1 C., which, however, will only cause an error of about ·002 per cent. To ascertain when the temperature of the wire is equal to that of the bath, the arrangement suggested by Chrystal of placing a properly insulated junction of a thermo-electric couple as near as possible to the wire, the other junction being fixed to the outer casing of the coil,

may be adopted. When the junctions are at the same temperature a galvanometer included in their circuit will show no current.

Example.—

(1) Left P	Right Q	Reading	598·0
,, Q	,, P	,,	490·2
	Temperature 12° C.	Difference	107·8

$\rho = \cdot 00022$. P correct at 16° C. Temperature Coefficient $= \cdot 03$.

$$Q - P = \cdot 00022 \times 107 \cdot 8 = \cdot 0237.$$
$$P = 1 - (4 \times \cdot 0003) = \cdot 9988.$$
$$Q = \cdot 9988 + \cdot 0237 = 1 \cdot 0225 \text{ ohm}.$$

(2) P at 12° C. Q at 27° C. Difference of Reading $= 123 \cdot 5$. Hence the coil Q of $1 \cdot 0225$ ohm has increased in resistance by $(123 \cdot 5 - 107 \cdot 8) \cdot 0002 = \cdot 00314$ ohms when its temperature has been raised from 12° to 27°, or through 15°. Hence the temperature coefficient is per cent per degree equal

$$\frac{\cdot 00314 \times 100}{1 \cdot 0225 \times 15} = \cdot 0204$$

Determination of ρ.—We must next consider some method for finding the value of ρ.

Method I.—Obtain a resistance Q somewhat less than that of the bridge wire, and balance it against a resistance P, consisting of a thick copper bar whose resistance may be neglected, then

$$Q = P + \rho(x_1 - x_2),$$

but $P = 0$ nearly, hence

$$\rho = \frac{Q}{x_1 - x_2} \quad . \quad . \quad . \quad . \quad . \quad (7)$$

If Q has a small value, some special method of determining its resistance must be used.

Method II.—As in equation (7), let us obtain

$$Q = \rho \delta_1 \quad . \quad . \quad . \quad . \quad . \quad (8)$$

where Q is a resistance somewhat smaller than that of the bridge wire, and δ_1 is the difference of readings.

Place Q in multiple arc with a *standard unit P*, and

balance the combination, as in the previous case, against a negligible resistance; then, since P and Q are in multiple arc, their united resistance will be $\frac{QP}{Q+P}$; and hence we shall have

$$\frac{QP}{Q+P}=\rho\delta_2 \quad . \quad . \quad . \quad . \quad . \quad (9)$$

where δ_2 is the difference of readings.

Eliminating Q between (8) and (9), we obtain

$$\rho=P\,.\,\frac{\delta_1-\delta_2}{\delta_1\delta_2}\,. \quad . \quad . \quad . \quad . \quad . \quad (10)$$

and thus ρ would be directly known, in terms of the standard unit.

Method III.—Measure the resistance of the bridge wire directly by means of another bridge in which r_1 and r_2 are known. Then find ρ by dividing the total resistance by the total equivalent length of bridge wire and ends.

Calibration of the Bridge Wire.—In the preceding methods it has been assumed that the resistance of the bridge wire is constant throughout. We cannot, however, be sure of this, and hence we must *calibrate* the wire. The following is the method devised by Carey Foster for this purpose.

The bridge wire EF, in order to be calibrated, must be connected with a second bridge wire E′F′, after the manner of Fig. 89. The gaps of the bridge at A and D

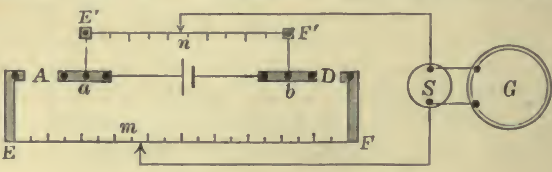


Fig. 89.—CALIBRATION OF BRIDGE WIRE.

are closed by two resistances; that at A, the "*connector*," is as small as possible, that at D is a small piece of wire whose resistance is equal to that of the length of EF,

which we wish to test. This is called the "*gauge.*" The movable contacts m and n are connected with the galvanometer. The process is as follows:—

(1.) m is placed very near F, and then n is moved until a balance is obtained.

(2.) The connector and gauge are now interchanged, and m is moved until a balance is restored.

(3.) The connector and gauge are restored to their first positions, and n moved so as once more to produce a balance.

These processes are repeated until m, by successive steps, is brought near to E and n near to E′, and then both EF and E′F′ will have been divided into short pieces of equal resistance, while between the resistance of one piece of the one bridge and that of one piece of the other there will be a constant ratio. This we shall now proceed to show.

Let the resistances of the several parts (see Fig. 90) be as follows:—

That of gauge $=$ G, of EF $=$ L.
Connector $=$ C, of E′F′ $=$ L′.
Permanent connectors between a and E $= e$.
,, ,, ,, a and E′ $= e'$.
,, ,, ,, b and F $= f$.
,, ,, ,, b and F′ $= f'$.

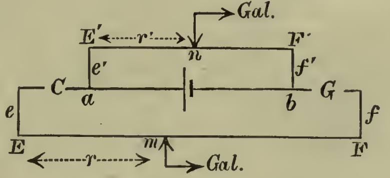


Fig. 90.

Call the resistance of E$m = r$ and of E′$n = r'$.

Let us first suppose that the connector and gauge are in the position figured; then we have

$$\frac{C+e+r}{G+f+L-r}=\frac{e'+r'}{f'+L'-r'} \quad . \quad . \quad . \quad . \quad (1)$$

Now, interchange G and C, and move n (m remaining undisturbed) until a balance is obtained; then we have

$$\frac{G+e+r}{C+f+L-r}=\frac{e'+r'_1}{f'+L'-r'_1} \quad . \quad . \quad . \quad . \quad (2)$$

where r'_1 is the resistance between E′ and the new position of n, so that $r'_1 - r'$ = resistance of the portion of the bridge E′F′ that n has passed over. From (1) and (2) we obtain

$$r'_1 - r' = (G-C)\frac{e'+f'+L'}{G+C+e+f+L} \quad . \quad . \quad . \quad (3)$$

If now C and G be again interchanged, and m be moved until the balance is again established, then by Foster's ordinary formula (5), (p. 163), G − C is equal to the resistance of that portion of EF that m passes over, also we see from (3) that G − C, as well as its multiplier, is constant. But, $r'_1 - r'$ being the resistance of the part moved over by n, it follows from (3) that this is necessarily constant, inasmuch as it is equal to the product of two constant quantities. Moreover $r'_1 - r'$ and G − C will be the resistances passed over at the successive steps of n and m. Accordingly both EF and E′F′ will be divided into parts of equal resistance, so that two bridges may be calibrated at the same time.

Example.—Two bridges, which had been in use for some time, were calibrated. It was found necessary to take special precautions to avoid thermo-electric currents, which had a tendency to be produced at the two sliding contacts and at the ends of the bridge. The effect of these currents at the sliding contacts was eliminated by method (3), p. 152, and that of the currents from the ends of the bridge by the use of a commutator in the battery circuit. The following table exhibits the mean of the readings:—

Reading Bridge EF.	Difference.	Reading Bridge E′F′.	Difference.
0		157	
89·7	89·7 (*a*)	312·2	155·2 (*a*)
180·6	90·9 (*b*)	467·8	155·6 (*b*)
273·8	93·2 (*c*)	623·1	155·3 (*c*)
363·6	89·8 (*d*)	776·5	153·4 (*d*)
453·3	89·7 (*e*)	930·9	154·4 (*e*)
545·0	91·7 (*f*)	1090	159·1 (*f*)
639·0	94·0 (*g*)	1247·7	157·7 (*g*)
730·5	91·5 (*h*)	1400·1	152·4 (*h*)
820·8	90·3 (*k*)	1555·9	155·8 (*k*)
914·0	93·2 (*l*)	1711·2	155·3 (*l*)

Taking the resistance of the bridge EF from 0 to 914 as ·213 ohm, this will be the sum of the resistances of the various parts (*a*) + (*b*) + (*c*), etc.

Now, since each of these has the same resistance, the resistance of any one of them will be $\frac{\cdot 213}{10} = \cdot 0213$.

We must now draw up a table exhibiting the value of the resistance of one division of the bridge at the various parts by dividing ·0213 by (*a*), (*b*), (*c*), etc. These values are given below.

Part of Bridge.	Value of ρ.
0 – 89·7	·0002374
89·7–180·6	·0002343
180·6–273·8	·0002286
273·8–363·6	·0002372
363·6–453·3	·0002374
453·3–545	·0002323
545 –639	·0002266
639 –730·5	·0002328
730·5–820·8	·0002359
820·8–914	·0002286

62. *Additional Exercises.*—(1.) Mount half a mètre of No. 34 B. W. G. platinum wire coiled in a spiral within an oil bath, and determine the resistance at various temper-

atures. Draw a curve of the results, also find the constants in the formula

$$R = \alpha\,\tau^{\frac{1}{2}} + \beta\,\tau + \gamma,$$

where R is the resistance at the temperature τ, reckoned from the absolute zero ($-273°$ C.), and α, β, and γ are the required constants. Compare your results with those of Siemens, whose paper should be consulted—"Electrical Resistance Thermometer and Pyrometer," by C. W. Siemens. See *Transactions of the Society of Telegraph Engineers*, 1875.

(2) Make an ohm coil by the *shunt method* as follows :— A length calculated to within ·5 cm. is cut off from a sample of thick insulated wire, of which the resistance per cm. is approximately known, 2 per cent being added to the length before the wire is cut off. Solder and compare with a standard unit by Foster's method. We want now to find what must be the resistance of a shunt coil in order that when combined with the coil that has been measured (and which is rather greater than an ohm), the two in multiple arc may be exactly 1 ohm in resistance. Let R_1 be the resistance of the first rough approximate coil, and S the resistance of a shunt coil which will reduce R_1 to unit resistance, then

$$1 = \frac{R_1 S}{R_1 + S},$$

or

$$S = \frac{R_1}{R_1 - 1}.$$

Thus 1·1 mètre of No. 24 B. W. G. German silver wire of ·56 cm. diameter was cut off and soldered. $R_1 = 1·0304$, temperature 17°·2, S = 33·9. Next 7·05 mètre of German silver of No. 36 B. W. G. at 20·8 cm. per ohm was cut off, and without further measurement soldered to the terminals of R_1. The combination measured 1·008 at 17°·3 C. See Professor S. P. Thompson "On the Adjustment of Resistance Coils," *Pro. Phys. Soc.*, vol. vi. p. 47.

63. *Modified Forms of the Slide Bridge.*—The form of bridge which we have called the slide bridge is due to Kirchhoff. This form being the most suitable for the comparison of standard coils, electricians have devoted much attention to the improvement of its construction. A few of their memoirs, with accompanying references, are outlined below :—

(1.) "On the Reproduction of Electrical Standards," by A. Matthiessen and C. Hockin, *B. A. Report*, 1864, p. 352.—A platinum-iridium wire used. Sledge of lead with platinum contact. Sledge ran on a tramway parallel to wire.

(2.) Siemens (Wiedemann, *Elektricität* (1882), vol. i. p. 453) uses the same method of moving the slider that we have given in the last lesson, and describes, in conjunction with Dahms, a mercury cup commutator.

(3.) "Electrical Balance for Reproduction of Exact Copies of the Standard of Resistance," by Fleeming Jenkin, *B. A. Report*, 1862.—The bridge being mainly required for making copies, has only a short length, and is used with proportional coils.

(4.) "A New Form of Resistance Balance adapted for comparing Standard Coils," by Dr. Fleming, *Pro. Phys. Soc.*, vol. iii. p. 174.—Here a circular disc of mahogany has a semicircular groove turned in its circumference. A platinum-iridium wire is laid evenly in the groove. Contact is made by a platinum-iridium wedge fixed to a radial arm. A series of mercury cups are used for the connections, and are so arranged that the interchange of the coils may be brought about by lifting the legs of the coils and replacing them in adjacent mercury cups. A very high degree of accuracy may be attained with this bridge, which has been much used for verifying copies of the legal ohm.

(5.) "On a Modified Resistance Balance," by Professor S. P. Thompson, *Pro. Phys. Soc.*, vol. vi. p. 121.—Returns to the straight form of bridge, with a length of two mètres. Two wires are stretched on the bridge, a thick German

silver one of ·24 ohm, and a finer platinum-silver one of 8·2 ohms. Either wire may be used at pleasure by the use of a switch. A series of mercury cups are used for the connections, arranged in the manner that has been described in the last lesson.

(6.) "On a Practical Point in connection with the Comparison of Resistance," by W. N. Shaw, *Pro. Phys. Soc.*, vol. vi. p. 71.—Describes how the coils may be interchanged by rotating a crank to the right or left.

64. *Rheostats.*—An arrangement permitting us to increase or diminish gradually the resistance in a circuit without the sudden changes that are occasioned when a box of coils is used has many and important applications in electrical measurements. Several electricians have attempted to produce a satisfactory **rheostat**, as this instrument is called. Wheatstone employed for this purpose two cylinders, one of brass and the other of wood, on the latter of which a spiral groove was cut, the resistance being gradually varied by winding an uninsulated wire from the wood, where the several coils were insulated from each other, to the brass where short-circuiting was produced. The arrangement is defective both on account of the injury to the wire in winding and the uncertainty of the contact with the brass. A less known but far better form was also used by Wheatstone,[1] in which the wire of the rheostat is laid once for all in a spiral groove turned in an insulating cylinder. The groove was of less depth than the diameter of the wire, so that the latter projected somewhat. When the cylinder was turned (it was mounted on a horizontal axis) a small contact wheel was made to travel by the screw action of the spirally laid wire along a fixed rod. In this way different lengths of the wire could be included in

[1] See the Physical Society's reprint of Wheatstone's *Scientific Memoirs* for a description of the two forms of Wheatstone's rheostat.

the circuit. The arrangement is not free from objection, owing to the fact that the contact between the wheel and the wire may not be always equally good. Several rheostats have been made on the principle of stretching two wires horizontally on a board, and short-circuiting more or less of the wire by means of a massive metal block or by means of a vessel containing mercury. Such is the principle of the **rheostats of Poggendorff and du Bois-Reymond.** Here the defects due to contact are very great. The same remark will apply to rheostats constructed on the principle of immersing different lengths of platinum wire in a vessel of mercury.

A rheostat was some time since devised by one of the writers of this work, and was likewise independently invented and described by Bidwell,[1] on a principle that seems to be free from the imperfections of those we have mentioned. An insulated cylinder (Fig. 91), consisting by preference of

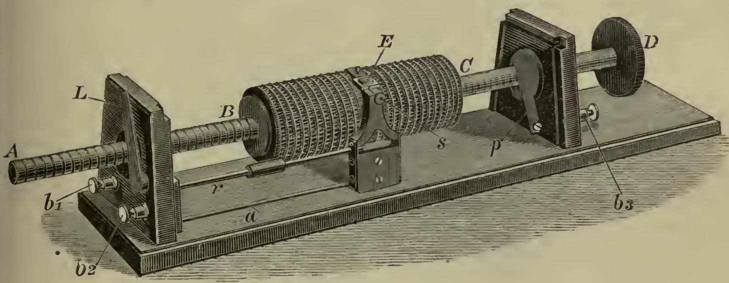


Fig. 91.—THE RHEOSTAT.

a hollow tube of ebonite with brass ends, has a spiral groove in which a German-silver wire is laid. So far the instrument resembles Wheatstone's second form, but, unlike it, instead of being mounted to revolve without

[1] *Phil. Mag.*, July 1886, "On a Modification of Wheatstone's Rheostat," by Shelford Bidwell, M.A. It appears that Jacobi used a similar form, see Pogg. *Ann.*, Bd. lix. s. 145 (1843).

other movement, the cylinder is provided with a long brass axle AD, upon one end, AB, of which a screw *of the same pitch* as that of the cylinder is cut. The axle revolves in two brass bearings, the distance between them being equal to twice the length of the cylinder; and one of the bearings has an inside screw corresponding with that upon the axle. Midway between the bearings is the contact E, provided with a notched plate of platinum. Both ends of the spiral wire are electrically connected with the brass axles, and thence with terminals b_1 and b_3 upon the base-board. The platinum contact is directly connected with a second terminal b_2 by an insulated wire *a*. When the cylinder is turned it travels, owing to the screw arrangement in the left-hand axis, backwards or forwards, while the point of contact of the platinum remains fixed in space. Thus more or less resistance is introduced between the two terminals. This arrangement obviates the several imperfections due to Wheatstone's rheostats, for (1.) the wire always remains fixed and does not leave its groove; (2.) there is no lateral stress, as in Wheatstone's second form, tending to force the wire out of the groove; and (3.) there is always good contact between the platinum and the wire. The instrument, shown in Fig. 91, which has been made for the Owens College Physical Laboratory, is provided with a fine adjustment screw at S, enabling the platinum contact to be moved through a small distance, which may be read off by the help of a small scale on the upper part of E. By pressing the lever L, a flat piece of metal that forms the nut of the screw is withdrawn from the thread, and simultaneously the rod *r* is turned which raises the contact from the wire, thus allowing the axles to be pushed through their bearings without the necessity of turning D. This enables the rheostat to be quickly set at any desired position.

65. *The Multiple Arc Box.*—By arranging a resistance

box so that the resistance coils may be placed in multiple arc instead of series, it is possible to make the alteration of resistance in a circuit as gradual as may be desired. A resistance box arranged in this way presents in a combined form many of the advantages of both a rheostat and a box of coils. For example, suppose that we have in circuit a coil of 10 ohms, and that it is wished to diminish somewhat the resistance. If we combine with the 10 ohms a coil in multiple arc of 10,000 ohms, the joint resistance is :—

$$\frac{1}{\frac{1}{10}+\frac{1}{10,000}}=\frac{1}{\cdot 1+\cdot 0001}=9\cdot 999001.$$

By the successive additions of coils in multiple arc, the resistance may be diminished until a particular resistance,

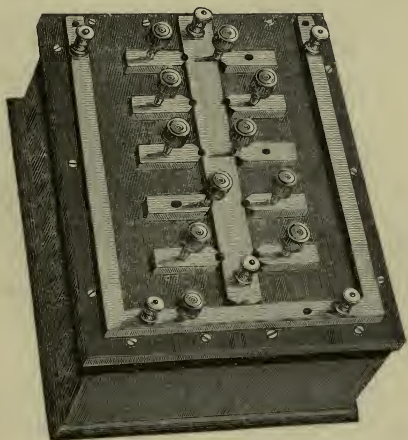


Fig. 92.

accurate perhaps to the hundred-thousandth of an ohm, is attained. The arrangement most generally used consists of a number of coils arranged in a series of powers of 2. Figs.

92 and 93 show 10 coils of 1, 2, 4, 8, 16, 32, 64, 128, 256, and 512 ohms conveniently mounted for use.

By placing any of the plugs in the holes on the right or on the left hand of EF, the corresponding coils will be

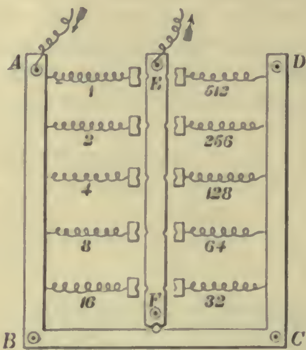


Fig. 93.

joined in multiple arc. The plug hole below F is for the purpose of short-circuiting the coils.

In calculating out the value of any combination, much time may be saved by the use of a table of reciprocals.[1] Thus, suppose that the coils 2, 8, and 16 were used, then the calculation becomes :—

	Reciprocal.	
2 ohms	·5	
8 ,,	·125	
16 ,,	·0625	
Sum	·6875	Reciprocal of sum = 1·454 ohm.

A table should accompany the box, exhibiting the effect of the more important combinations.

[1] Such as Barlow's Tables of Squares, Cubes, Square Roots, Cube Roots, and Reciprocals (Spon & Co.)

PART IV.—MEASUREMENT OF VERY LOW RESISTANCES.

66. The Wheatstone's bridge is unsuitable for this purpose, for when the resistance to be measured becomes small, the error introduced by the unavoidable resistance of the contacts becomes more important, and hence it is desirable to make use of some method in which the resistance of the contacts may vary considerably without affecting the result. Three such methods that have been devised are—

(1.) The method of **Comparison of Potentials.**
(2.) The method of **Projection of Equi-Potentials.**
(3.) The method of **Auxiliary Conductors.**

The first of these methods was in use before the invention of the bridge. Wheatstone (*Phil. Trans.*, 1843, p. 323) attributes the principle of it to Petrini; the second is due to Matthiessen and Hockin, and was employed by them in determining the resistance of metals in the form of bars (*Laboratory*, 1867, p. 423), while the third was invented by Sir Wm. Thomson ("New Electrodynamic Balance for Resistance of Short Bars or Wires," *Phil. Mag.*, 4th series, vol. xxiv., 1862, p. 149).

We shall describe these methods in order in the following lessons.

LESSON XXX.—Method of Comparison of Potentials.

67. *Exercise.*—To find the resistance of ·5 mètre of No. 20 S. W. G. copper wire, and thence to determine its specific resistance.

Apparatus.—A high resistance reflecting galvanometer, ten $\frac{1}{10}$ ohm coils with copper connecting pieces, mercury cups for connecting the coils in multiple arc. A Grove's cell in good condition, keys, and commutator. Also knife-

edges for making connections, which we shall describe presently.

Theory of the Method.—The student who has grasped the principle involved in the proof of Ohm's law given in Lesson XX. will see that the same principle may be utilised for the comparison of resistances. Let AC, CB (Fig. 94) be two resistances connected together, the length of the lines being proportional to and thus denoting the resistance, and suppose that we wish to compare the resistance denoted by *ab* with the resistance denoted by *cd*.

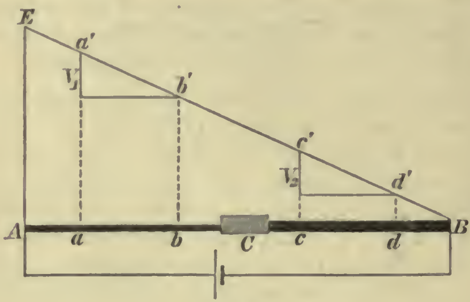


Fig. 94.

Connect A and B with a battery, then between these points there will be a fall of potential which may be graphically represented by BE, the slope of which will denote the rate of change of potential. The difference between the potentials at *a* and *b* will thus be represented by the difference between *aa'* and *bb'*; let us call this difference V_1. In like manner V_2 may be taken to represent the difference of potential between *c* and *d*. But evidently (since EB is a straight line by Ohm's law, that is to say, since difference of potential is proportional to resistance)

$$\frac{V_1}{V_2}=\frac{ab}{cd}.$$

To find the ratio between V_1 and V_2 it is sufficient, first, to place at the point *a* one of the electrodes of a galvanometer whose resistance is very great compared to that between *a* and *b*, the other electrode being at *b*. A deflection D_1 will be produced. On transferring the electrodes to *c* and *d*, let the deflection now be D_2. Then

$$\frac{D_1}{D_2}=\frac{V_1}{V_2}=\frac{ab}{cd}.$$

Here evidently the resistance of the contacts need not be made a matter of special concern, for in the circuit including these contacts there is the high resistance of the

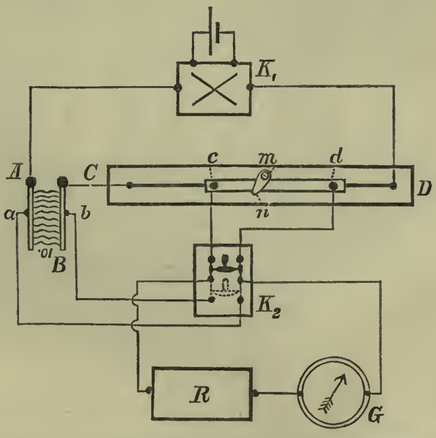


Fig. 95.—Connections for Fall of Potential Method.

galvanometer. It will be remarked too, that the resistances at A, B, and C are not involved in the measurements.

Practice of the Method.—Make connections as seen in Fig. 95. Here the battery is a Grove's cell, K_1 is a commutator, K_2 a switch key, *ab* the ·01 ohm standard, consisting of ten ·1 ohms placed in multiple arc. The arrange

ment of these coils will best be seen in Fig. 96, where the ten coils are seen mounted on wooden reels, and provided with copper straps at their lower ends. The two copper straps belonging to each coil dip into the channels of mercury terminating in two large mercury cups *m*, *n*. The mercury channels and cups have well amalgamated sheet copper fixed to their bottoms. Two slotted uprights of metal, having a wooden rod attached to them by thumb

Fig. 96.—STANDARD ·01 OHM.

screws *rr*, constitute an arrangement for supporting the coils, and keeping their copper straps well pressed against the bottom of the channels. Two lateral binding screws, one of them being seen at the front of the figure, are in connection with the channels. Stretched on a board CD (Fig. 95) is the wire whose resistance is required. It is provided with two large binding screws at its ends. *cd* (Fig. 95) is a rod of wood having two binding screws *bb′*, whose shanks project through the wood, as seen in Fig. 97.

The end of each of these screws is filed to a knife-edge. One of the binding screws is movable in a slot s, and may be clamped in any position by means of a screw. The knife-edges are placed so as to rest on the wire, the rod being held in place by the arm mn (Fig. 95), so that the knife-edges just press on the wire. R is a coil of high resistance placed in the circuit of the high resistance galvanometer G.

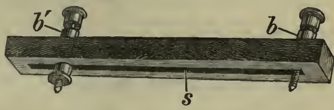


Fig. 97.

The connections will require little explanation, the letters *abcd* (Fig. 94) corresponding to the theoretical diagram. The purpose of the switch K_2 is that the galvanometer may readily be connected either with a and b or with c and d by moving the switch in the one direction or the other.

Precautions in using the Method.—There are two sources of error. (1.) The rise of resistance caused by the increase of temperature produced by the battery current, and (2.) inconstancy of the battery. To avoid these as much as possible the battery current must be made for only the time actually required to take the readings. It will be obvious that the more sensitive the galvanometer the less need will there be to use a strong current.

Example.—5 ohms were placed in the circuit of the Grove's cell, and 5000 ohms in that of the galvanometer.

Galvanometer connected with *ab*. Mean readings 201 divisions.
,, ,, *cd*. ,, 130 ,,

Length between knife-edges, 25 cm. Mean area of cross-section of wire = ·00657 sq. cm. Temperature 15° C.

Resistance of standard = ·01 ohm. Hence

$$\text{Resistance of length of wire between knife-edges } \frac{\cdot 01}{201} \times 130 = \cdot 00647.$$

$$\text{Specific resistance at } 15^\circ = \frac{\cdot 00647 \times \cdot 00657}{25}$$

$$= 1{\cdot}700 \times 10^{-6} \text{ ohms per cubic centimètre.}$$

Lesson XXXI.—The Method of Auxiliary Conductors.

68. *Exercise.*—To compare the resistances of two copper wires of different thickness.

Apparatus.—A low resistance galvanometer, a Grove's cell. The wires whose resistances are to be compared must be soldered together, stretched on a board, and provided with binding screws at the ends. Two knife-edge contact makers are necessary, one may have both contacts fixed, the other should have one contact movable. Two resistance coils of two or three ohms will be required. They should be fitted in a box, as shown in Fig. 98, where *abc* is a resistance coil bisected at *b*, and *def* a second coil bisected at *e*. The ends and points of bisection are connected with binding screws. The coils

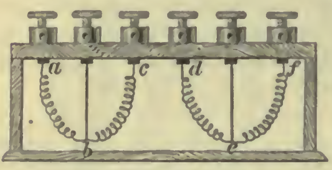


Fig. 98.—Bisected Coils.

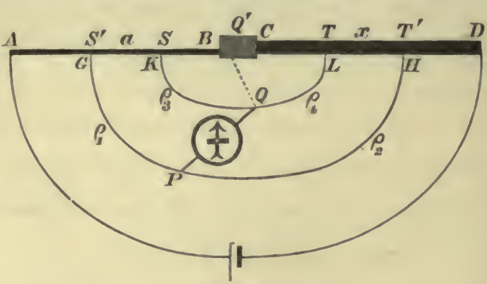


Fig. 99.

must be wound so as to be free from self-induction; for sake of clearness this is not shown in the figure.

Theory of the Method.—Let the conductors AB and CD

(Fig. 99) to be compared be united together by a connection of low resistance, and have a current passing through them, and let the ends of two other conductors be applied as in the figure. These auxiliary conductors are KQL and GPH, and the latter is called the *primary* and the former the *secondary* auxiliary conductor. Let the resistance of $SS' = a$ and that of $TT' = x$. At the points P and Q the terminals of a galvanometer are connected. The resistances of the several parts we shall call as follows: resistance of $GPH = r_1$, of $KQL = r_2$, of $GP = \rho_1$, of $PH = \rho_2$, of $KQ = \rho_3$, of $QL = \rho_4$; whence $r_1 = \rho_1 + \rho_2$ and $r_2 = \rho_3 + \rho_4$. We shall suppose that the resistances of the contacts at S′G and HT′ are negligible compared with ρ_1 and ρ_2, and those of the contacts at SK and TL negligible compared with ρ_3 and ρ_4. Let $SBCT = C$. The whole resistance c in the double channel SBCT and SQT will be

$$c = \frac{1}{\frac{1}{C} + \frac{1}{r_2}} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The combined resistance from S′ to T′ (not including the circuit S′PT′) will therefore be

$$a + c + x \quad . \quad . \quad . \quad . \quad . \quad (2)$$

This whole resistance may be supposed to be divided into two parts by a line passing from Q to a point in S′BCT′, which is at the same potential as Q. Let Q′ be such an imaginary point. Call the resistance $SQ' = \alpha$ and $Q'T = \beta$. The resistance on the left of QQ′ is therefore

$$a + \frac{1}{\frac{1}{\rho_3} + \frac{1}{\alpha}} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

But by the principle of Wheatstone's bridge

$$\frac{\rho_4}{\rho_3} = \frac{\beta}{\alpha} \text{ or } \frac{\rho_4 + \rho_3}{\rho_3} = \frac{\alpha + \beta}{\alpha}.$$

But $a + \beta = C$ and $\rho_4 + \rho_3 = r_2$, hence

$$\frac{1}{a} = \frac{r_2}{C\rho_3} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

therefore the resistance to the left of QQ′ (as far as S′) is, substituting the value of (4) in (3), and putting C in terms of c by the aid of (1)

$$a + \frac{1}{\frac{1}{\rho_3} + \frac{r_2}{C\rho_3}} = a + \frac{\rho_3 c}{r_2} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

In like manner the resistance to the right of QQ′ is

$$x + \frac{\rho_4 c}{r_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

So far we have been considering the resistance of the inner joint circuit, and have taken no account of the primary circuit. Now let us suppose that

The Potential at S′ is assumed for convenience to be zero,
,, T′ ,, V_1,
,, Q ,, V_2,
,, P ,, V_3,

then, from the principles regulating the fall of potential, we have

$$V_2 = V_1 \frac{a + \frac{\rho_3 c}{r_2}}{a + c + x},$$

also

$$V_3 = V_1 \frac{\rho_1}{r_1}.$$

Hence by subtraction

$$V_2 - V_3 = V_1 \frac{\frac{a(r_1 - \rho_1)}{r_1} - \frac{x\rho_1}{r_1} + c\left(\frac{\rho_3}{r_2} - \frac{\rho_1}{r_1}\right)}{a + c + x} \quad . \quad . \quad . \quad (7)$$

Now let the position of the galvanometer electrodes be varied until $V_2 - V_3 = 0$, that is, until there is no deflection, then from (7) we obtain

$$x = a\frac{(r_1 - \rho_1)}{\rho_1} + c\left(\frac{\rho_3 r_1}{\rho_1 r_2} - 1\right) \quad . \quad . \quad . \quad (8)$$

This last expression will be simplified by arranging the values so that the second term vanishes. We shall then have

$$x = a\frac{(r_1 - \rho_1)}{\rho_1} = a\frac{\rho_2}{\rho_1} \text{ or } \frac{x}{a} = \frac{\rho_2}{\rho_1} \quad . \quad . \quad . \quad (9)$$

which is equivalent to saying that the required ratio between the two resistances is equal to that into which the primary conductor is divided.

To ensure the disappearance of the second term of (8) we must have either

$$c = 0 \quad . \quad . \quad . \quad . \quad . \quad (10)$$

or

$$\frac{\rho_3 r_1}{\rho_1 r_2} = 1 \quad . \quad . \quad . \quad . \quad . \quad (11)$$

or both of these conditions combined.

Now the expression (11) states the condition that the secondary conductor must be divided in the same ratio as the primary, for we have

$$\frac{\rho_3}{\rho_1} = \frac{r_2}{r_1} = \frac{\rho_3 + \rho_4}{\rho_1 + \rho_2} \text{ or } \frac{\rho_2}{\rho_1} = \frac{\rho_4}{\rho_3} \quad . \quad . \quad . \quad (12)$$

In order, then, that we may employ the simplified formula (9) in the usual arrangement of the experiment, where c is not $= 0$, it is necessary that the two auxiliary conductors should be divided in the same ratio, and it then follows from (9) that this ratio is likewise that between the resistances under comparison.

Practice of the Method.—The requisite connections are shown in Fig. 100. The auxiliary conductors are seen at r_1 and r_2, their points of bisection being connected with the galvanometer, which is provided with the shunt S. The battery, being a cell of low resistance, must be connected with the commutator K_1. The distance between T and T′ must be adjusted until no deflection is produced in the unshunted galvanometer. The same precautions must be taken in this method as in the previous ones,

namely—(1.) The battery circuit must remain complete only so long as is necessary for observing a deflection of the galvanometer. (2.) Precautions must be taken to eliminate the effects of thermo-currents, and to avoid

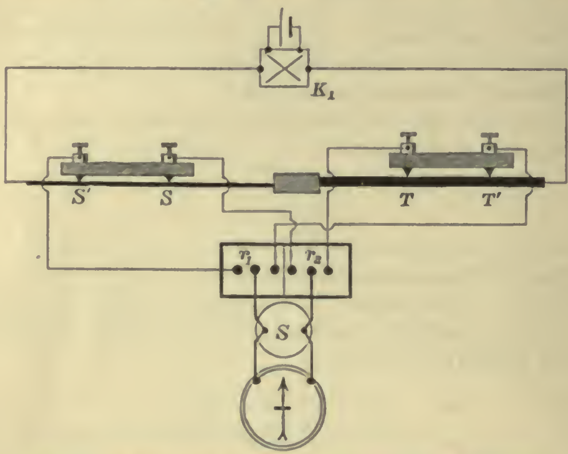


Fig. 100.—CONNECTIONS FOR METHOD OF AUXILIARY CONDUCTORS.

false readings due to self-induction. (3.) The galvanometer must be sufficiently distant from the rest of the testing apparatus not to be sensibly influenced by the latter.

Example.—The ratio of the arms of r_1 and r_2 was measured, the connecting wires being included in the measurement. It was found that

$$\frac{\rho_1}{\rho_2}=\frac{1{\cdot}7255}{1{\cdot}7455}={\cdot}9885,$$

$$\frac{\rho_3}{\rho_4}=\frac{1{\cdot}1445}{1{\cdot}1515}={\cdot}9939.$$

Now since C, and therefore c, is very low, the condition $\frac{\rho_1}{\rho_2}=\frac{\rho_3}{\rho_4}$

is sufficiently nearly fulfilled to enable us to assume, according to equation (9), that

$$\frac{a}{x} = \cdot 9885.$$

The diameter of SS′ was found on an average to be ·104 inch, and its fixed length was 100 mm.; the average diameter of TT′ was ·048 inch, while its length was varied until a balance was obtained—this occurred when TT′ = 23 mm. Hence calling K_1 the specific resistance of SS′ and K_2 that of TT′, we have

$$\frac{\text{Resistance of SS}'}{\text{Resistance of TT}'} = \frac{\frac{100}{(\cdot 104)^2}K_1}{\frac{23}{(\cdot 048)^2}K_2} = \cdot 9266\,\frac{K_1}{K_2},$$

hence

$$\frac{K_1}{K_2} = \frac{\cdot 9885}{\cdot 9266} = 1\cdot 067.$$

Lesson XXXII.—Projection of Equi-Potentials.

69. *Exercise.*—To compare the resistance of two copper wires.

Apparatus.—A German silver wire about a mètre long, mounted on a board and provided with a millimètre scale (a Wheatstone's bridge will answer), a Grove's cell, a low resistance galvanometer, shunt keys, two knife-edge connectors. The wires to be tested must be soldered together, stretched straight on a board, and provided with binding screws.

Theory of the Method.—Suppose that AC (Fig. 101) represents the two conductors connected together, and that it is wished to compare the resistance of the portion SS′ of the one with the portion TT′ of the other. Let the ends A and C be connected with a battery, which is also in connection with a graduated wire DE. Place one terminal of a galvanometer at S and move the other terminal along DE

until a point s is found which is at the same potential as S. Transfer the galvanometer terminal from S to S′, T, and T′, and find corresponding equi-potential points s', t, and t'. Our object will be to show that

$$\frac{\text{Resistance of SS}'}{\text{Resistance of TT}'}=\frac{ss'}{tt'}.$$

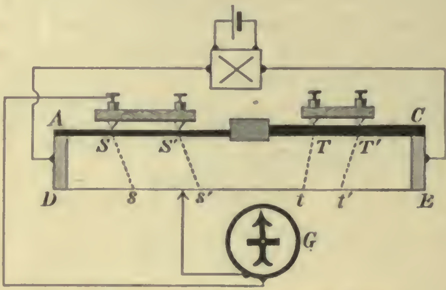


Fig. 101.—MATTHIESSEN AND HOCKIN'S METHOD.

This will be best seen by aid of the graphical method. Let OR (Fig. 102) be the resistance from A to C, and OR′

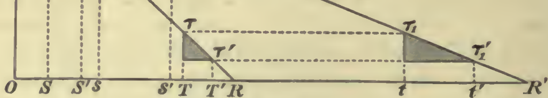


Fig. 102.

the resistance from D to E. Let OV represent the potential at A or D, and suppose, for the sake of simplicity, that the potential at C or E is zero. The rate of

fall of potential in the two branches AC and DE will be represented by the slope or rate of fall of the lines VR and VR′ respectively. Represent the resistances of AS, AS′, etc., by OS, OS′, etc. The potentials at S, S′, etc., will be σS, σ'S′, etc. Now we wish to find in VR′ the points which have equal potentials with these. For this purpose draw the parallels $\sigma\sigma_1$, $\sigma'\sigma_1'$, $\tau\tau_1$, $\tau'\tau_1'$ so as to project these potentials on VR′, and where the parallels meet VR′ drop perpendiculars on OR′, then s, s', t, t' will indicate the points in DE, of which the potentials are equal respectively to those of S, S′, T, T′. Now by a simple inspection of the diagram we see that

$$\frac{\text{Fall of potential between S and S}'}{\text{Fall of potential between T and T}'}=\frac{\text{Resistance SS}'}{\text{Resistance TT}'},$$

also

$$\frac{\text{Fall of potential between } s \text{ and } s'}{\text{Fall of potential between } t \text{ and } t'}=\frac{\text{Resistance } ss'}{\text{Resistance } tt'}.$$

But by construction the fall of potential between S and S′ is equal to that between s and s', and the fall of potential between T and T′ equal to that between t and t'. Hence it follows that

$$\frac{\text{Resistance SS}'}{\text{Resistance TT}'}=\frac{\text{Resistance } ss'}{\text{Resistance } tt'}.$$

Now if the wire DE be well drawn, the ratio of the resistance of ss' to that of tt' will be indicated by the readings on the scale. Since, however, these conditions cannot be assumed where accurate work is required, a calibration of the wire will be necessary.

Practice of the Method.—The student, who has already been instructed in the use of the slide Wheatstone's bridge, will understand the necessary manipulations. He may, however, be reminded of several essential precautions :—

(1.) The galvanometer must be sufficiently far removed

from the rest of the testing apparatus to remain unaffected by the currents in the latter.

(2.) The battery current must only be completed when absolutely necessary, so as to avoid all heating effects as much as possible.

(3.) The usual precautions must be taken to avoid false readings due to thermo-currents.

Example.—

Reading.	Position on Graduated Wire.	Position on Wires under test.
(1)	56·3	S
(2)	133·8	S′
(3)	552	T
(4)	904·7	T′

Hence difference of readings for S and S′ = 77·5. Do. for T and T′ = 352·7, and hence

$$\frac{\text{Resistance of SS}'}{\text{Resistance of TT}'} = \frac{77{\cdot}5}{352{\cdot}7}\,\frac{\rho}{\rho'} = {\cdot}2197\,\frac{\rho}{\rho'},$$

where ρ, ρ' represent the mean values of the resistance of one division of the scale between the readings (1) and (2), and of one division between the readings (3) and (4).

The length of SS′, also of TT′, was 100 mm. The average diameter of SS′ was ·104 inch, and of TT′ ·048 inch. We have therefore, calling K_1 and K_2 the specific resistance of the wires,

$$\frac{\text{Resistance of SS}'}{\text{Resistance of TT}'} = \frac{K_1\dfrac{100}{({\cdot}104)^2}}{K_2\dfrac{100}{({\cdot}048)^2}} = \frac{K_1}{K_2}{\cdot}213,$$

hence we have

$$\frac{K_1}{K_2} = 1{\cdot}0314\,\frac{\rho}{\rho'}.$$

A second experiment with a different length gave

$$\frac{K_1}{K_2} = 1{\cdot}0370\,\frac{\rho_1}{\rho'_1}.$$

The graduated wire had, however, been previously calibrated, and it was found that

$$\frac{\rho}{\rho'} = \frac{90\cdot3}{93} \text{ and } \frac{\rho_1}{\rho'_1} = \frac{90\cdot1}{93}.$$

Making the correction indicated, we finally find $\frac{K_1}{K_2} = 1\cdot0015$ from the first and $1\cdot0047$ from the second experiment.

70. *Comparison of the Three Methods.*—We see that Methods II. and III. are null or zero methods, and require a low resistance galvanometer, whilst Method I. is a deflection method requiring a high resistance galvanometer. Method III. requires a calibrated wire, and Method II. an adjustable knife-edge contact, unless we substitute for the bisected coils two calibrated wires, which would add much to the complication of the method. It will thus be seen that the choice of one or other method will depend upon whether a zero or a deflection method is preferred, and also upon what apparatus is at the disposal of the experimenter.

Part V.—Measurement of High Resistances.

71. The Wheatstone bridge not being adapted to measure resistances greater than one megohm, it is customary to make use of a direct deflection method when the resistance exceeds this limit. This is especially applicable for finding the *insulation resistance* of wire, etc., that is to say, the resistance of the insulator.

Lesson XXXIII.—The Direct Deflection Method.

72. *Exercise.*—To find the resistance of some telegraphic insulators.

Apparatus.—A number of various specimens of insulators made of brown earthenware, porcelain, and ebonite.

A trough for testing them having a sheet of zinc at the bottom. A battery possessing an electromotive force of about 100 volts. A high resistance galvanometer with shunts. A box of coils and a megohm resistance. This last may be made after the manner of Hittorf by using amylic alcohol containing a 10 per cent solution of cadmium iodide.[1] This is contained in a glass tube provided with two electrodes of cadmium. Such an arrangement is found to be a good substitute for the very expensive German silver megohm. Fig. 103 shows the resistance suitably mounted.

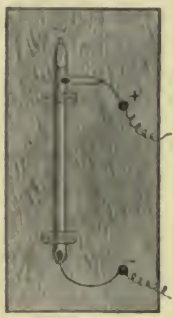


Fig. 103.
HITTORF'S HIGH RESISTANCE.

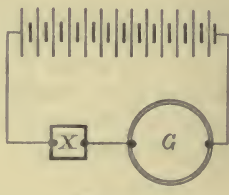


Fig. 104.

Theory of the Method.—Arrange in series (Fig. 104) a battery B of many cells, a delicate high resistance galvanometer G, and the very high resistance X, whose value we wish to determine. Let E denote the electromotive force of the battery, and suppose that a deflection of D_1 divisions is obtained, then

$$\frac{E}{X+G+B}=kD_1 \quad . \quad . \quad . \quad . \quad (1)$$

where k is the constant which converts the readings of the

[1] See Appendix D. F 9.

galvanometer into units of current. Now let us substitute for X a very high resistance R (Fig. 105), whose value is

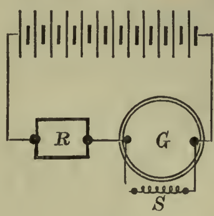


Fig. 105.

known. We shall suppose that in order to obtain a readable deflection from the galvanometer it will be necessary to use a shunt S. Hence

$$\frac{S}{G+S} \cdot \frac{E}{R+\frac{GS}{G+S}+B} = kD_2 \quad . \quad . \quad . \quad (2)$$

D_2 being the new deflection.

From (1) and (2), eliminating E and k by division, and solving for X, we obtain

$$X = \frac{D_2}{D_1}\left\{\frac{G+S}{S}(R+B)+G\right\} - (G+B) \quad . \quad . \quad (3)$$

Usually G + B is small compared with X. When this is the case we may write

$$X = \frac{D_2}{D_1}\left\{\frac{G+S}{S}(R+B)+G\right\} \quad . \quad . \quad . \quad (4)$$

In practice (3) may be further simplified by making $\frac{G+S}{S} = 1000$ (*i.e.* using the $\frac{1}{1000}$ shunt). Now let R = 1,000,000, which is so high that B may be neglected in comparison with it.

Further, by regulating the directing magnet of the galvanometer, and altering the number of cells in the

battery, D_2 may be made = 100. Hence, neglecting G, (4) will become

$$X = \frac{100}{D_1}\left\{1,000,000,000\right\} = \frac{100,000}{D_1} \text{ megohms} \quad . \quad . \quad (5)$$

This last formula is sufficiently accurate and very convenient for many practical purposes.

Practice of the Method—Treatment of the Insulators.—The value of this will depend greatly upon the exact method of preparation of the insulators, and on the exact method of applying the test. We shall chiefly adopt the *Instructions for Testing Insulators* followed in the Indian telegraphic service.[1]

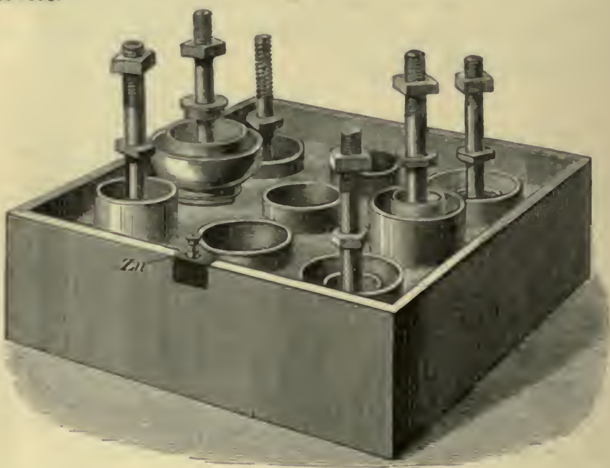


Fig. 106.—Insulators Arranged for Testing.

(1.) The insulators should be placed inverted in the box lined with zinc (Fig. 106). The box must contain

[1] See *Instructions for Testing Telegraphic Lines*, by L. Schwendler, vol. i. Appendix xi.

water to a depth of not more than 2 inches. Water is also poured into the cups in sufficient quantity to reach within an inch of the rims. The insulators must be soaked in water for twenty-four hours before testing.

(2.) The test should not be made on a damp day. The rims should be quite dry. It is advisable to hold a hot iron over them before making a test.

(3.) One pole of the battery being connected with the zinc lining of the boxes, the other pole is joined to one terminal of the galvanometer. The other terminal of the galvanometer is then to be joined to one end of an insulated wire (Fig. 107). Gutta-percha covered wire should

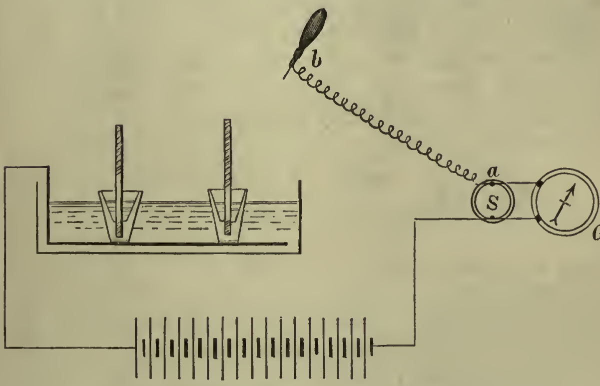


Fig. 107.—Scheme of Method of Testing Insulators.

be used for this purpose. There should now be no deflection on the galvanometer if the wire *ab* is perfectly insulated. To ensure this the wire had better not come in contact with the ground or wall of the room. It should be an *air line* as much as possible, and only supported, if necessary, by ebonite penholders. The insulation of *ab* should be tested at the commencement of each set of observations.

(4). The insulation of the leading wire being thus rendered certain, touch with the free end of this wire (supported by an ebonite penholder) the stalks of the insulators which are being tested, the galvanometer being shunted with the $\frac{1}{999}$ shunt. In cases where there is no reading the insulation has stood the test. Such cases should then be tried with the $\frac{1}{99}$ shunt; then when this test has been stood, with the $\frac{1}{9}$ shunt, and finally without any shunt whatever.

(5.) Those insulators showing a high insulation should be finally tested by connecting all the stalks together with bare copper wire, the deflection being observed both with the positive and negative current. From this the average resistance of each insulator should be ascertained. Before the final test with reversed currents the insulators must be thoroughly discharged, for the high electromotive force used is sufficient to electrify the insulators.

Example.—

$$R=1{,}000{,}000, \quad B=400, \quad G=5000.$$
$$\frac{G+S}{S}=1000, \qquad D_2=100, \quad D_1=50.$$

Then, by formula (3),

$$\begin{aligned} X &= \frac{D_2}{D_1}\left\{\frac{G+S}{S}(R+B)+G\right\}-(G+B) \\ &= 2[1000(1{,}000{,}000+400)+5000]-5400 \\ &= 2{,}000{,}804{,}600. \end{aligned}$$

By formula (4)

$$X=\frac{D_2}{D_1}\left[\frac{G+S}{S}(R+B)+G\right]=2{,}000{,}810{,}000.$$

By formula (5)

$$X=\frac{100{,}000}{50}=2000 \text{ megohms.}$$

This value, 2000 megohms, is the minimum insulation resistance allowed by the Indian telegraph service.

73. *Additional Exercises.*—The method of this lesson has several other interesting applications, of which examples will now be given.

(1.) A number of wooden reels, each of the same size, were wound each with two separate layers of covered copper wire of the same thickness, but insulated differently. The insulation resistance between the two layers was measured, in order to compare the value of the different protecting coatings. A resistance = 1·0785 megohm and a shunt were employed with the following result:—

Coating.	Shunt.	Deflection.	Resistance in Megohms.
Double silk . . .	∞	82	1275
Double silk soaked in paraffin .	∞	2	5227
Single cotton . .	$\frac{1}{999}$	250	·4
Double cotton . .	$\frac{1}{999}$	140	·7
Single cotton soaked in paraffin .	$\frac{1}{99}$	45	23

(2.) The insulation resistances of two condensers were found to be—

Condenser insulated with paper soaked in paraffin .	1058 megohms.
,, ,, mica	5292 ,,

(3.) It was desired to ascertain the faulty places in a coil of gutta-percha covered wire, and to determine the insulation resistance per yard. The wire was placed coiled in a zinc trough (Fig. 108), filled with water; one end of the wire *a* being raised above the water, it was dried and covered with paraffin to the length of 2 inches—this end was connected with the galvanometer. The other end was also raised out of the water, dried, and insulated. Now if there should be any faults in the covering of the wire the circuit would be complete on touching the trough with the end *b* of the battery wire, in which case the galvanometer (which ought to have a shunt) would be strongly affected. If, however, the deflection be small the insulation resistance of the whole wire should be measured (*b* being now con-

nected with a binding screw T fixed to the trough) by the method of the above lesson. To find the insulation resistance per yard it is only necessary to *multiply the observed total resistance by the number of yards.* The reason for this is obvious if we reflect that the leakage or current through two yards

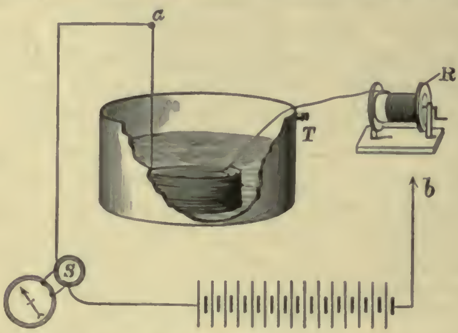


Fig. 108.—TESTING INSULATION OF WIRE.

is double that through one. When the wire is faulty it will be necessary to wind the wire from the coil on to a reel R, the end b being held in a wet cloth, which is also folded round the wire under test. As the wire is drawn through the cloth the galvanometer is watched. When a deflection occurs a faulty place is passing through the cloth.

PART VI.—RESISTANCE OF BATTERIES AND ELECTROLYTES.

74. The accurate measurement of the resistance of batteries and electrolytes demands special methods adapted to avoid the influence of back electromotive force or **Polarisation.** The measurement of a battery or electrolyte resistance is further complicated by variation in the resistance of the liquids brought about by electrolytic transfer-

ence. This source of disturbance has been called **Transition Resistance.**

It is therefore clear that no definite meaning can be attached to the term battery resistance, for since the polarisation and transition resistance depend upon the passage of the current, the internal resistance will be some complicated function of the current. The value of the battery resistance deduced by methods depending upon Ohm's law have therefore in many cases but little value. Thus, suppose that with one current we have

$$C=\frac{E-p}{B+R\pm t} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where p is the polarisation and t the transition resistance. If the external resistance is increased so as to decrease the current, the new condition of affairs will be represented thus—

$$C'=\frac{E-p'}{B+R_1\pm t'} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

From these two equations no information can be obtained regarding B unless something is known about the values of p, p', t, t'. The ordinary methods of measurement assume that the polarisation and transition resistance are constant during the experiment. Now the higher the value of the external resistance the less is the change of p and t, so that by making the external resistance sufficiently high and experimenting rapidly, sufficiently consistent results might be obtained, but the number so obtained would be very different with stronger currents. It is, however, in practice a particular *working resistance* which is desired rather than that portion of it which is independent of the disturbances due to the passage of the current.

The determination of the resistance of electrolytes presents difficulties very similar to those experienced with batteries, for it will be necessary to provide the electrolyte with metallic electrodes of negligible resistance in order to

find the intervening liquid resistance. At the electrodes polarisation will take place, giving rise to the same uncertainty as in the case of the battery.

To enable the student better to appreciate the nature of polarisation, the next lesson will be devoted to experiments on the subject.

Lesson XXXIV.—Study of Polarisation.

75. *Apparatus.*—A very dead-beat galvanometer. The best form is that of Deprez and D'Arsonval,[1] inasmuch as it is *aperiodic.*[2] A strong horse-shoe magnet AA (Fig. 109) is fixed vertically on a base-board provided with levelling screws. Between the poles of the magnet is a light rectangle C, consisting of many coils of insulated wire. This coil is held by two silver wires HJ and DE, the upper wire is attached at H to the end of a screw, which may be raised and lowered, while the lower wire is fixed to a tongue of metal FG, provided with an adjustable screw at G. These wires prevent the coil from taking a swinging motion, and resist by their torsion a movement in azimuth. The ends of the wires are

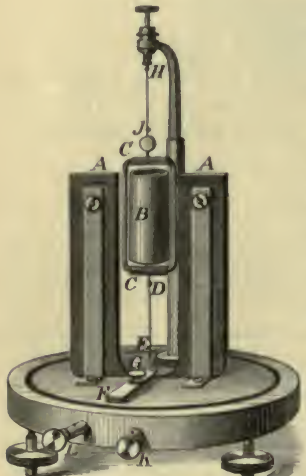


Fig. 109.—Galvanometer of Deprez and D'Arsonval.

[1] See *Comptes Rendus* 94, p. 1347 (1882).

[2] By *aperiodic* is meant having no reference to time of vibration, that is to say, the movable portion of the instrument takes up immediately without oscillation its new position. The dead-beat character of this instrument is due to induced currents (see Chap. VII.)

in connection with the binding screws K and L. When a current is passed through the coil the magnetic moment produced is resisted by the torsion of the wires, and hence the coil takes up a position of equilibrium. Now, since the coil is in the strong magnetic field of the horse-shoe magnet, which is intensified in the region of the coil by having a cylinder of soft iron B placed inside the rectangle (supported free from the coil), it will be brought to rest with great rapidity. A mirror J is attached to the coil, by which the readings may be observed in the usual manner.

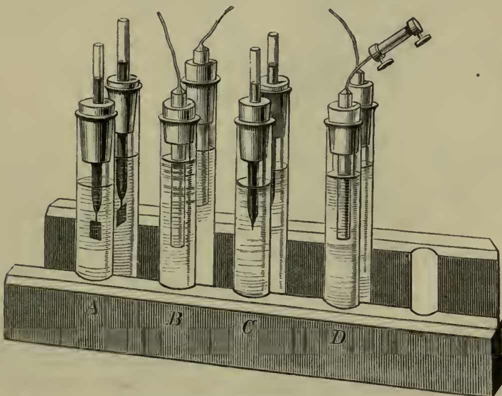


Fig. 110.—U Tubes for Polarisation Experiments.

The electrolytes and electrodes whose polarisation is to be studied should be placed in U tubes arranged in a stand (see Fig. 110). The most interesting cases will be (1.) platinum points in dilute sulphuric acid; (2.) platinum foil in dilute sulphuric acid; (3.) copper in solution of copper sulphate; (4.) zinc in solution of zinc sulphate; (5.) amalgamated zinc in zinc sulphate. It is convenient to have the U tubes fitted with corks having holes, through which the electrodes, in the cases of the zinc and copper, pass.

These electrodes have connecting wires soldered to them. In the case of the platinum the electrodes are fused in glass tubes, which are filled with mercury for the purpose of making the connection. It will be necessary to have in addition a commutator, a *two-way* key, and several Daniell's cells.

Method.—We wish to be able first to arrange in series the battery, provided with a commutator, the galvanometer, and U tube, and then at any moment to leave out the

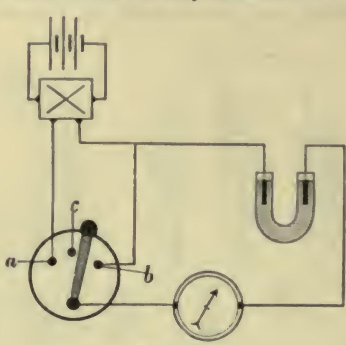


Fig. 111.

battery circuit. This is readily accomplished by a *two-way* key. The requisite connections are shown in Fig. 111. On the contact at a being made, that at b being broken, we obtain the first condition; and when b is made and a is broken the battery is put out of the circuit. The first will be spoken of as the *charge*, the second as the *discharge* position. When contact is made at c, both the U tube and battery are thrown out of circuit. The student should now make a series of experiments, such as are given below, with the key in the two positions.

Example of Experiments on Polarisation.—The Deprez and D'Arsonval galvanometer was first tested and found to be only absolutely dead-beat when the resistance of the outer circuit was below a certain limit. It was

therefore decided to always use it with a shunt, in order that the combined resistance external to the galvanometer might be always sufficiently low. Two Daniell's cells were used as the polarising battery. A high resistance was added to the external circuit, in comparison with which the battery resistance was negligible. Now, when the battery is suddenly switched into circuit with the polarising cell, there will be a sudden deflection; this deflection will, however, rapidly diminish, owing to the polarisation of the cell setting in, and will, when the polarisation is a maximum, reach its minimum value. If we now reverse the battery, so as to send the battery current in the same direction as the polarisation, the first value of the new and increased deflection will be a maximum one, which will ultimately fall, owing to the depolarisation and subsequent repolarisation in the opposite direction, once more to a minimum. Finally, if the battery be cut out of circuit, the deflection will be due to polarisation alone. The following results were thus obtained :—

	Pt Foil in H_2SO_4.		Pt Points in H_2SO_4.		Zn in $ZnSO_4$.		Cu in $CuSO_4$.	
	Deflection.	Time of Waiting.	Deflection.	Time of Waiting.	Deflection.	Time of Waiting.	Deflection.	Time of Waiting.
Battery in series with U tube	80	..	50	..	53	..	77	..
After waiting till constant	32·75	8′·5	3	2′	64	3′	79·6	3′
Reversed battery	149	..	25	..	66	..	78	..
After waiting till constant	33·2	18′·5	3·5	..	68	2′	78·5	2′
Battery cut out of circuit	58	..	17	..	..	..	..	..
After waiting till constant	..	4′	..	30″	..	..	..	..

Time observations were made of the rate of increase of polarisation and the rate of depolarisation. Experiments were also made upon the influence of varying strengths of current, and also upon the effect of shaking and warming the U tube.

76. *Electrolyte Resistance—List of Methods.*—The chief methods that will be given in the following lessons are :—

(1.) Reduced Deflection Method (Ohm).
(2.) Shunt Method (Thomson).
(3.) Horsford's Method.

(4.) Mance's Method.[1]
(5.) Kohlrausch and Nippoldt's Method.

Other methods will be described in the chapter on the condenser.

LESSON XXXV.—Electrolyte Resistance—Methods 1, 2, 3.

77. *Exercise.*—To find the resistance of a Daniell's cell by Ohm's and Thomson's methods; and to find the specific resistance of a solution of copper sulphate by Horsford's method.

Apparatus.—It is very convenient to be provided with a dead-beat mirror galvanometer, whose sensibility can be readily varied within wide limits without the use of shunts. Fig. 112 shows an instrument—a modification of a Wiedemann's galvanometer—suitable for this purpose. On a base-board two uprights are capable of sliding. These uprights support two reels, C and C′, each wound with a number of coils. These reels form two independent galvanometer coils, and the same current may be made to pass through the two reels in series or in opposition, or only one reel may be used. Between C and C′ there is a third upright, M, supporting (in addition to the circular framework and rod holding the directing magnet) a brass cell, containing a suspended magnet and mirror which may be levelled so as to swing freely within the confined space of the cell by the screws at the base. The deflection of the mirror is observed by means of the usual graduated scale, the light from the lamp passing through the central hole in one of the reels. Usually one of the reels may be dispensed with, the two being of service only when the instrument is used differentially. It will also be necessary to have a well

[1] *Pro. Royal Society*, January 19, 1871.

varnished wooden trough with two copper plates, one being fixed and the other movable. The trough should be

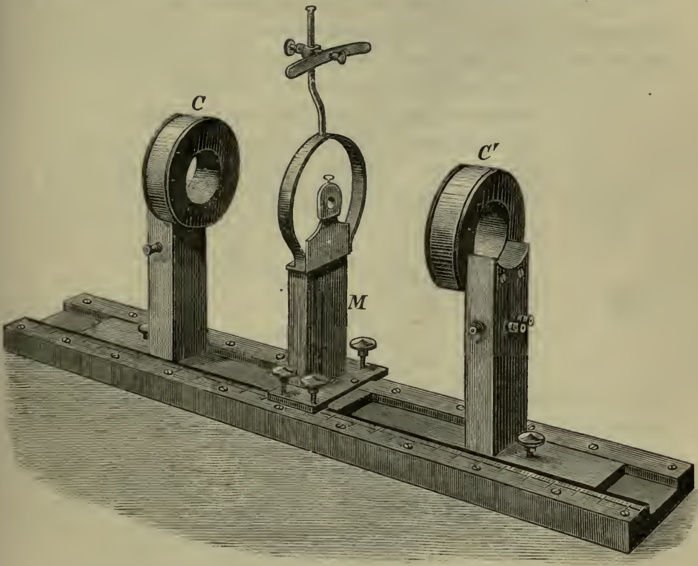


Fig. 112.—Modified Wiedemann's Galvanometer.

graduated along one of its top edges, as in Fig. 113. Boxes of coils and keys will be required.

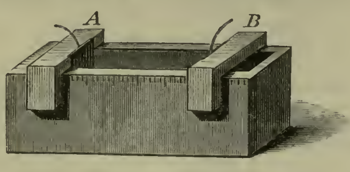


Fig. 113.—Cell for Horsford's Method.

Ohm's Method.—(1.) The galvanometer must be set up and adjusted. (2.) A circuit comprised of the battery,

galvanometer, and box of coils in series should be made. (3.) Slide the coil that is in use back until a readable deflection d_1 is obtained on the galvanometer scale, a resistance R_1 being included in the circuit. (4.) Increase the resistance to R_2, when the deflection will be reduced to d_2.

Let E = electromotive force of the battery.

C_1 = current when B, the battery resistance, G, the galvanometer resistance, and R_1 are in circuit.

C_2 = the current when the circuit includes the resistance R_2 instead of R_1.

Then

$$C_1 = \frac{E}{B+G+R_1}, \quad \text{and } C_2 = \frac{E}{B+G+R_2},$$

hence

$$\frac{C_1}{C_2} = \frac{B+G+R_2}{B+G+R_1},$$

but the deflections on the scale being taken to be proportional to the currents producing them, we have

$$\frac{d_1}{d_2} = \frac{B+G+R_2}{B+G+R_1},$$

from which

$$B = \frac{d_2R_2 - d_1R_1}{d_1 - d_2} - G.$$

Example.—

$$R_1 = 0, \quad d_1 = 200, \quad d_2 = 110, \quad R_2 = 10$$

(G and resistance of connecting wires negligible),

$$B = \frac{110 \times 10}{200 - 110} = \frac{1100}{90} = 12{\cdot}2.$$

Thomson's Method.—Here a deflection produced by an arrangement (in series) is reproduced by a shunt arrangement, the nature of which will be understood from the diagram (Fig. 114). A shunt S is placed across the galvanometer terminals when the plug key K_1 is closed, while in the main circuit, also provided with a plug key, there is a resistance box R. In order to make the test (1.) open

K_1 and close K_2, then slide the coil (of Fig. 112) until a large but readable deflection d_1 is obtained; (2.) close K_1 and observe the reduced deflection d_2; (3.) open K_1 and add a resistance R to the main circuit until the same deflection d_2 is reproduced; (4.) calculate the battery resistance B from the formula

$$B=\frac{RS}{G},$$

where G is the resistance of the galvanometer. It will be instructive to prove this formula graphically.

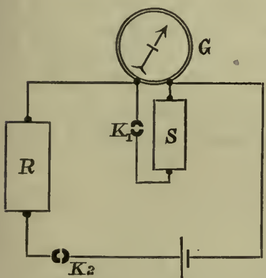


Fig. 114.

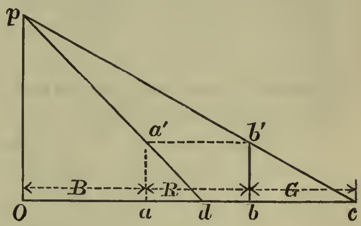


Fig. 115.

Let us represent the difference of potentials at the terminals of the battery by Op (Fig. 115), and the resistance of the battery by Oa, the resistance R by ab, and the resistance of the galvanometer by bc. Then bb' will be the difference of potentials at the galvanometer terminals which causes the deflection d_2. Now when the galvanometer is shunted the only resistance external to the battery will be the combined resistance of the galvanometer and the shunt, for at first we shall assume $R=0$. This, by the rule of combined resistances, will be

$$\frac{GS}{G+S}.$$

Make *ad* equal to this resistance, then the fall of potential under these new circumstances will be represented by the line *dp*, and the difference of potential causing the deflection d_2 will be *aa'*. But *aa'* must be equal to *bb'*, for the same current must be produced by the same difference of potential. Hence

$$B : \frac{GS}{G+S} = B+R : G,$$

or

$$B = \frac{RS}{G},$$

an expression not involving the galvanometer deflections; hence the test can be made without consideration of the kind of galvanometer in use; indeed, a simple galvanoscope would serve for the purpose. The test is often made by placing the shunt across the battery terminals instead of across the galvanometer terminals. This will not change the formulæ.

Example.—A set of 6 Daniell's were tested. Two boxes of coils adjustable to units were used—

$$d_1 = 4, \quad S = 15, \quad d_2 = 27{\cdot}5, \quad G = 12, \quad R = 70,$$

hence

$$B = \frac{70 \times 15}{12} = 87{\cdot}5.$$

Horsford's Method.—Place in the trough a solution of copper sulphate, and arrange in series the trough, with its plates some distance apart, a constant cell, a resistance box, and the galvanometer. Slide the coil of the latter back until a readable deflection is obtained, which must be accurately observed. Now bring the movable plate of the trough nearer the other, so as to include a smaller column of liquid between the electrodes. The deflection will be increased, but by inserting resistance R from the box it may be brought to be the same as before. When this is the case the inserted resistance R will be equal to the

resistance of the column of liquid included between the first and second positions of the movable electrode, hence, knowing the dimension of the trough, it will be quite easy to calculate the specific resistance of the electrolyte. Here the current being the same in the two measurements, Horsford assumes that the polarisation remains unchanged, but it must be remembered that the passage of the current may alter the state of the liquid in the neighbourhood of the electrodes, and thus possibly produce a difference in the polarisation and transition resistance. On this account the experiment should be expeditiously made. The method is interesting as being the best of the early attempts to measure accurately the resistance of an electrolyte.

Example.—One Grove cell was placed in series with the galvanometer coil of resistance ·35, and the cell containing the copper sulphate with the plates 13·5 cm. apart. When the coil was slid about 9 cm. from the suspended needle the deflection was 200 divisions. One plate was now moved to the position marked 5·1 cm. On adjusting the resistance, which could be readily done within ·1 ohm, it was found that 15 ohms were required to give the same deflection as before. The depth of liquid in the cell was 3·8 cm., and breadth of cell was 5·4 cm., hence—

$$\rho=\frac{15\times 3{\cdot}8\times 5{\cdot}4}{13{\cdot}5-5{\cdot}1}=36{\cdot}6 \text{ ohms per cm.}$$

The copper sulphate had a specific gravity of 1·12. The result obtained here was compared with that of Ewing and Macgregor (Jenkin, 3d edition, p. 259), and found to agree approximately with their result.

Lesson XXXVI.—Mance's Method.

78. *Exercise.*—To find the resistance of a sand battery.

Apparatus—A mirror galvanometer, box of coils, etc.

Theory of the Method.—In the Wheatstone diagram (Fig. 116), if the battery be placed in the X arm so as always to send a current through the galvanometer, then, by the principle of the bridge which we have already explained, when $\frac{P}{Q}=\frac{R}{X}$, the opening or closing of the key can have no influence on the current in the galvanometer, inasmuch as the two points A and D are at the same potential. This is the principle of Mance's method, in which adjustments are made of P, Q, and R until the current in the galvanometer remains the same, whether the key is open or closed (see Appendix).

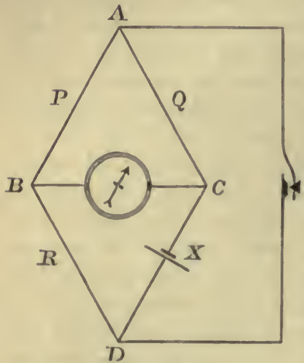


Fig. 116.

Practice of the Method.—Owing to the current that is always passing through the galvanometer branch it will be necessary to adopt some means of balancing this current. This is usually done by means of a directing magnet, but a better method in many cases is to use in addition to the magnet a differential galvanometer, through one of whose coils a compensating current may be sent from an independent battery, such as was used in the precisely similar case of p. 142.

Fig. 117 exhibits the necessary connections with a Post Office box of coils. Here the screws D and A′ should be joined by a short piece of wire. The battery to be tested is B_1, while the counteracting battery is B_2; the latter may be conveniently made by bending a long glass tube into U shape, and placing a layer of sand at the bend so as to form the porous partition of a Daniell's cell. By raising the zinc and copper more or less out of the weak solutions of zinc sulphate and copper sulphate contained in the limbs,

the strength of the battery may be varied considerably. The order of the operations in making the test will be seen from the following examples :—

Example I.—Four Minotto cells were tested. The arms P and Q were each made equal in resistance to 1000 ohms, while that of R was 200 ohms; K_3 was closed.

The galvanometer was shunted (shunt $\frac{1}{1000}$), then the key K_2 was closed. A deflection to the right quite off the scale was produced. In order to balance this the directing

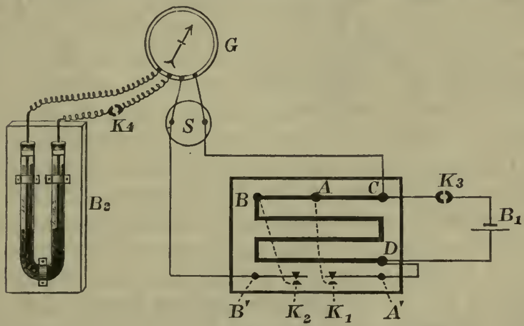


Fig. 117.

magnet was placed at the bottom of its rod and turned through about 45°, which was sufficient to bring the spot of light to the middle of the scale. Key K_1 was now pressed down, producing a deflection to the right. On making $R = 100$ it was found again necessary to alter the magnet to bring the light to the middle of the scale. The deflection was now to the left on closing K_1, showing that the resistance of the battery was between 100 and 200 ohms. Further tests showed that this lay between 120 and 125 ohms, but the arrangement was not sufficiently delicate to give a closer result. K_4 was then closed, it having been previously ascertained that B_2 gave a current in a direction

opposite to that of B_1, and the resistance of the battery was increased by raising the copper rod so as to be almost sufficient to balance the current without the aid of the directing magnet, which was accordingly placed higher in order to make the galvanometer more sensitive. The spot of light being made central by the help of the magnet, it was now found easy to obtain the resistance of the battery within a single ohm.

Example II.—A single Daniell's cell, whose resistance was known to be about 3 ohms, was introduced. An approximate balance was obtained with $P = 1000$, $Q = 10$, $R = 368$, or $X = 3·68$, but it was found difficult to obtain a true balance, owing to the polarisation of the battery. This gave rise to the following effect: On pressing down the key K_1 there was first a quick deflection to the right, but this was directly followed by a slow movement to the left. The former was taken to be the true deflection showing a want of balance, while the latter was presumed to be due to the polarisation and transition resistance of the battery. The true explanation of the cause of this evidently lies in the fact that when the key K_1 is closed the resistance external to the battery is considerably decreased, in consequence of which there will be a much greater current. Now we have already explained that the resistance of a battery is a function of the current passing through. A change of the current will therefore cause a change of resistance, which may not, however, take place immediately. By inserting a resistance in the battery circuit these effects may be diminished.

The best method of procedure, when the battery is polarising, is that indicated by Chrystal.[1] "Arrange the bridge until the deflection, owing to deviation from the balance, is opposite to that due to change in the E. M. F.; then by gradual adjustment work down the initial jerk to

[1] *Ency. Brit.*, Art. "Electricity," p. 50.

nothing, so that the needle appears to start off on its slow swing without any perceptible struggle. When this state of matters is reached there is a balance."

Example III.—On attempting to measure the resistance of a bichromate cell by Mance's method, the variation due to polarisation was so great as to make the measurements entirely without value. This method is therefore not suitable for the determination of the resistance of batteries that are not fairly constant, for the adjustment cannot be made with sufficient rapidity.

Example IV.—A copper sulphate rheostat was taken (Fig. 118), consisting of a wide tube provided with two copper plates, *a b*, soldered to copper rods, which pass through india-rubber corks at the ends of the tube; the plates could thus be placed at any distance apart. To avoid polarisation both plates were coated with copper by electro-deposition from the solution of copper sulphate with which the tube was filled, and the copper rods were enclosed in glass tubes. The details of the arrangement are shown in the small diagram of Fig. 118, exhibiting *a* or *b* enlarged, where to the copper plate *p* a shoulder *s* is soldered, being of the same diameter as the glass tube *g*; an india-rubber cork *c* connects the glass tube to the shoulder and prevents liquid from reaching the wire *w*. A paper scale was fastened to the tube. The tube was placed in circuit with a constant cell, whose resistance had been previously measured. The plates were set at various distances apart and the resistance measured.

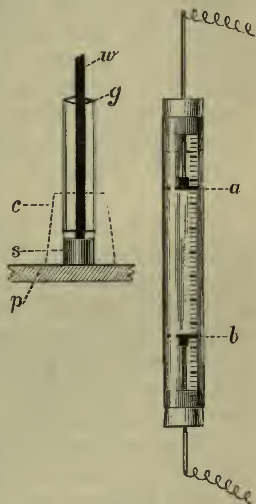


Fig. 118.

79. *Use of Alternate Currents.*—The problem of determining the resistance of electrolytes was only at length satisfactorily solved by Kohlrausch and Nippoldt, who used the alternating currents produced by an elementary form of dynamo. Let us here consider for a moment what happens when alternating currents are sent through a cell containing acidulated water and provided with two precisely similar platinum electrodes. The current going from left to right through the cell would, let us suppose, cause a liberation of oxygen at pole A and a liberation of hydrogen at pole B. Immediately afterwards it would be succeeded by an equal current passing from right to left, which would cause hydrogen to be liberated at A and oxygen at B. The polarising tendency of the gas produced in the first case would therefore be neutralised by that produced in the second, if we suppose that these gases continue to exist side by side. On the other hand, if they combine to form water, all polarisation will be eliminated. This destruction of polarisation will be aided (1) by platinising the electrodes, and (2) by increasing their size.

80. *The Electro-dynamometer.*—In experiments with alternate currents we cannot use an ordinary galvanometer, for the effect of the rapidly succeeding currents will be equal and opposite, and hence no deflection will be produced. We may, however, replace the magnet of the galvanometer by a small coil suspended by two fine wires which are in connection with the ends of the small coil. The instrument now becomes an electro-dynamometer, and we shall suppose that the same current circulates through the galvanometer and through the small coil. Now it will at once be seen that if we double the intensity of the current in the galvanometer we double likewise that in the small coil, so that the action of the one upon the other will be increased, not *twice*, but *fourfold*,—in other words, this action will vary as the square of the current. If,

therefore, θ denote the angle of deflection of the small coil, this angle being supposed small, and C be the current, we shall have

$$C=k\sqrt{\theta},$$

where k is a constant quantity. Now let the current be reversed in the galvanometer, then it must likewise be reversed in the suspended coil. The result will be that the deflection will remain in the same direction as before, so that if a series of reversals follow each other with sufficient rapidity, the deflection will be constant. If we have an electro-dynamometer, and a means of producing alternate currents, they can be directly applied to Mance's method.

81. *Use of the Telephone and Induction Coil.*—F. Kohlrausch[1] has made the method of alternate currents easier in application by using the telephone and the induction coil.

A telephone consists essentially of a magnet, with a coil of insulated wire wrapped round it. One of its poles is close to a diaphragm made of soft iron. Vibrations of the soft iron diaphragm to and from the coil will cause induction currents in that coil, and likewise variations in a current circulating in the coil will cause vibrations of the diaphragm, and these vibrations, if rapid enough, will constitute a sound. When alternating currents are passed through the coil a buzzing sound is heard from the telephone. Now when, by means of a zero arrangement, there are no alternating currents, there will of course be no sound, so that in such arrangements the telephone may be made a substitute for the electro-dynamometer.

The machines generally used for the production of alternating currents require some motive power to cause the armature to rotate at a high speed. This, however, may

[1] F. Kohlrausch, Wied. *Ann.*, 11, p. 653 (1880).

be altogether avoided by the use of the ordinary induction coil, which, by means of its automatic make and break arrangement, produces alternating currents of brief duration and high electromotive force by the use of a primary battery of a few cells. For an account of the construction of the induction coil ordinary text-books may be consulted. The practice of Kohlrausch's method will form the next lesson.

LESSON XXXVII.—Electrolytic Resistance by Alternating Currents.

82. *Exercise.*—To find the specific conductivity of a solution of copper sulphate and also the resistance of several cells by Kohlrausch's method.

Apparatus.—A slide bridge with its associated apparatus, a U tube or other receptacle for the copper sulphate, an induction coil with metallic contact breaker giving ¼ inch spark, telephones, batteries.

Method.—A Bunsen's or Grove's battery of one or two cells should be charged in order to work the induction coil. The induction coil should be in a neighbouring room, or outside the window, at any rate sufficiently far away to prevent the noise of the contact breaker being heard. It will be necessary to insulate very completely the wires coming from the induction coil which carry the secondary current, inasmuch as this current, having a very high E. M. F., will break through cotton and silk-covered wires, and even gutta-percha will be found to insulate it insufficiently. It will be best, therefore, to make the secondary wires air lines as much as possible, supporting them where necessary by rods of ebonite.

A scheme of the connections is shown in Fig. 119, where I is the induction coil, T the telephone, R a box of coils, and X the unknown electrolytic resistance (shown in a U tube enlarged at its ends to admit the copper electrodes).

The observation consists in moving the contact-piece and altering the resistance coils until either no sound can be heard in the telephone or the sound is reduced to a minimum. In order to hit the exact point some practice will be necessary. It will be of great assistance to place a second telephone in the circuit, and hold a telephone to each ear. Should this be done the contact-piece must be slowly moved by a second observer, who should note down

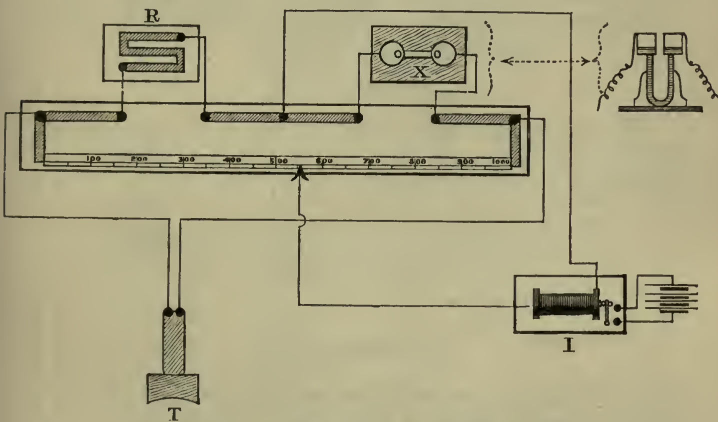


Fig. 119.—Connections for Kohlrausch's Method.

the readings of the bridge when the listener announces that a point of minimum sound has been reached. To prevent the latter from being biassed, he should be kept in ignorance of the position of the slides. It is necessary that the room should be free from noise when making the observations.

Example.—The U tube employed was about 2 cm. diameter, and with limbs 28 cm. long; its ends were passed through two large india-rubber corks which were fitted into two short cylinders of glass 2 inches in diameter and 3 inches high. The ends of the tube came flush with the

top of the india-rubber. In each cylinder was placed a copper disc (cut with a tag for connection), nearly of the same diameter as the cylinder.

By running distilled water into the U tube from a burette, the volume was found to be 183 cc. Next the internal diameter was measured by callipers, and found to be for one limb 10·35 mm., other limb 10·25 mm., hence the mean radius is 1·03 cm., and the section is $\pi \times (1{\cdot}03)^2$. These data will enable us to find the length of the tube, which could not readily be found otherwise. The length is—

$$\frac{183}{\pi \times (1{\cdot}03)^2} = 54{\cdot}91 \text{ cm.}$$

The electrolyte was formed by dissolving 20 grms. of $CuSO_4$ (not pure) in 200 cc. of distilled water. Its density at 15° C. was 1·0627.

With R = 800, balance found at 99·7, hence X = 804·81.
,, R = 600, ,, ,, 42·7, ,, X = 805·05.
Mean value of X = 804·93.

We have therefore—

$$\text{Specific resistance} = \text{Whole resistance} \times \frac{\text{Section}}{\text{Length}} = 48{\cdot}86 \text{ ohms per cm.}$$

With practice it was found that an alteration of $\frac{1}{2000}$ of length of slider made a difference in the sound of the telephone. Better results were obtained by using two 10-ohm coils at the end of the bridge to make the arms more nearly equal.

83. *Results of Kohlrausch's Experiments.*—These experiments were made with a convenient rheostat[1] consisting of an ebonite cylinder on which a screw had been cut. Within the trace of the screw was a wire of German silver. On turning the rheostat a movable contact will cause the wire to be divided in any desired ratio. The liquids to be experimented on were contained in small vessels with

[1] Similar to Wheatstone's second form, see p. 172.

platinised platinum electrodes. A, B, C, Fig. 120, show the chief kinds of those vessels. A and B are adapted for liquids of good conductivity, and C for liquids of very bad conductivity. In using these vessels their resistance

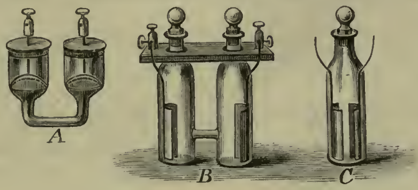


Fig. 120.—VESSELS FOR ELECTROLYTES.

capacity must first be ascertained by filling them with a solution of zinc sulphate of known specific resistance. The specific resistance of this zinc sulphate was ascertained by the use of *a straight* graduated tube of known cross-section.

The following table gives the electric conductivity (reciprocal of the resistance) in a few useful cases taken from Kohlrausch's experiments :—

TABLE H.

SPECIFIC CONDUCTIVITIES OF ELECTROLYTES.

Per cent of Solution.	Sodium Chloride.		Zinc Sulphate.		Copper Sulphate.		Sulphuric Acid.	
	Sp. Gravity at 18° C.	Conductivity at 10° C.	Sp. Gravity at 18° C.	Conductivity at 18° C.	Sp. Gravity at 18° C.	Conductivity at 18° C.	Sp. Gravity at 18° C.	Conductivity at 18° C.
5	1·0345	628	1·0509	179	1·0513	177	1·0331	1952
10	1·0707	1132	1·1069	301	1·1073	300	1·0673	3665
15	1·1087	1535	1·1675	389	1·1675	395	1·1036	5084
20	1·1477	1830	1·2323	439	...	...	1·1414	6108
25	1·1898	1996	1·3045	450	...	...	1·1807	6710

Here the scale is such that the conductivity of mercury is reckoned equal to 10^8 units.

Knowing that the specific resistance of mercury in C. G. S. units is 96146, the conductivities given above may be converted into specific resistances, as under :—

$$\text{Specific resistance of 10 per cent solution of copper sulphate} = \frac{96146 \times 10^8}{300} = 3{\cdot}205 \times 10^{10}.$$

84. *Additional Exercises.*—(1.) Determine the conductivity of an electrolyte at different temperatures. Plot a curve exhibiting the results. Determine the coefficients α and β in the formula—

$$k_t = k_0 (1 + \alpha t + \beta t^2),$$

where k_t is the conductivity at $t°$, and k_o that at 0°.

(2.) Determine the conductivity of different percentages of a salt dissolved in water, and plot a curve exhibiting the results.

(3.) All Kohlrausch's results are expressed in the form given in the above table. It would be useful to compile complete tables of the resistance of electrolytes in C. G. S. units. The following references taken from Wiedemann (*Elektricität, Erster Band*, 1882) will be useful for this purpose :—

F. Kohlrausch and Nippoldt, Pogg. Ann., **138**, p. 379 (1869); Grotrian for H_2SO_4 and HCl, Pogg. Ann., **151**, p. 378 (1874); F. Kohlrausch and Grotrian, HNO_3, Pogg. Ann., **154**, p. 1, 215 (1875); F. Kohlrausch, HCl, HBr, HI, H_2SO_4, H_3PO_4, $C_2H_2O_4$, $C_2H_4O_2$, $C_4H_6O_6$, Pogg. Ann., **159**, p. 233 (1876); F. Kohlrausch, hydrates and salts of alkali metals, $CuSO_4$, $ZnSO_4$, and $AgNO_2$, Wied. Ann., **6**, p. 145 (1879); F. Kohlrausch, temperature coefficients, Wied. Ann., **11**, p. 653 (1880).

85. *Compensation Methods.*—Methods, in which one battery is balanced against another, can be applied with

advantage to inconstant cells, inasmuch as it is only necessary to close the circuit momentarily, as in that of Beetz.

Lesson XXXVIII.—The Method of Beetz.

86. *Exercise.*—To find the resistance of a Leclanché cell.

Apparatus.—A slide-mètre bridge; two boxes of coils, each with 1, 2, 2 and 5 ohms; a double contact key; low resistance galvanometer; a Daniell's cell.

Theory of the Method.—Suppose the cell (electromotive force = E) whose resistance x is required has its poles connected with P and S, the – pole going to P (Fig. 121), so

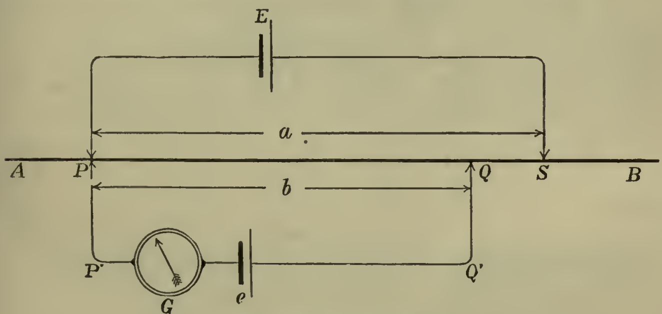


Fig. 121.—Method of Beetz.

as to include in circuit a portion of the resistance between A and B. Also let there be a second cell of electromotive force $= e$ (less than E), with a galvanometer G in its circuit, and let this cell have its – pole at P and its + pole at Q. The cells will thus be in *opposition.* Suppose, in the first place, that this is an open circuit, the extremities of which, P and Q, are gradually brought up to AB. Now, in virtue of the electric separation at e, all the points of the left-hand wire of this open circuit P′P will have a certain fixed

potential, and all the points of the right-hand wire Q′Q will have another fixed potential different from the former by the amount e. Now let us imagine that things are so arranged that when this open circuit is brought up to AB it will touch it at points which, owing to the upper current, have exactly the same potential as the wires P′P and Q′Q. It is clear that under these circumstances no change will take place in the electric condition of P′P, Q′Q, and of the whole under circuit; in other words, no current will pass, and the circuit may still be regarded as an open one.

Thus virtually the current of the upper circuit is not affected by bringing up to it this under circuit.

Under this condition of things the galvanometer of the under circuit will remain undeflected, there being no current in this circuit. Now let the resistance of $PS = a$, $PQ = b$; also let the current in the upper circuit be C. Hence, neglecting the resistance of the conducting wires,

$$C = \frac{E}{x+a} \qquad . \quad . \quad . \quad . \quad . \quad (1)$$

also

$$C = \frac{e}{b} \qquad . \quad . \quad . \quad . \quad . \quad (2)$$

From (1) and (2)

$$\frac{E}{e} = \frac{x+a}{b} \qquad . \quad . \quad . \quad . \quad . \quad (3)$$

If a fresh pair of values a' b' be found, which likewise leave the galvanometer undeflected, then

$$\frac{E}{e} = \frac{x+a'}{b'} \qquad . \quad . \quad . \quad . \quad . \quad (4)$$

hence from (3) and (4)

$$x = \frac{ab' - a'b}{b - b'} \qquad . \quad . \quad . \quad . \quad . \quad (5)$$

thus giving the resistance in terms of known quantities.

Practice of the Method.—In the working diagram (Fig. 122) MN is the slide-mètre bridge with the resistance boxes

R and R′ at its ends. The connections are made through the double key, arranged so as to prevent the circuit of *e* being closed when that of E is open, which would cause a deflection of the galvanometer due to the former battery only. When *a* is pressed against *b* the current of E is closed, and by still further pressing the key *b* is brought into contact with *c*, and the circuit of *e* is closed.

Should there now be a deflection, the key should be raised, the sliding contact must be moved, and a fresh experiment made. If no balance can be obtained it is because

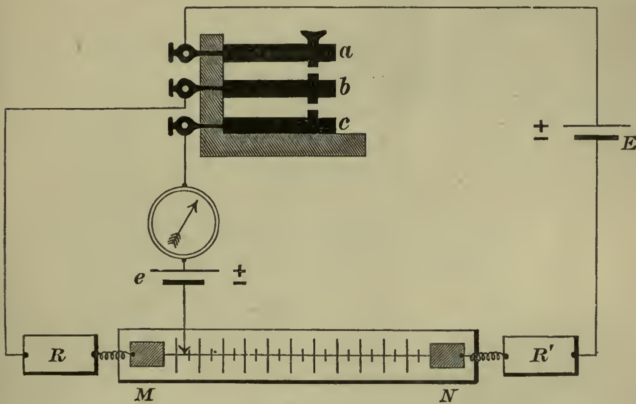


Fig. 122.—Method of Beetz.

we are neglecting the conditions demanded by equations (3) and (4). The resistance boxes at the ends are provided in order that it may be possible to fulfil these conditions.

Example.—A trial of the method was made by balancing two Groves against one Grove. A rheostat of ·56 ohm was used.

I.	R = 1	R′ = 0	Rheostat Reading =	·1076	a = 1·56	b = 1·1076.
II.	R = ·5	R′ = 0	,, ,,	·3367	a' = 1·06	b' = ·837.
III.	R = ·2	R′ = 0	,, ,,	·4876	a'' = ·76	b'' = ·688.

From I. and II. $x = \cdot 691$.
,, I. and III. $x = \cdot 650$.
,, II. and III. $x = \cdot 625$.

Assuming that $E = 2$ and $e = 1$, we find—

From I. $x = \cdot 655$.
,, II. $x = \cdot 614$.
,, III. $x = \cdot 616$.

It was noticed that the battery was changing its resistance during the test. A dead beat galvanometer should have been used, in order that the test might have been made rapidly.

CHAPTER V.

THE TANGENT GALVANOMETER.

87. WE shall prove in the chapter on electro-magnetism that the action of a current in a circular coil on a very small magnetic needle placed at its centre is expressed by the formula

$$C = \frac{Ha}{2n\pi} \tan \alpha \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where C = strength of the current.

H = horizontal intensity of the earth's magnetism.

a = radius of coil.

n = number of windings.

α = angle of deflection.

$\pi = 3{\cdot}1416$.

The quantity $\frac{a}{2n\pi}$ we shall call Γ, or the *true constant* of the instrument. Hence (1) becomes

$$C = H\Gamma \tan \alpha \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Now, for a particular locality, H is sensibly constant, hence it will be convenient to use one symbol for $H\Gamma$. Let us call it K, which may be known as the *working constant*. We may then for most purposes simply write

$$C = K \tan \alpha \quad . \quad . \quad . \quad . \quad . \quad (3)$$

In the lessons which immediately follow it will be

shown how the tangent galvanometer may be employed for measuring

(*a*) The internal resistance of batteries.
(*b*) The resistance of wires.
(*c*) The electromotive force of batteries.

Lesson XXXIX.—The Tangent Galvanometer—Battery Resistance.

88. *Exercise.*—To measure the internal resistance of several cells.

Apparatus.—A tangent galvanometer. Fig. 123 shows a pattern of the instrument which is well suited for most purposes. It consists of a wooden hoop A with flanges. Inside the hoop and fixed centrically at right angles to it is a compass box B with a glass lid. The hoop is supported by the base C, on which are the binding screws. On the top of the hoop is a directing magnet D, supported by a graduated standard, up and down which it may slide. (1.) *The hoop* is wound with several coils of about the following resistances, ·2, 1, 4, 10, 25, 50, and 1500 ohms. These seven coils are connected with binding screws after the manner exhibited in Fig. 124, in which the seven coils are represented by the loops A to G inclusive. The ends of A are brought to the binding screws 1 and 2, the ends of B to the binding screws 2 and 3, and so on. This arrangement will allow of the use of any number of the coils in

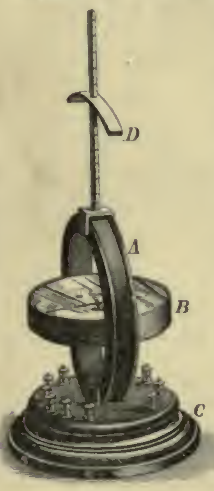


Fig. 123.
THE TANGENT GALVANOMETER.

series; thus, if screws 4 and 7 be used, the coils D, E, and F, with a total resistance of 85 ohms, will be in circuit. For the purpose of measuring strong currents an additional coil H, consisting of a single turn of copper ribbon, is provided, having its ends connected with two terminals 9 and 10 of larger size and separate from the others. (2.) *The compass box* is of wood. Into the centre of its bottom is screwed the pivot, consisting of a hard well-polished steel needle fixed vertically in a brass support. Covering the greater portion of the bottom of the box are two

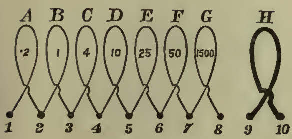


Fig. 124.
ARRANGEMENT OF COILS.

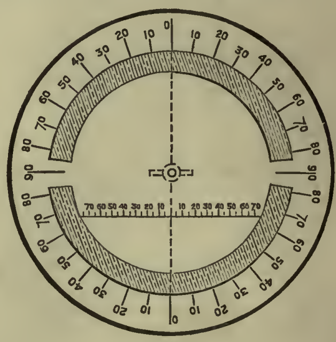


Fig. 125.
TANGENT GALVANOMETER SCALE.

pieces of mirror glass. On the top of the glass is fixed a scale of cardboard or metal, divided into degrees, but having two sectors through which the mirror glass is exposed. The method of graduating the circle will be perceived from Fig. 125. Here on one side of the circle is a linear scale stretching from about 70° on one side to 70° on the other. This serves the purpose of a tangent scale, the readings of that part of the needle pointer which crosses this scale being proportional to the tangent of the angles of deflection from the zero. The shaded portion is the mirror glass. (3.) The *needle* NS (Fig. 126) is of hardened

steel, 30 mm. long, 5 mm. broad, and 1 mm. thick. At its centre is a jewelled cap. At right angles to the needle there is a pointer *pp′* of fine brass wire, having its extremities filed to knife-edges. On the pointer are two vanes *a* and *a′* of aluminium foil for the purpose of helping to bring the needle to rest. Additional details relating to the construction of the instrument will be found in Appendix D.

Fig. 126.—Needle of Tangent Galvanometer.

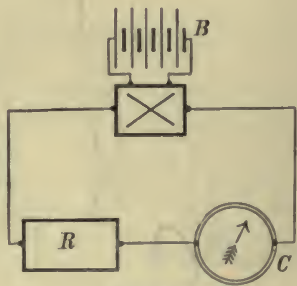


Fig. 127.

The other apparatus required will be a box of coils, a commutator, the cells to be tested, and connecting wires.

Theory of the Method.—Let the battery to be tested, along with its commutator, the galvanometer, and the box of resistance coils, be placed in series (see Fig. 127).

Let E = Electromotive force of the battery,
B = Resistance of battery,
G = Resistance of galvanometer and connecting wires,

Let R = Resistance of the coils,
α = Angle of deflection,
then, from Ohm's law, the current C passing through the galvanometer will be

$$C=\frac{E}{B+G+R} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

But

$$C=K \tan \alpha \quad . \quad . \quad . \quad . \quad . \quad (2)$$

by the theory of the tangent galvanometer. Hence

$$\frac{E}{B+G+R}=K \tan \alpha \quad . \quad . \quad . \quad . \quad (3)$$

Let now R be changed to R_1, causing α to be changed to α_1, then

$$\frac{E}{B+G+R_1}=K \tan \alpha_1 \quad . \quad . \quad . \quad . \quad (4)$$

Hence, dividing (3) by (4), we obtain

$$\frac{B+G+R_1}{B+G+R}=\frac{\tan \alpha}{\tan \alpha_1} \quad . \quad . \quad . \quad . \quad (5)$$

Hence

$$B=\frac{R \tan \alpha - R_1 \tan \alpha_1}{\tan \alpha_1 - \tan \alpha}-G \quad . \quad . \quad . \quad . \quad (6)$$

a general formula for the battery resistance.

The expression (6) may be simplified in approximate measurements by making R = 0, $\alpha_1 = 45°$, then

$$B=\frac{R_1}{\tan \alpha - 1}-G \quad . \quad . \quad . \quad . \quad (7)$$

or, still better, by making $\tan \alpha_1 = \frac{1}{2} \tan \alpha$, and then

$$B=R_1-(2R+G) \quad . \quad . \quad . \quad . \quad (8)$$

When this last formula is used the method is called the *half-deflection method*.

Practice of the Method—(1.) *The best values of α and α_1.*—Having completed the connection as figured above, and

placed the galvanometer in the magnetic meridian with the pointer at 0°, the first consideration will necessarily be which is the best coil of the galvanometer to use, and what is the best value of deflection to obtain. On consulting a table of tangents it will be found that the effect of making an error in the reading will be greater at the extremities of the scale than at the middle, thus

tan 10° = ·1763	tan 45° = 1·000	tan 80° = 5·671
11° = ·1944	46° = 1·0355	81° = 6·314
Difference ·0181	Difference ·0355	Difference 0·643
= 9·8 per cent	= 3·5 per cent.	= 10·7 per cent.

of the whole mean effect.

We see from these examples that both small and large deflections must be avoided.

If we consult our tables more minutely we shall find that the effect of making an error is actually smallest at 45°, a result which is likewise in accordance with theory.

In testing the resistance of a battery by the method of this lesson we shall require to observe two deflections. It follows as a corollary to the result just given that these ought to be at equal distances on either side of 45°, the best part of the scale being between 30° and 60°. Hence the first deflection should not be greater than 60°. Now the less external resistance we place in the circuit the greater will be the effect of the battery resistance on the whole current, hence if a coil of the galvanometer can be found which with $R = 0$ gives a not greater than 60°, that will be the best coil to take, provided that the battery continues constant.

(2.) *Adjustment of Coil and Needle.*—A suitable coil having been found, the student should proceed to test the galvanometer. Let him first take the deflection as given by both ends of the needle. In taking the reading the eye must be placed so that the pointer covers its image in the mirror; in this way error due to parallax may be

avoided. In the next place let him reverse the current, when the readings should be nearly the same. If this should not be the case the error may be due to three causes—

(1.) The coil may not be in the magnetic meridian.
(2.) The zero point of the scale may be wrongly placed.
(3.) The pointer may not be straight, and not at right angles to the needle.

To remedy this, take the needle off its pivot, and having previously ruled two lines on cardboard at right angles to one another, test by their means the straightness of the pointer, and its position with regard to the magnet. Straighten and bend the pointer if necessary. On replacing the needle after allowing it to come to rest, one may judge by the eye whether the plane of the coil is approximately at right angles to the pointer. We may now proceed to make a more accurate adjustment of the plane of the coil. Suppose, for instance, that the pointer now stands at $+5^\circ$, and that the reading is $+65^\circ$ when deflected by a current in one direction, and -52° when deflected by the same current in the opposite direction, that is to say, the actual deflections are $+60^\circ$ and -57°. If now the plane of the coil be turned through about $\frac{60^\circ - 57^\circ}{2} = 1^\circ{\cdot}5$, the real deflections on either side of the point of rest should be equal. Test whether this is the case, and make another adjustment if necessary. Now turn the scale until the pointer stands at zero, when the adjustment should be complete. Any error made in this adjustment may be eliminated by taking double readings with the commutator, so that there is no absolute necessity that this adjustment should be made at all. It is, however, convenient in rapid measurements to assume that when the pointer stands at zero, the galvanometer coil lies in the magnetic meridian.

(3.) *Care of the Pivot and Cap.*—The student should be

very careful of the pivot and cap, for should the point of the former become blunted or the inside of the latter covered with dust, the friction resulting from these causes will be fatal to accuracy. Some simple arrangement is provided with the best galvanometers in order to lift the needle from its pivot when not in use, or when the instrument has to be removed. This greatly assists the preservation of the pivot and cap. The state of the pivot and cap should be tested by displacing the needle from its position of rest, and ascertaining whether it returns to the same position. It should do so to within ·1 or ·2 of a degree.

(4.) *The Magnetism of the Needle.*—Although, as we have proved in the chapter on magnetism, the amount of deflection of the needle is quite independent of its magnetic moment, yet the power of overcoming friction will depend upon the strength of the magnet. Hence we should have a magnet as strong as possible, since in this instrument friction cannot be entirely avoided. There are many causes tending to weaken the magnet, such as the influence of strong currents, changes of temperature, etc. It is therefore important to ascertain from time to time the magnetic state of the needle. This may be simply done by ascertaining its time of vibration, which should not be allowed to become greatly different from what it was when the magnet was freshly magnetised.

The method of working with the galvanometer will be seen from the following examples :—

(I.) To find the resistance of two Grove's cells in series. Copper strap used. Six yards of connecting wire = ·07 ohm. The connecting wires were carefully twisted together so that they had no direct effect upon the galvanometer. This was tested by moving the wires and ascertaining that this did not produce any movement of the needle.

The following results were obtained :—

Expt.	Resistance.	Commutator up.		Commutator down.		Mean.
		East end of needle.	West end of needle.	East end of needle.	West end of needle.	
I.	0	53·8	54	56	56	54·95
II.	1	30	30	31·2	31·1	30·57
II.	2	20	20	21	21	20·5

Here we have to make use of the formula

$$B = \frac{R \tan \alpha - R_1 \tan \alpha_1}{\tan \alpha_1 - \tan \alpha} - G,$$

also G + connecting wires = ·07.

Now from the tables we find

$$\tan 54^\circ{\cdot}95 = 1{\cdot}4255$$
$$\tan 30^\circ{\cdot}51 = {\cdot}5906$$
$$\tan 20^\circ{\cdot}50 = {\cdot}3739.$$

Hence, from Experiments I. and II.,

$$B = \frac{1 \times {\cdot}5906}{1{\cdot}4255 - {\cdot}5906} - {\cdot}07 = {\cdot}64 \text{ ohm}.$$

From Experiments I. and III.,

$$B = \frac{2 \times {\cdot}3739}{1{\cdot}4255 - {\cdot}3739} - {\cdot}07 = {\cdot}64 \text{ ohm},$$

while from Experiments II. and III.,

$$B = \frac{2 \times {\cdot}3739 - {\cdot}5890}{{\cdot}5890 - {\cdot}3739} - {\cdot}07 = {\cdot}66 \text{ ohm},$$

which gives a mean result of ·65, or the average resistance per cell is nearly $\frac{1}{3}$ ohm.

(II.) The resistance of twelve Daniell's cells, of the type known as the *Chamber Daniell*,[1] used in the English Postal Telegraph Department, was next tested. The battery had been standing for some days, so that copper

[1] See Preece's *Telegraphy*, p. 21.

had become deposited on the zinc. The following readings were taken :—

Expt.	Resistance.	Mean of Readings.	Tangent of Angle.	Remarks.
I.	$R_1=0$	$64°\cdot6$	$\tan\alpha_1=2\cdot1060$	The battery was variable as to its current.
II.	$R_2=125$	$55°\cdot62$	$\tan\alpha_2=1\cdot4605$	
III.	$R_3=250$	$46°\cdot72$	$\tan\alpha_3=1\cdot0612$	Current steady.
IV.	$R_4=375$	$39°\cdot12$	$\tan\alpha_4=\cdot8127$	
V.	$R_5=575$	$30°\cdot12$	$\tan\alpha_5=\cdot5797$	
VI.	$R_6=975$	$20°\cdot65$	$\tan\alpha_6=\cdot3759$	

Resistance of galvanometer and wires $=9\cdot6$ ohms.

It was thought that correct values of the resistance could best be obtained from III. and V., III. and VI., and IV. and VI.

From III. and V. we have

$$B=\frac{R_3\tan\alpha_3-R_5\tan\alpha_5}{\tan\alpha_5-\tan\alpha_3}-9\cdot6=131\cdot9.$$

In like manner from III. and VI.,

$$B=137\cdot9.$$

From IV. and VI.,

$$B=133\cdot9.$$

The resistance of the battery was therefore about 134 ohms, or about eleven ohms per cell.

The values of B obtained from expressions involving I. and II. gave widely different results. This example shows the importance in certain cases of making the external resistance high, in order that the constancy of the battery may be ensured.

(III.) The same battery, after being in use for several days in short-circuit, was again tested by the half-deflection method. The needle of the galvanometer was accurately set to zero, when it was ascertained that the galvanometer was in good adjustment, giving equal deflections on both

sides of the scale, so that single readings would be sufficient. The following results were obtained :—

Reading on Tangent Scale.	Resistance.	Remark.
152	40	Current quite constant, G=10.
76	260	

Hence

$$B = R_1 - (2R + G) = 260 - (80 + 10) = 170;$$

or

$$\frac{170}{12} = 14 \text{ ohms for each cell very nearly.}$$

(IV.) Four Daniell's cells of the Minotto type with wet sand gave, without external resistance ($G = 10$), $\alpha = 42°$. Hence

$$\tan \alpha = \tan 42° = \cdot 9004.$$

Now, from the tables, $\frac{1}{2} \tan \alpha = \cdot 4502 = \tan \alpha_1$, or $\alpha_1 = 24° \ 15'$.

Resistance was then put into the circuit until a deflection of 24° 15′ was obtained; this required 500 ohms. Hence

$$B = 500 - 10 = 490 \text{ ohms, or } 123 \text{ ohms per cell.}$$

89. *Use of Tangent Galvanometer for determining Wire Resistance.*—The method of determining resistance by the Wheatstone's bridge being a zero method, and therefore independent of the variations of the battery, leaves nothing further to be desired. It is, however, well that the student should know how to use the tangent galvanometer for the purpose of approximately measuring wire resistance, for it may happen that it is the only instrument available.

Lesson XL.—Wire Resistance by Tangent Galvanometer.

90. *Exercise.*—To determine by the tangent galvanometer the resistance of two coils of about ·5 and 1500 ohms respectively.

Apparatus.—The same as before. The battery, however, should be capable of variation from one to twenty cells.

Theory.—We have, precisely after the method of last lesson,

$$\frac{B+G+x}{B+G+R}=\frac{\tan \alpha}{\tan \alpha_1} \quad . \quad . \quad . \quad . \quad (1)$$

where α_1 is the angle of deflection with the unknown resistance (x) in circuit. From (1) we have

$$x=\frac{\tan \alpha}{\tan \alpha_1}(B+G+R)-(B+G) \quad . \quad . \quad . \quad (2)$$

Practice of the Method.—In using formula (2) in practice we must remember that B may vary while the test is being made, thus causing the result to be uncertain. To avoid this source of error as much as possible we must choose—

(1.) As constant a battery as possible.
(2.) A battery of low resistance, for the lower the resistance of the battery the less is the influence of variation on the whole circuit.
(3.) An external resistance sufficiently high in comparison with the resistance of the battery, so as not to endanger the constancy of the latter.

Assuming the constancy of B, the accuracy of the determination will depend upon the correctness of the ratio $\frac{\tan \alpha}{\tan \alpha_1}$. Now we have proved that errors have least influence when the pointer is at 45°, hence the best result would be obtained when $\alpha=\alpha_1=45°$; when this is the case (2) becomes

$$x=R \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

which simple formula is very convenient.

Examples.—(A.) Resistance of a wire known to be about ·5 ohm. Copper strap used.

	External Resistance.	Mean Reading.	Tangent.
I.	0	38·8	·804
II.	x	29·6	·568
III.	1	24	·445

From I. and III., by formula (6), p. 229,

$$B+G=\frac{\cdot 445}{\cdot 804-\cdot 445}=1\cdot 24.$$

From II. and III., by formula (2), p. 236,

$$x=(1\cdot 24+1)\frac{\cdot 445}{\cdot 568}-1\cdot 24=\cdot 52.$$

(B.) Resistance of coil known to be 1830 ohms. Five Grove's cells used. Galvanometer terminals 1-6.

$$B=1\cdot 4,\ G=71\cdot 25,$$

with

$$R=1640 \text{ ohms},\ \alpha_1=46°,$$
$$x \quad ,, \quad ,, \quad \alpha_2=43°.$$

Hence

$$x=\frac{\tan 46°}{\tan 43}(1640+72\cdot 65)-72\cdot 65=1829\cdot 25 \text{ ohms}.$$

On adjusting R until $\alpha_1=\alpha_2$, so as to make $x=R$, the value obtained for x was 1820 ohms, the arrangement not permitting a more delicate adjustment. The error in this last case is within ·55 per cent.

91. *Determination of E. M. F. by the Tangent Galvanometer.*—There are two kinds of methods that can be applied for finding the E. M. F. of a battery in terms of that of a standard element, namely, (1.) methods involving a knowledge of battery resistance; (2.) methods in which the battery resistance is eliminated. Under the first head we have

(*a*) The Method of Unequal Resistances and Deflections.
(*b*) The Method of Equal Resistances.
(*c*) The Method of Equal Deflections.

Under the second head we have

(*d*) Wheatstone's Method (*Phil. Trans.*, 1843, p. 313).

(*e*) Method of Sum and Difference (often in England called Wiedemann's Method).

As these various methods should be known to the student they will be made the subject of the next lesson.

Lesson XLI.—E. M. F. by the Tangent Galvanometer.

92. *Exercise.*—To discuss the various methods for comparing E. M. F.

Apparatus.—As in the previous lesson.

Theory.—(*a*) *Method of Unequal Resistances and Deflections.*—Call the E. M. F. of the two batteries E_1 and E_2, and let their internal resistances be B_1 and B_2, while R_1 and R_2 are the external resistances in the two circuits. Let G be the resistance of the galvanometer, and let the resulting deflections be a_1 and a_2. Then with the one battery we have

$$K \tan a_1 = \frac{E_1}{B_1 + G + R_1} \quad . \quad . \quad . \quad . \quad (1)$$

while with the other

$$K \tan a_2 = \frac{E_2}{B_2 + G + R_2} \quad . \quad . \quad . \quad . \quad (2)$$

Hence dividing (1) by (2) we have

$$\frac{E_1}{E_2} = \frac{(B_1 + G + R_1)\tan a_1}{(B_2 + G + R_2)\tan a_2} \quad . \quad . \quad . \quad . \quad (3)$$

an equation which gives us the relation between the two values of E. M. F.

(*b*) *Method of Equal Resistances.*—If in (3) we arrange the resistances so that $B_1 + G + R_1 = B_2 + G + R_2$, then we have

$$\frac{E_1}{E_2} = \frac{\tan a_1}{\tan a_2} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

(*c*) *Method of Equal Deflections.*—If R_1 and R_2 be adjusted until $\alpha_1 = \alpha_2$, then

$$\frac{E_1}{E_2} = \frac{B_1 + G + R_1}{B_2 + G + R_2} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

(*d*) *Wheatstone's Method.*—Suppose that ϵ_1 represents the total resistance in the circuit of E_1, when the deflection is α_1, or

$$K \tan \alpha_1 = \frac{E_1}{\epsilon_1} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Suppose this resistance to be increased by the addition of ρ_1, then

$$K \tan \alpha_2 = \frac{E_1}{\epsilon_1 + \rho_1} \quad . \quad . \quad . \quad . \quad (2)$$

where α_2 is the new deflection. In like manner to get the same deflection (α_1) with the battery for which E. M. F. is E_2, we shall have

$$K \tan \alpha_1 = \frac{E_2}{\epsilon_2} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and

$$K \tan \alpha_2 = \frac{E_2}{\epsilon_2 + \rho_2} \quad . \quad . \quad . \quad . \quad (4)$$

From (1) and (3) we have

$$\frac{E_1}{E_2} = \frac{\epsilon_1}{\epsilon_2} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

and from (2) and (4)

$$\frac{E_1}{E_2} = \frac{\epsilon_1 + \rho_1}{\epsilon_2 + \rho_2} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Hence from (5) and (6), by taking differences of numerators and denominators,

$$\frac{E_1}{E_2} = \frac{\rho_1}{\rho_2} \quad . \quad . \quad . \quad . \quad . \quad (7)$$

that is to say, the electromotive forces are as the added resistances.

(*e*) *Method of Sum and Difference.*—Place the batteries to be compared in series, then

$$K \tan \alpha_1 = \frac{E_1 + E_2}{\epsilon} \quad . \quad . \quad . \quad . \quad (1)$$

where ϵ is the total resistance in the circuit.

Now interchange the poles of one of the batteries so as to cause E_1 and E_2 to oppose each other, and let the result be as follows :—

$$K \tan \alpha_2 = \frac{E_1 - E_2}{\epsilon} \quad . \quad . \quad . \quad . \quad (2)$$

The resistance in circuit being the same as before. From (1) and (2) we find

$$\frac{E_1 + E_2}{E_1 - E_2} = \frac{\tan \alpha_1}{\tan \alpha_2} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and from (3) we obtain

$$\frac{E_1}{E_2} = \frac{\tan \alpha_1 + \tan \alpha_2}{\tan \alpha_1 - \tan \alpha_2} \quad . \quad . \quad . \quad . \quad (4)$$

or the electromotive forces are to one another as the sum and difference of the tangents of the angles of deflection when the cells are in conjunction and opposition.

Examples.—The practice of the above methods will best be understood from the following examples :—

Two Grove's cells were compared against one. Copper strap used.

Determination of Battery Resistance—

		Resistance.	Deflection.	Tangent.
1 *Cell.*	I.	R = 0	55·25	1·441
	II.	= ·3	35·3	·708
	III.	= ·2	40·75	·861

I. and II. give ·289, I. and III. ·297. Mean value of battery resistance = ·293.

	Resistance.	Deflection.	Tangent.
2 *Cells.*	R = 0	60·5	1·768
	= ·6	40·5	·854

Mean value of battery resistance = ·56.

Comparison of E. M. F.

Method (*a*).—Galvanometer terminals 1-2.

2 Cells. $R_1=27$, $B_1=\cdot 56$, $G=\cdot 17$, $\alpha_1=40\cdot 7$, $\tan\alpha_1=\cdot 8601$.
1 Cell. $R_2=17$, $B_2=\cdot 29$, $G=\cdot 17$, $\alpha_2=34\cdot 4$, $\tan\alpha_2=\cdot 685$.

$$\frac{E_1}{E_2}=\frac{27\cdot 73\times\cdot 8601}{17\cdot 46\times\cdot 685}=1\cdot 995.$$

Method (*b*).—Terminals 1-4.

2 Cells. $R_1+B_1+G=140\quad+\cdot 56+15\cdot 28=155\cdot 84$, $\alpha_1=58$.
1 Cell. $R_2+B_2+G=140\cdot 3+\cdot 29+15\cdot 28=155\cdot 87$, $\alpha_2=39$.

$$\text{Log}\tan\alpha_1-\text{Log}\tan\alpha_2=10\cdot 2042-9\cdot 9084=\cdot 2958.$$

$$\frac{E_1}{E_2}=\frac{\tan\alpha_1}{\tan\alpha_2}=1\cdot 976.$$

Method (*c*).—Terminals 1-3.

2 Cells. $R_1+B_1+G=66\cdot 5+\cdot 56+1\cdot 48=67\cdot 54$, $\alpha_1=45$.
1 Cell. $R_2+B_2+G=32\quad+\cdot 29+1\cdot 48=33\cdot 77$, $\alpha_2=45$.

$$\frac{E_1}{E_2}=\frac{67\cdot 54}{33\cdot 77}=2\cdot 00.$$

Method (*d*).—Terminals 1-6.

1 Cell. $R_1+B_1+G\quad=140+\cdot 29+71\cdot 29=211\cdot 58$, $\alpha_1=59$.
$R_1+B_1+G+\rho_1=211\cdot 58+200$ $\qquad\alpha_2=41$.
2 Cells. $R_2+B_2+G\quad=344+\cdot 56+71\cdot 29=415\cdot 85$, $\alpha_1=59$.
1 Cell. $R_2+B_2+G+\rho_2=415\cdot 85+396$ $\qquad\alpha_2=41$.

$$\frac{E_1}{E_2}=\frac{396}{200}=1\cdot 98$$

Method (*e*).—Terminals 1-4. Total resistance in circuit, 220 ohms.

Cells in conjunction. $\alpha_1=60\cdot 4$, $\tan\alpha_1=1\cdot 76$.
,, opposition. $\alpha_2=31\cdot 0$, $\tan\alpha_2=\ \cdot 6$.

$$\frac{E_1}{E_2}=\frac{1\cdot 76+\cdot 6}{1\cdot 76-\cdot 6}=\frac{2\cdot 36}{1\cdot 16}=2\cdot 03.$$

Assuming the true value of $\frac{E_1}{E_2}$ to be 2, we see that in

(*a*) the error is $-\ \cdot 25$ per cent.
(*b*) ,, ,, $-1\cdot 2$,,
(*c*) ,, ,, $0\cdot 0$,,
(*d*) ,, ,, $-1\cdot 0$,,
(*e*) ,, ,, $+1\cdot 5$,,

93. *Standard Tangent Galvanometer.*—In the formula for the tangent galvanometer which we have discussed the two following assumptions have been made—(1.) The needle has been assumed to be very small, and (2.) the positions of the several turns of the coil have been supposed to be the same. It is beyond the scope of this work to give proofs of the complicated formulæ that result when

departures from these assumptions are taken into consideration. For this the student must be referred to the work of Clerk Maxwell (vol. ii., new edition, chap. xv.), from which we take the following expression for a tangent galvanometer, consisting of a single coil with a magnet at its centre.

Let a = *mean* radius of coil.
ξ = depth of coil.
η = breadth of coil.
n = number of windings.
H = horizontal intensity of the earth's magnetism.
C = strength of current.
α = angle of deflection.
$2l$ = length of magnet.

Then, as a near approximation,

$$\mathrm{C}\left\{1+\frac{1}{12}\frac{\xi^2}{a^2}-\frac{1}{8}\frac{\eta^2}{a^2}\right\}\left\{1+\frac{3l^2}{4a^2}(1-5\sin^2\alpha)\right\}=\frac{\mathrm{H}a}{2\pi n}\tan\alpha \quad . \quad (1)$$

when a is very great compared with ξ and η.

The correcting factor

$$1+\frac{1}{12}\frac{\xi^2}{a^2}-\frac{1}{8}\frac{\eta^2}{a^2} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

is of less importance than

$$1+\frac{3l^2}{8a^2}(1-5\sin^2\alpha) \quad . \quad . \quad . \quad . \quad (3)$$

and by making the depth of the coil bear to the breadth the ratio of $\sqrt{3}$ to $\sqrt{2}$, expression (2) may be made to become unity.

The value of (2) when $a = 10\xi = 10\eta$ will be

$$1+\tfrac{1}{12}\cdot\tfrac{1}{100}-\tfrac{1}{8}\cdot\tfrac{1}{100}=\cdot999584,$$

which differs from unity by less than ·05 per cent.

The value of (3) will depend on α. If

$$\alpha=0 \text{ the correcting factor is } 1+\frac{3l^2}{4a^2}.$$

If

$1-5\sin^2\alpha=0$, *i.e.*, if $\alpha=26°\ 34'$, the factor becomes unity.

If

$$\alpha=45 \text{ it becomes } 1-\frac{9}{8}\frac{l^2}{a^2}.$$

If

$$\alpha=90 \quad ,, \quad 1-3\frac{l^2}{a^2}.$$

For a case in which the diameter of the coil is ten times the length of the needle, and $\alpha = 45°$, the term amounts to

$$1-\tfrac{9}{8}\cdot\tfrac{1}{100}=\cdot989,$$

being about 1 per cent less than unity.

We may make the correction much smaller by adopting two equal coils after the manner of Helmholtz and Maxwell. Here the coils must be placed symmetrically with regard to the needle, at a distance from the needle equal to half the radius of either coil. We shall prove in Chap. VII. that

$$C=\frac{H(a^2+x^2)^{\frac{3}{2}}}{2n\pi a^2}\tan\alpha \quad . \quad . \quad . \quad . \quad (1)$$

in the case of a single coil of n windings at x distance from the needle. In the above double coil we have $x=\frac{a}{2}$, and the current would have a double influence, hence

$$C=\frac{H\left(a^2+\frac{a^2}{4}\right)^{\frac{3}{2}}\tan\alpha}{4n\pi a^2}=\frac{5\sqrt{5}Ha}{32n\pi}\tan\alpha \quad . \quad . \quad (2)$$

This is the formula, disregarding the length of the needle and the depth and breadth of the coil. By making $\frac{\xi}{\eta}=\sqrt{\frac{36}{31}}$ the corrections due to these dimensions nearly vanish, the complete formula being

$$C\left(1-\frac{1}{60}\frac{\xi^2}{a^2}\right)=\frac{5\sqrt{5}Ha}{32n\pi}\tan\alpha \quad . \quad . \quad . \quad (3)$$

This will be very nearly the same as (2), when ξ is very

small compared with a. When $\xi = \frac{a}{10}$ the factor becomes

$$1 - \tfrac{1}{60} \cdot \tfrac{1}{100} = \cdot 99984,$$

which is virtually the same as unity.

94. *Methods of ascertaining K.*—The construction of a standard galvanometer and the determination of its constant are points of very great importance; for having once made the necessary measurements with a properly constructed instrument, the constants of other galvanometers may easily be determined by comparing them with the standard. We shall therefore consider at some length the various methods of obtaining the constant of a galvanometer. The following are the chief methods available:—

(1.) Direct calculation from the dimensions of the coil and the known value of H in the locality by making use of the above formulæ.

(2.) Determination of the current producing an observed deflection by means of its electro-chemical effects, such as—

- (*a*) Deposition of metal by the current—(α) Deposition of copper; (β) deposition of silver.
- (*b*) Decomposition of water and either—(α) Measuring the volume of gas (volume voltameter), or (β) obtaining the loss of weight (weight voltameter).

(3.) Determination of the current producing an observed deflection by the measurement of its heating effect.

(4.) Determination of the current producing an observed deflection by calculation from the known constants of a standard cell.

95. *Construction of Standard Tangent Galvanometer.*—A standard tangent galvanometer consists essentially of one or more hoops of large size, having a known number of turns of well insulated wire wound round them, and provided with a small magnetic needle freely movable about a central point, where the magnetic field due to a current in the coils is as uniform as possible. A suitable arrangement of

sufficient accuracy for measuring the deviations of the needle is likewise necessary. In constructing a standard instrument the first consideration must be the making of the hoop or reel. This may be made sufficiently well of wood. The diameter should be at least 24 inches, which is too large to admit of the hoop being turned on an ordinary lathe; hence the hoop is usually made in sections, which are put together with the grain of the wood in the adjacent pieces crossed in order to avoid warping. A suitable hoop having been thus obtained, it should be mounted upon an axis for convenience in winding, and the insulated wire (which should be silk-covered) must then be evenly and regularly laid on, with no greater strain on the wire than is necessary to keep it straight. During the winding the number of turns must be counted and the wire well examined for any bare places, which should be coated with tissue paper steeped in paraffin. As the value of the instrument greatly depends on the accurate knowledge of the number of turns, and a complete certainty of the absence of short-circuiting between the several coils, the importance of these two matters cannot be over-estimated. Since the process of winding the wire tends to alter its resistance, no certain conclusion can be obtained as to the perfect insulation by measuring the resistance of the wire before and after winding. This, however, should be done in order to discover whether any extensive short-circuiting has occurred. The insulation is greatly improved by giving to each layer a coating of melted paraffin, which should be laid on with a brush, and then the upper surface of the solidified paraffin made smooth by means of a wedge-shaped piece of wood. This process is likewise valuable, inasmuch as it produces an even surface on which the next layer may be laid. The number of turns and the thickness of the wire used will depend entirely upon the magnitude of the currents to be measured. It is well to provide the instrument with several coils—one consisting of very

many turns of fine wire for weak currents, and one consisting of a single turn of copper strap for strong currents.

A standard instrument should be provided with a suspended rather than with a pivoted needle, for the friction of the pivot is always an objectionable element. When an ordinary compass box is used the needle may be similar in construction to that already described, but suspended from a suitable torsion-head. In order to bring the needle rapidly to rest it should have a damper attached to it that may swing in a vessel of water or light oil. A good arrangement is to provide the needle with a small mirror, so that readings may be taken on a suitable galvanometer scale. Fig. 128 shows a standard galvanometer with a

Fig. 128.—STANDARD GALVANOMETER.

reflecting arrangement centrally situated at *h* in a brass cell *d*, so as to be almost dead beat. The cell is held in position by being inserted in a slit in the tube *t* that fits the hole *h*.

Lesson XLII.—Use of Standard Galvanometer—Determination of E. M. F.

96. *Exercise.*—To find the E. M. F. in volts of a Grove's cell.

Apparatus.—An absolute galvanometer with commutator; two Grove's cells; a box of coils.

Theory.—The battery, the box of coils, and the galvanometer being connected in series, let the total resistance in the circuit be R, also let E denote the electromotive force of the cells, and C the current produced, then, by Ohm's law,

$$C=\frac{E}{R}. \qquad (1)$$

Now let the resistance be increased by r, then

$$C_1=\frac{E}{R+r} \qquad (2)$$

From (1) and (2), eliminating R, we find

$$E=\frac{CC_1r}{C-C_1} \qquad (3)$$

The values of C and C_1 are determined in C. G. S. measure by the standard galvanometer, for we have respectively

$$C=H\Gamma\tan\alpha \qquad (4)$$

and

$$C_1=H\Gamma\tan\alpha_1 \qquad (5)$$

where α and α_1 are the angles of deflection observed. Inserting these values in (3), there results

$$E=H\Gamma r\times10^9\frac{\tan\alpha\tan\alpha_1}{\tan\alpha-\tan\alpha_1}. \qquad (6)$$

This will give the value of E in C. G. S. units, since for this purpose the value of the resistance in ohms must be multiplied by 10^9. But since also one volt $\doteq 10^8$ C. G. S. units, the value of E in (6) must therefore be divided by

10^8 in order to reduce its value to volts. Hence finally

$$\text{E (in volts)} = \frac{\text{H}\Gamma r \tan \alpha \tan \alpha_1}{\tan \alpha - \tan \alpha_1} \times \frac{10^9}{10^8} = \frac{10\text{H}\Gamma r \tan \alpha \tan \alpha_1}{\tan \alpha - \tan \alpha_1} . \quad (7)$$

Method.—Make a circuit consisting of the galvanometer provided with a commutator, the box of coils, and the battery arranged in series. Take sets of double readings with varying resistances, and calculate out the values of E from different pairs of observations.

Example.—The coil of the standard galvanometer used consisted of a single turn of thick copper $\frac{1}{16}$ inch thick. The inner diameter of the hoop was 22·5 inches, and the hoop itself was $\frac{1}{2}$ inch thick. Hence the radius of the copper strap was 11·5 inches. Therefore

$$\Gamma = \frac{a}{2n\pi} = \frac{11{\cdot}5 \times 2{\cdot}539}{2 \times 1 \times 3{\cdot}1416},$$

where 2·539 is the multiplier which converts inches into centimètres. The value of H had been previously determined by the magnetometer and found to be ·17 C. G. S. units, so that

$$\text{H}\Gamma = \frac{{\cdot}17 \times 11{\cdot}5 \times 2{\cdot}539}{2 \times 1 \times 3{\cdot}1416} = {\cdot}797.$$

Two Grove's cells provided with a commutator were connected with the copper strap by means of *adynamic leads;* that is to say, leading wires so twisted together that they had no direct effect upon the galvanometer when they conveyed a current. A box of resistances of the values ·5, ·2, ·2, and ·1 ohms, made of thick German silver wire, was also in circuit. It was found that the current did not perceptibly heat the coils. Readings were taken to a tenth of a degree. One of the observations gave $\alpha = 34°{\cdot}5$, $\alpha_1 = 23°$, $r = {\cdot}4$. Hence $\tan \alpha = {\cdot}6873$, $\tan \alpha_1 = {\cdot}4245$; and hence also

$$\text{E} = \frac{10 \times {\cdot}797 \times {\cdot}4 \times {\cdot}6873 \times {\cdot}4245}{{\cdot}6873 - {\cdot}4245} = 3{\cdot}51,$$

or 1·76 per cell.

The result was lower than that expected, and the difference was found to be due to the fact that H had a higher value than ·17, owing to the presence of local iron. This is the most fruitful source of inaccuracy in measurements of this kind when made in laboratories not specially constructed with a view to avoid the error.

97. *Electro-Chemical Equivalents.*—When an electrolyte is placed in a galvanic circuit the amount of chemical decomposition in unit time for unit strength of current is a fixed quantity. The mass of substance liberated at the electrode is called its **electro-chemical equivalent.** The equivalents for certain important elements have been determined with great accuracy by Lord Rayleigh and others. Thus the amount of silver deposited from silver nitrate in one second by one ampère of current is ·001118 gramme. By the aid of this number the electro-chemical equivalent of any element can be found if we know its **chemical equivalent**; for if the same current be sent in succession through different electrolytes the amount of the elements deposited at the electrodes would be in the ratio of the chemical equivalents. Thus we have

Chemical equivalent of silver		$=108$
,,	,, hydrogen	$=\ \ 1$

Hence the electro-chemical equivalent of H is

$$\frac{\cdot 001118 \times 1}{108} = \cdot 00001035.$$

Again, since the chemical equivalent of copper in the *cupric* state is $\frac{63}{2} = 31 \cdot 5$, the electro-chemical equivalent is

$$\frac{\cdot 001118 \times 31 \cdot 5}{108} = \cdot 000326.$$

Let W denote the mass in grammes liberated *at one*

electrode by a current of strength C in t seconds, and let ϵ be the electro-chemical equivalent, then we have

$$W = C\epsilon t, \text{ or } C = \frac{W}{\epsilon t} \quad . \quad . \quad . \quad . \quad (1)$$

This will enable us to measure currents by ascertaining the mass in grammes liberated in a given time.

In the case of water it is customary to determine the total weight of water decomposed. Now an ampère in one second—in other words, a **coulomb**—will liberate ·00001035 gramme of hydrogen and eight times the weight of oxygen, or ·00008280 gramme. Hence a coulomb will decompose ·00009315 gramme of water. If instead of weighing the water we measure the volume of one or both the gases liberated, it will be convenient to know the relation between the volume of these and their weight. Now 1 gramme of hydrogen measures at 0° C. and 760 mm. pressure 11170 cubic centimètres. By aid of this number it will be easy to convert the weights into volumes. The resulting values and others that are also useful are exhibited in the following table :—

TABLE I.

ELECTRO-CHEMICAL EQUIVALENTS.

	Grammes per Coulomb.	Normal cubic centimètres per Coulomb.
Hydrogen	·00001035	·1156
Silver	·001118	...
Copper (cupric) . .	·000326	...
Mercury (mercuric) .	·001035	...
Zinc	·0003364	...
Oxygen	·0000828	·0578
Water	·00009315	·1734

Lesson XLIII.—Galvanometer Constant by Copper Deposition.

98. *Apparatus.*—The depositing cell of Lesson XV., with accompanying liquids, etc.; a Daniell's battery; box of coils; commutator; chemical balance; stop-watch; the galvanometer whose constant is required.

Method.—Adjust the number of cells in the battery and the resistance until the deflection of the galvanometer is not greater than 60°, the connections being as in Fig. 129,

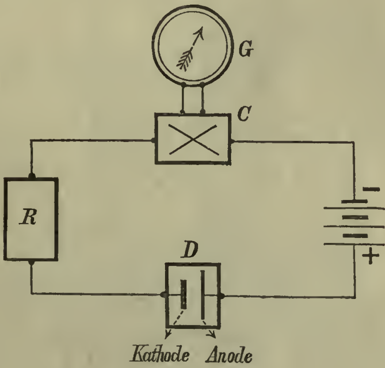


Fig. 129.

where D is the depositing cell. Next thoroughly clean the anode and the working cathode—dry them in a current of hot air and weigh them to within half a milligramme. Now fix them in their position in the depositing cell, the circuit being still incomplete. Set the stop-watch to an exact hour, and then simultaneously start the current and the watch. Read the galvanometer, rapidly reverse the commutator so as not to lose time, and read again. Take readings from time to time and adjust the resistance in

circuit if the deflection does not keep constant. The deposition should be continued for some time, at least two hours, and at the end of the time the current should be discontinued, and the clock stopped. Remove the cathode, wash it well first in common water and then in distilled water, dry it in a current of hot air and weigh it. From the gain in weight in the observed time calculate the average current that has been circulating. This having been determined, deduce the constant of the galvanometer from the formula

$$K = \frac{C}{\tan a},$$

where a is the average deflection.

Precautions.—The battery chosen for this purpose should be a constant one. When the constant to be determined is small, Daniell's may be employed. The Daniell's battery should be left short-circuited through a resistance some time before use, so that it may be in a normal working condition.

If it be necessary to have a strong current, a Grove's or Bunsen's battery should be used, and it will then be necessary to employ plates in the depositing cell of a large size; for when a certain *density* of current (that is to say, number of units of current to unit area of electrode) is exceeded the deposit is in the form of a powder and does not adhere. It is ascertained that the loss of weight of the anode cannot be used as an accurate measure of the current, owing to secondary corrosive chemical action and disintegration producing loss of weight, which would vitiate the determination of real electrolyte loss.

Example.—

Weight of cathode at commencement . . .	10·425 grms.
,, ,, at end	11·219 ,,
Gain in weight	·794

Mean deflection 47°. Tan 47° = 1·0724. Time 125 minutes.

Hence weight of copper in grammes per second $= \dfrac{\cdot 794}{60 \times 125}$.

Strength of current in ampères $= \dfrac{\cdot 794}{60 \times 125 \times \cdot 000326}$.

Constant of galvanometer $= \dfrac{\cdot 794}{60 \times 125 \times \cdot 000326 \times 1 \cdot 0724} = \cdot 3028$.

Lesson XLIV.—Galvanometer Constant by Silver Deposition.

99. *Apparatus.*—A Poggendorff's voltameter, Fig. 130. In this form the anode and cathode are horizontal, the former being placed above the latter. The cathode is a

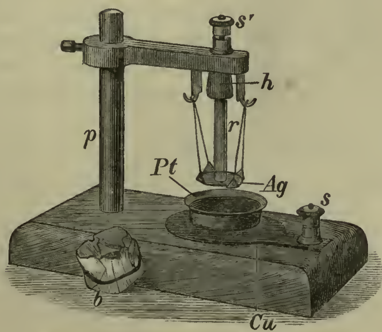


Fig. 130.—Poggendorff's Voltameter.

platinum basin P*t* fitting within a copper ring, which is provided with a terminal *s*. The platinum basin is easily removable from the ring, and yet fits sufficiently firmly to give good electric contact. The anode is a square plate of silver A*g*, with its four corners turned up. The plate is supported by platinum wires passing through small holes in the corner of the plate, and secured to hooks that are in metallic connection with the terminal *s*′. The anode is kept firm by the help of a glass rod *r*, having a cork *h* at

the top, which is forced between the upper surface of the anode and the lower surface of the supporting arm. The arm is movable up and down the ebonite pillar *p*. Silver nitrate will be required for charging the voltameter. The other apparatus necessary will be the galvanometer, etc., as in the last lesson; also a desiccator, *i.e.*, a glass chamber containing calcium chloride, in which the platinum basin may be placed.

Method.—The only details requiring notice beyond those given in the previous lesson are those relating to the voltameter. Make a solution containing from 5 to 30 per cent of silver nitrate, according to the strength of the current employed—the stronger the current the stronger should be the solution. Lord Rayleigh and Mrs. H. Sidgwick,[1] who have investigated the precautions necessary to obtain the highest degree of accuracy with a silver voltameter, find the following conditions satisfactory :—

TABLE K.

WORKING CONDITIONS OF SILVER VOLTAMETER.

Max. strength of Current in Ampères.	Percentage of Silver Nitrate in Solution.	Length of time of Deposit in Minutes.	Diameter of Platinum Dish in Inches.
·25	4	...	...
1	15	60	3
2	30	15	...
5	30	15	9[2]

To make use of this table some information should be obtained as to the strength of the current required by means of a tangent galvanometer whose constant is approxi-

[1] *Phil. Trans.*, 1884, p. 411, on "The Electro-Chemical Equivalent of Silver," and on "The Absolute E. M. F. of Clark's Cells," by Lord Rayleigh and Mrs. H. Sidgwick.

[2] Formed by three 3-inch bowls in multiple arc.

mately known. The platinum basin must be thoroughly cleaned with nitric acid, well washed with distilled water, and then placed in a porcelain evaporating basin and heated by the flame of a spirit lamp. When dry the hot basin must be placed in the desiccator, and accurately weighed when cool.

The anode should be a piece of fine sheet-silver, about $\frac{1}{8}$ inch thick, and should be cut of such a size as to leave a good space between its edges and the side of the bowl. When suspended by the platinum wires in the way we have described, the plate should be enclosed as in a bag by a piece of the best filter paper, which may be secured in its place by means of an elastic band, see *b* Fig. 130. This bag of filter paper serves to protect the cathode from disintegrated silver, which is invariably formed on the anode. When the deposition is complete the silver solution should be returned to the stock bottle, and the anode and cathode well washed with distilled water. After rinsing the dish a few times with water it should be filled up with water and left to soak for an hour, and then enclosed within a porcelain basin and heated by a spirit lamp. Finally, the dish should be well washed and dried, enclosed within the basin over the spirit lamp for an hour, and then weighed.

The chemical student will perceive that the precautions are exactly those adopted in quantitative analysis.

Lesson XLV.—Galvanometer Constant by Decomposition of Water (weight voltameter).

100. *Apparatus.*—A weight voltameter. This consists essentially of two tubes, one containing acidulated water and platinum electrodes, and the other a material for desiccating the evolved gas. Three various arrangements, which we have found satisfactory, are figured below. In A Fig. 131 the tube T is placed in connection with a calcium chloride tube *rt* by means of a paraffin

joint P, and not india-rubber.[1] The electrodes *e e* are soldered through the glass; they are protected from a tendency to break off at the glass by means of a piece of india-rubber tubing, which is slipped on at *i*. The joint P is avoided in the arrangement B. Here the two tubes are formed by contracting at the middle and

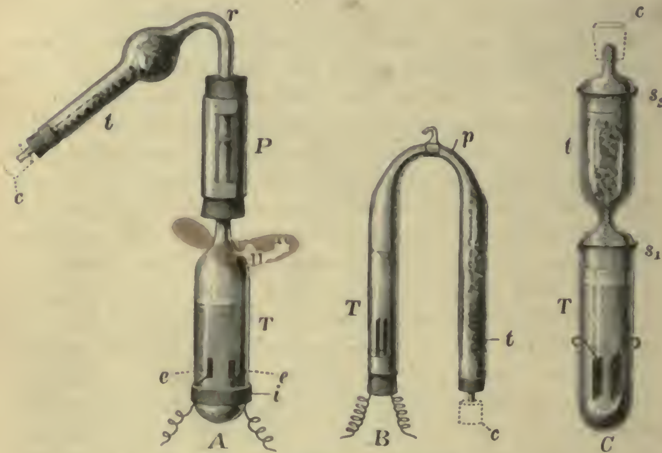


Fig. 131.—WEIGHT VOLTAMETERS

bending in U form a piece of wide glass tubing. The electrodes pass through holes in a cork soaked in paraffin. The open end of *t* is also closed by means of a cork soaked in paraffin, having a small tube passing through it by which the permanent gas may escape. The neatest arrangement is that shown in C, which can be constructed, by any one familiar with glass working, out of two weighing tubes *t* and T that have wide ground-glass perforated stoppers s_1 and s_2.

[1] India-rubber cannot be used for a weight voltameter, as it is rapidly destroyed by the action of the evolved ozone.

The best drying material is asbestos soaked in strong sulphuric acid.

Method.—The apparatus, being charged by placing dilute sulphuric acid in T and drying material in t, is suspended from the balance and weighed, the open end being meanwhile closed with a cork c that has been soaked in paraffin. Next the voltameter is placed in circuit and the decomposition allowed to proceed during a known time. When the current is broken the open end is again stopped, the apparatus allowed to cool, if necessary, and then reweighed.

Example.—Verification of the constant of a current measurer (an electro-dynamometer) in which the following relation was supposed to hold :—

$$C = K\sqrt{a},$$

where C is the current, K the constant, and a the reading.

Mean value of a	$=80{\cdot}7$
Loss of weight of voltameter . . .	${\cdot}5746$
Time	$=60$ minutes.

$$C = \frac{{\cdot}5746}{60 \times 60 \times {\cdot}00009315} = 1{\cdot}714 \text{ ampère.}$$

Hence

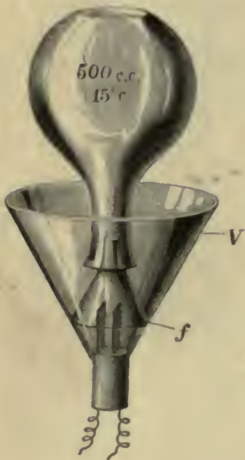
$$K = \frac{1{\cdot}714}{\sqrt{80{\cdot}7}} = {\cdot}1907,$$

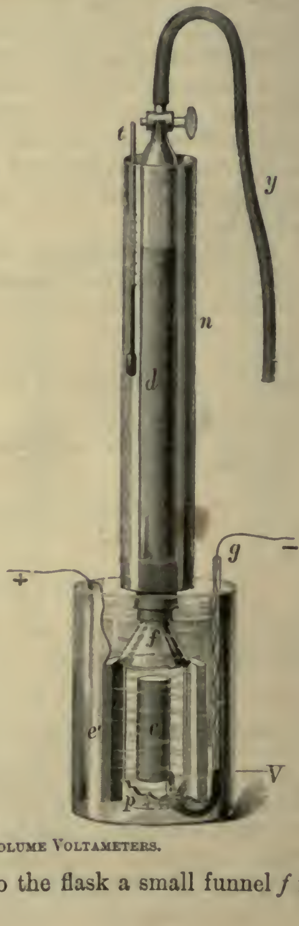
which agreed very well with the certificate of the maker, who gave $K = {\cdot}1902$.

Lesson XLVI.—Use of the Volume Voltameter.

101. *Apparatus.*—Two different types of volume voltameters are shown in Figs. 132 and 133. In Fig. 132 the two gases are evolved from an ordinary funnel voltameter V, whose construction has already been described (see Chap. III.), and are collected in a quarter or half-litre flask provided with an etched millimètre scale on its neck (see Vol.

I. p. 106). The volume of the flask up to the various marks should be accurately ascertained (see Vol. I. p. 107).





Figs. 132 and 133.—VOLUME VOLTAMETERS.

To lead the evolved gas into the flask a small funnel *f* is placed over the electrodes.

The voltameter is charged with water acidulated with phosphoric acid obtained by dissolving phosphoric pentoxide in water, for this acid has the property of preventing the formation of ozone,[1] which would interfere with the operation by diminishing the volume of oxygen.

In the apparatus of Fig. 133 the hydrogen only is collected. This form has the advantage of giving a greater length of time for collecting a given volume of gas, also avoiding the error caused by the formation of ozone. Here it is necessary to make arrangements to isolate the oxygen. This is done by interposing the porous pot p between the electrodes e and e'. The electrode e is of platinum fused into the glass tube g, which contains mercury for the purpose of making the connection with the negative pole of the battery. The electrode e' is a hollow cylinder of carbon, such as is used in some batteries of the Bunsen form. The whole is enclosed in the outer glass vessel V. Over the mouth of the porous pot is a small funnel for leading the gas into the burette d. Surrounding the burette is a wider glass tube n, which is filled with water for keeping the liberated gas at a constant temperature, as recorded by the thermometer t. The burette is supported by a wooden stand not shown in the figure. To the end of the burette there is attached an india-rubber tube y for the purpose of raising the liquid in the burette by suction. The voltameter is charged with dilute sulphuric acid. The best strength is that which contains 30 per cent of acid, as this gives the maximum degree of conductivity; but it will, as a rule, be unnecessary to use a stronger acid than one containing 15 per cent.

Method of Reducing the Observations.—The problem is to ascertain the volume of H, or of H and O, reduced to normal temperature and pressure, which is evolved in a known time. Let us suppose that at first the measuring

[1] Mascart.—This is specially true of small electrodes.

vessel is quite full of liquid, and that the amount of gas given off in n seconds has the apparent volume V. This volume exists at the pressure H, as given by the barometer, *less* the back pressure due to the height h of liquid that still remains above the level of the reservoir (see Fig. 134). The pressure due to h must be converted into inches of mercury, in order that it may be expressed in the same units as the barometric pressure. If we call Δ the relative density of the liquid, then $\frac{\Delta h}{13 \cdot 6}$ will be the equivalent pressure in inches of mercury. The value of Δ for several strengths is exhibited in the following table :—

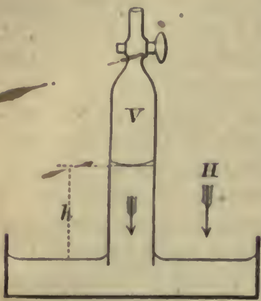


Fig. 134.

TABLE L.

RELATIVE DENSITY OF DILUTE SULPHURIC ACID.

Percentage by weight of Sulphuric Acid.	Density = Δ
5	1·03
10	1·07
15	1·11
20	1·14
25	1·18
30	1·22

A further correction will require to be made, for the tension of the vapour above the dilute acid must be deducted. For very dilute acid the tension may be considered to be that due to water alone; in other cases the correct value must be used (see Appendix G, Table Q).

If the vapour tension be called T, the corrected barometric pressure H′ will be expressed as follows :—

$$H' = H - \frac{\Delta h}{13 \cdot 6} - T.$$

The true volume V′ of the gas will be, for the normal temperature and pressure (0° C. and 760 mm.),

$$V' = V \cdot \frac{H'}{760} \cdot \frac{1}{1 + \cdot 003665t'}$$

where t is the temperature in centigrade degrees. The true volume of the gas being thus ascertained in cubic centimètres, the current in ampères will be, if the hydrogen alone is collected,

$$C = \frac{V'}{n \times \cdot 1156};$$

while if both gases are collected it will be

$$C = \frac{V'}{n \times \cdot 1734},$$

where n denotes the time during which the gases are liberated in seconds.

Practice of the Method.—When the apparatus of Fig. 133 is used it will be necessary to ascertain once for all the volume between the top graduation mark on the burette and the space up to the stopcock. This may be done by weighing the amount of mercury or water which will fill the space (see Vol. I. p. 107).

The burette, being replaced in the voltameter, is then filled with liquid by suction, and at a known time the circuit is completed. Decomposition is allowed to proceed until the burette is nearly filled with gas, when the circuit is broken and the time again noted. The height of the liquid in the burette above the general surface of the liquid is obtained by means of a glass millimètre scale, or it may be read off directly on the divisions of the burette, which should be compared with a millimètre scale. The barometer must be read at the end of the experiment, and also the thermometer inserted in the water of the outer tube.

The flask (Fig. 132) when about to be used must be filled

with acidulated water free from air-bubbles. When filled with gas it may be removed from the voltameter to a glass trough containing water, where water from the tap may be allowed to flow over it so as to bring the gas to a constant temperature. The height of the water in the flask should in this case be adjusted to that of the trough, in order to avoid the h correction. The gas being measured over water, the correction for vapour tension will be that due to water only.

Example.—It was wished to compare the constant of a standard galvanometer obtained by calculation with that obtained by the use of the voltameter.

Hydrogen alone was collected in the voltameter, which was of the burette form. Height of barometer $= 766{\cdot}7$ mm. $= \mathrm{H}$. Mean temperature $= 13^\circ{\cdot}5 = t$. The galvanometer, provided with a commutator, was placed in series with the voltameter and four Grove's cells. Observed deflection on galvanometer $= 42^\circ{\cdot}7$, $43^\circ{\cdot}9$, $41^\circ{\cdot}1$. Mean, $42^\circ{\cdot}6$. $35{\cdot}8$ cc. ($= \mathrm{V}$) of hydrogen were collected in 27 minutes and 2 seconds $= 1622$ seconds $= n$. Height of liquid in burette at end of experiment $= 14{\cdot}2$ cm. $= h$. Liquid in voltameter had a density of $1{\cdot}1 = \Delta$, which corresponded with a vapour density at 13·5 of $11{\cdot}5$ mm. $= \mathrm{T}$.

Here

$$\mathrm{H}' = 766{\cdot}7 - \frac{142 \times 1{\cdot}1}{13{\cdot}6} - 11{\cdot}5 = 743{\cdot}7$$

and

$$\mathrm{V}' = 35{\cdot}8\,\frac{743{\cdot}7}{760}\cdot\frac{1}{1 + ({\cdot}003665 \times 13{\cdot}5)} = 33{\cdot}38,$$

therefore

$$\mathrm{C} = \frac{33{\cdot}38}{1622 \times {\cdot}1156} = {\cdot}178 \text{ ampères},$$

and

$$\mathrm{K} = \frac{{\cdot}178}{\tan 42{\cdot}6} = {\cdot}1938.$$

This number agreed with that obtained by calculation

from the dimensions of the coil and the local value of H, which gave ·1923.

102. *Joule's Law.*—The work W done by an electric current in moving Q units of electricity from a point where the electric potential is V_2 to another where it is V_1 is

$$W = Q(V_2 - V_1) \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The truth of this statement will be at once recognised if we reflect that as a matter of definition there is unit difference of potential between two points when it requires unit of work to convey unit of electricity from the one point to the other against electric repulsion. The amount of work in the above formula is thus jointly proportional to the quantity of electricity and to the difference of potential or electric level. But we have also

$$Q = Ct \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where C is the strength of the current and t the time during which it flows; hence, remembering that for $V_2 - V_1$ we may write E, the electromotive force between the points, we have

$$W = CEt \quad . \quad . \quad . \quad . \quad . \quad (3)$$

But, by Ohm's law, if R be the resistance between the two points,

$$E = CR \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Hence we have

$$W = C^2Rt \quad . \quad . \quad . \quad . \quad . \quad (5)$$

If the electrical energy be converted into heat H, then

$$W = JH \quad . \quad . \quad . \quad . \quad . \quad (6)$$

J denoting the mechanical equivalent of heat. Inserting the value of W in (5), we obtain

$$C = \sqrt{\frac{JH}{Rt}} \quad . \quad . \quad . \quad . \quad . \quad (7)$$

This last equation expresses Joule's law, and enables a

current to be determined in terms of the heat given out in a circuit of resistance R during the time t. The value of J is $4{\cdot}2 \times 10^7$ ergs per gramme-degree.

LESSON XLVII.—Application of Joule's Law.

Fig. 135.—APPARATUS FOR OBSERVING HEATING EFFECT.

103. *Exercise.*—To determine the constant of a galvanometer by Joule's law.

Apparatus.—A calorimeter, consisting of a wooden box B (Fig. 135), containing a tin can, the space between it and the can being filled with lightly packed saw-dust. Within the can a smaller one C of thin copper fits with a lipped edge, which enables the can C to rest on and to be supported by a thick felt washer C′ fastened to the top of the box. C is provided with a lid which supports the stirrer S, consisting of a ring of thin copper provided with two handles, by means of which the stirrer may be moved up or down. A thick wire R of German silver is wound in a loose spiral, and has its ends soldered to large binding screws. A delicate thermometer T passes through a hole in the centre of the lid. The thermometer

should be capable of being read to at least $\frac{1}{100}$ of a degree centigrade. Four cells of Grove's battery, a stop-watch, and a key.

Method.—Arrange the calorimeter in circuit with the battery, the galvanometer whose constant is required, and the key. The calorimeter can C must be weighed, and a weighed quantity of water placed within it sufficient to cover the German silver wire when the lid is in position. It is desirable that the water should be as much below the temperature of the room as at the end of the experiment it will be above that temperature. This should be done in order that the correction due to cooling may be small, for the calorimeter will, on the whole, under these circumstances, have received during the experiment as much external heat as it subsequently loses. The initial temperature θ_1 must be accurately observed, and at a given moment the circuit closed, the agitator being set in motion during the whole time in which the calorimeter is receiving heat. The galvanometer should likewise be read from time to time. When the temperature of the water has sufficiently risen, the circuit should be broken and the final temperature θ_2 observed.

Let W be the weight of the water, w that of the stirrer and can, s the specific heat of the stirrer and can, then the whole amount of heat received is

$$\mathrm{H} = (\theta_2 - \theta_1)(ws + \mathrm{W}).$$

Inserting this value in equation (7), the current C should be calculated, and from it the constant of the galvanometer determined.

Example.—The method was used to find the constant of an instrument adapted for measuring strong currents.

The weight of calorimeter	= 533·5	grms.
,, ,, stirrer	= 191·2	,,
or w	= 724·7	

The specific heat of the can and stirrer was taken as $\cdot 1 = s$.

Weight of water $= 3060\cdot 5$ grms. $=$ W.
Initial temperature, $\theta_1 = 14°$ C.
Final ,, $\theta_2 = 14°\cdot 31$ C.

Hence

$$H = (14\cdot 31 - 14)\,(3060\cdot 5 + 72\cdot 5) = 971\cdot 2.$$

$$t = 5 \text{ minutes} = 300 \text{ seconds.}$$

Loss of temperature due to cooling found to be negligible.

Resistance of spiral $= \cdot 274$ ohm $= \cdot 274 \times 10^9$ absolute units $=$ R.

We have

$$C^2 = \frac{4\cdot 2 \times 10^7 \times 971\cdot 2}{\cdot 274 \times 10^9 \times 300}, \text{ or } C = \cdot 704 \text{ absolute unit.}$$

Hence the current passing through the circuit, comprising four Grove's cells, the calorimeter, and the measuring instrument, was 7·04 ampères. The mean reading of the current measurer was 20°, or its constant is

$$\frac{7\cdot 04}{20} = \cdot 352 \text{ ampère per degree.}$$

Assuming the deflection to be simply proportional to the current as certified by the maker.

104. *Comparison of Constants of Coils.*—The constant of a standard coil having once been accurately determined may be used for obtaining the constant of any other coil when this does not differ greatly from that of the standard. The method of operation consists in placing the two coils in series with a battery. The same current thus being passed through the two coils, the constants will be in the ratio of the tangents of the angles of deflection. For in the one galvanometer we have

$$C = K \tan \theta_1,$$

while in the other, the constant of which is k and deflection θ_2,

$$C = k \tan \theta_2.$$

Hence

$$k = K \frac{\tan \theta_1}{\tan \theta_2}.$$

In this determination the instruments must be placed

sufficiently far apart to prevent them from influencing each other. If we desire to obtain the true constants (*i.e.* the constants not involving the horizontal magnetic force) from the working constants, it will be desirable to obtain H or the horizontal force separately for the situation of each instrument. Of course this does not imply that the earth's horizontal force is appreciably different at these two situations, but that the disposition of local iron may

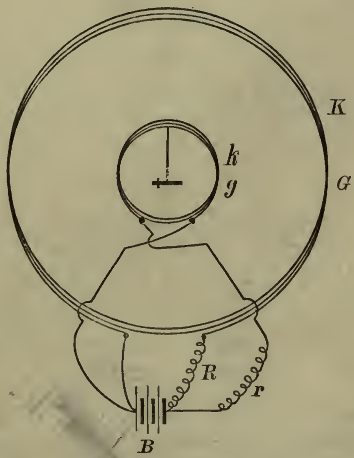


Fig. 136.

render the value of the horizontal magnetic force at the one situation sensibly different from that at the other.

Where the standard coil is much larger than the instrument under comparison it will only be necessary to know the value of H in one place, for in this case the smaller coil may be placed inside and concentric with the larger one, and, by a divided current, the one balanced against the other. The method then becomes a zero one. Fig. 136 shows the necessary arrangement. Here the current

from the battery B is split between the two coils, and so supposed to pass round them in opposite directions, and the resistances R and r are adjusted until there is no deflection of the centrally suspended magnet. If C denote the current in the large and c that in the small coil, then

$$\frac{c}{C}=\frac{R+G}{r+g},$$

G and g being the respective resistances of the two galvanometers, but

$$c=k\tan\theta,$$

and

$$C=K\tan\theta,$$

θ being the deflection which would be produced if either coil acted alone, but whose value we do not require to know. Hence

$$k=K\frac{R+G}{r+g}.$$

From this expression the true constant, that is to say, the constant independent of locality, or Γ, may be obtained by dividing by the local value of H.

Lesson XLVIII.—Galvanometer Constant from Known Current.

105. *Exercise.*—To find the value of K for the several coils of a tangent galvanometer.

Apparatus.—Bunsen's or Grove's cells; a cell of known E. M. F. for use as a standard, as a Latimer Clark cell. The apparatus of Lesson XIX., for comparing the E. M. F. of the cells by the method of high resistance should be accessible to the student. The other apparatus required will be that of the immediately preceding lessons.

Method.—(1.) Determine, by the method of high resistance, the relative E. M. F. of the Clark and Bunsen (or Grove), and calculate the E. M. F. of the latter in volts on the assumption that the Clark = 1·45 volt. (2.) Find the resistance of the Bunsen or Grove by the half-deflection

method or ordinary method (Lesson XXXIX.) (3.) Place the Bunsen in circuit with the coil of the galvanometer whose constant is required and a box of resistance coils. Vary the latter until the deflection is at or near 45°. These three processes will give all the needed data.

Example.—

Determination of E. M. F. of the Bunsen.—

12,000 ohms and $\frac{1}{999}$ shunt of high resistance reflecting galvanometer in circuit with Clark = 68 divisions.

12,000 ohms and $\frac{1}{999}$ shunt of high resistance reflecting galvanometer in circuit with Bunsen = 90 divisions.

Hence Bunsen $= \dfrac{1{\cdot}45 \times 90}{68} = 1{\cdot}92$ volt.

Determination of Internal Resistance of the Bunsen—Copper strap used.—

Resistance.	Deflection.	Tangent.
0	48·3	1·122
·4	29·9	·575

$$B+G = \frac{{\cdot}4 \times {\cdot}575}{{\cdot}547} = {\cdot}42.$$

Constant K of Copper Strap.—

$$K = \frac{E}{(B+G+R)\tan\alpha} = \frac{1{\cdot}92}{{\cdot}42 \times 1{\cdot}122} = 4{\cdot}074.$$

Constant K_1 of Coil I.—G = ·17. With 11·3 ohms gave 45° deflection.

$$K_1 = \frac{1{\cdot}92}{{\cdot}17 + {\cdot}42 + 11{\cdot}3} = {\cdot}1615.$$

106. *Additional Exercises on the Use of the Tangent Galvanometer.*—A tangent galvanometer, whose constants are known, is of great value in the laboratory for ascertaining the current required for telegraphic instruments, for ascertaining rapidly the condition of a battery, and for the graduation of simple galvanometers. These uses will furnish the student with additional exercises, such as we give below.

(1.) Ascertain the current in ampères that will be sufficient to ring an electric bell.

(2.) Find the figure of merit of a Morse telegraphic instrument, *i.e.*, the current which will be sufficient to give distinct and recognisable signals.

Example.—The instrument, which was of the type of Siemens and Halske, was adjusted according to the directions in Preece's *Telegraphy*, p. 69. It was put in circuit with a Morse key, box of coils, and twelve Daniell's cells, and tangent galvanometer provided with commutator. With 1700 ohms out of the box the instrument would just work, the spring being adjusted until it was as weak as possible, and the magnet being very near the armature. The mean deflection α on the tangent galvanometer was $13°\cdot3$, the constant of galvanometer = $\cdot0871$. Hence $C = K \tan \alpha = \cdot0871 \times \cdot236 = \cdot0206$ ampère nearly, or $20\cdot6$ milliampères.

(3.) Ascertain the current that a bichromate cell will give from time to time when working in short-circuit.

Example.—A bichromate cell was placed in circuit with the copper strap of a tangent galvanometer, the total external resistance being ·15 ohm. The following readings were taken :—

Time.	Deflection α.	$K \tan \alpha =$ Ampères.	
45	36	2·96	
50	35·9	2·95	
53	35·1	2·86	
54	35	2·85	
55	34·7	2·82	$K = 4\cdot074$
56	34·4	2·79	
57	34	2·75	
58	33·5	2·70	
59	33	2·65	

On stirring the liquid of the cell the deflection rose to $52°\cdot5 = 5\cdot31$ ampères, but in ten minutes later the deflection was $25\cdot7 = 1\cdot96$ ampère. The bichromate cell is thus seen to be under these conditions extremely inconstant. By keeping the liquid continually stirred, or by blowing air through the cell, it became very constant.

107. *Use of a Tangent Galvanometer of High Resistance.*—A tangent galvanometer wound with fine German silver wire is sometimes called a *Potential Galvanometer* or *Voltmeter*, on account of its use for the comparison of potentials (*i.e.* electromotive forces). A galvanometer of this kind should have at least the resistance of 5000 ohms, consisting of 6-7000 turns of fine German silver wire wound on a hoop about 60 mm. diameter. The principle of the instrument has already been employed by the student (see Lesson XX.), where the problem has been to measure the difference of potential at two points in a circuit through which a current is flow-

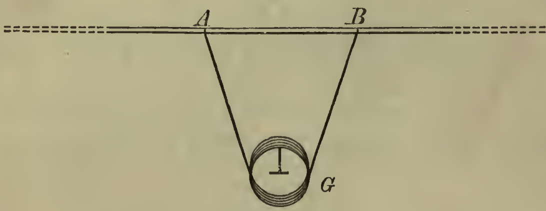


Fig. 137.

ing. Thus, suppose A, B (Fig. 137) to be two points in a circuit through which a current is flowing, and that we wish to find the difference of potential E between these points. This may be done by shunting an infinitesimal amount of the current through a galvanometer G, of very high resistance compared with that between A and B. Call

$$\begin{aligned} \text{Resistance of AB} &= r, \\ \text{Resistance of G} &= \text{R}, \\ \text{Current flowing in circuit} &= \text{C}. \end{aligned}$$

By Ohm's law

$$\text{C}r = \text{E} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

when the galvanometer is out of circuit, but on placing it in circuit the combined resistance between A and B is

$$\frac{r\text{R}}{r+\text{R}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

hence the difference of potential between A and B will be somewhat lower. Call it E_1. Then

$$C\frac{rR}{r+R}=E_1 \quad . \quad . \quad . \quad . \quad . \quad (3)$$

or on combining (3) with (1)

$$E_1=\frac{ER}{r+R} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

But E_1 must be nearly equal to E when, as we have supposed, r is small compared with R. Hence for many purposes we write

$$E_1=E \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Whether this may be assumed or not, if only we can measure E_1, we shall be able to deduce E. To find E_1 we must be able to measure the current C_1 that this difference of potential causes through the circuit of the galvanometer, and here

$$E_1=C_1R \quad . \quad . \quad . \quad . \quad . \quad (6)$$

but if the galvanometer be a tangent one

$$E_1=(K \tan \alpha)R \quad . \quad . \quad . \quad . \quad (7)$$

Hence to determine E_1 it will be necessary to know both the constant K, the resistance R, and the deflection α. The next lesson will give examples of the determination of the constants and use of a high resistance tangent galvanometer.

Instead of an ordinary tangent galvanometer we may employ the instrument of Sir William Thomson, in which the compass box slides along a graduated platform, after the manner of Fig. 35. This instrument has been so well described elsewhere[1] that no further description will be given.

[1] See *Absolute Measurements in Electricity and Magnetism*, by Professor Andrew Gray. Macmillan and Co.

Lesson XLIX.—Use of Potential Galvanometer.

108. *Examples.*—(I.) A standard tangent galvanometer whose constant was known to be ·0023 was placed in series with a potential galvanometer (Thomson's pattern) whose resistance was 6460 ohms, and with 48 Daniell's cells. The deflection on the standard instrument was 63°·5, and that on the potential galvanometer, when the compass was placed at position marked $\frac{1}{8}$ on the platform, was 22. Hence the current was $·0023 \tan 63°·5 = ·0046$ ampère. Hence the E. M. F. at the potential galvanometer terminals was $6460 \times ·0046 = 29·7$ volts.

Now the Thomson instrument is graduated, so that

$$\text{Number of volts (V)} = \frac{\text{Strength of magnetic field (H)} \times \text{deflection (D)}}{\text{Platform reading (P)}}.$$

Taking strength of magnetic field as ·17, we find

$$\text{Number of volts} = \frac{·17 \times 22}{\frac{1}{8}} = 29·9,$$

which agrees well enough with the ascertained value.

(II.) A Daniell's cell whose E. M. F. was said to be 1·072 volt was connected with a Thomson's potential galvanometer, and the following readings taken, when H was ·18 :—

Division in Platform (P).	Deflection (D).	E. M. F. calculated.
6·52	38·5	1·063
4	23·6	1·062
2	11·8	1·062

(III.) The range of a potential galvanometer may be increased by using a strong magnet to strengthen the magnetic field of the compass needle. The following example will show how the number representing the increased strength of field (M) may be found.

A constant battery of 24 Daniell's cells gave the following readings :—

$$\text{Without magnet } P = \tfrac{1}{8}, \quad D = 30, \quad \text{or } V = \frac{H30}{\frac{1}{8}}.$$

$$\text{With magnet } P = 6{\cdot}52, \quad D = 25{\cdot}9, \quad \text{or } V = \frac{(H+M)25{\cdot}9}{6{\cdot}52}.$$

Hence

$$\frac{H30}{\frac{1}{8}} = \frac{(H+M)25{\cdot}9}{6{\cdot}52},$$

or

$$M = 59{\cdot}4H,$$

$$\text{taking } H = {\cdot}172 \text{ C. G. S unit, } M = 10{\cdot}22.$$

(IV.) It was desired to find the E. M. F. at the terminals of an arc lamp.

Whilst the lamp was burning its terminals were connected with a high resistance tangent galvanometer whose resistance was 5000 ohms. The directing magnet was lowered until the deflection was 45°. A table of constants belonging to the galvanometer showed that in this position $K = {\cdot}01$, hence

$$V = 5000 \times {\cdot}01 \times \tan 45° = 50 \text{ volts.}$$

(V.) An incandescent lamp had a difference of potential equal to 40 volts at its terminals as measured by a high resistance galvanometer. A second tangent galvanometer of low resistance, placed in the main circuit, showed that a current of 1·2 ampère was passing. From these data the resistance of the lamp hot must be

$$R = \frac{E}{C} = \frac{40}{1{\cdot}2} = 33{\cdot}3 \text{ ohms.}$$

CHAPTER VI.

DETERMINATION OF THE MAGNETIC ELEMENTS.

109. This chapter will be devoted to a description of the instruments employed in the English magnetic observatories, and to the method of using these for the purpose of determining the three terrestrial magnetic elements, namely :—

(1.) Dip or Inclination, this being the angle which the magnetic axis of a magnet freely suspended in the plane of the magnetic meridian makes with the horizon.

(2.) Horizontal Intensity.

(3.) Declination, or the angle which the magnetic axis of a freely suspended horizontal magnet makes with the geographical meridian.

Lesson L.—Magnetic Inclination by the Dip Circle.

110. *Apparatus.*—The dip circle, exhibited in Fig. 138, the most essential part of which is the magnetic needle nn', about 3 inches in length, with pointed extremities. It is represented in Fig. 139 suspended on its supports. The axles of the needle consist of two very fine cylinders of hardened steel at right angles to the plane of the needle, and the perfect condition of these axles as to polish and dryness is a point of essential importance. The axles are

fitted so as to rest on two horizontal agate rounded edges *aa′*, the one axle lying on the one edge, and the other axle on the other. The instrument is provided with two

Fig. 138.—THE DIP CIRCLE.

needles, which we shall distinguish as No. 1 and No. 2. The poles are lettered with, say, α and β, the lettered side being called *the face.* When the needles are not in use

they should be kept in a box containing quicklime to prevent the oxidation of the axles. The needle in use is enclosed within a mahogany box ABCD, having a plane glass front, through which the ends of the needle are seen, and a ground-glass back, that gives the proper kind of illumination for viewing the ends of the needle distinctly. By turning the milled head *a* (Fig. 138) the arm *v′* (Fig. 139) is rotated, causing two Y-shaped pieces of metal (one of which is seen below *v*, Fig. 139) to lift the needle from the agate planes. A pair of bar magnets and a wooden frame for holding the needles during magnetisation are provided with the dip circle. For recording the observations blank schedules, as used at Kew, should be adopted.

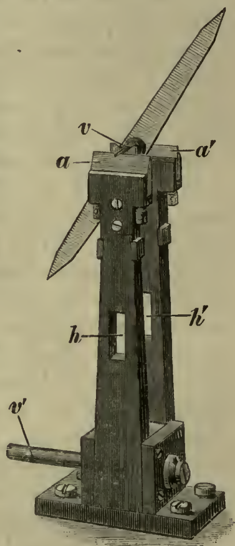


Fig. 139.
DIP NEEDLE AND SUPPORT.

If the vertical plane in which the needle swings be the magnetic meridian, if the centre of gravity of the needle coincides exactly with its axis of motion, and the axis of figure of the needle with its magnetic axis, and if there be no friction or adhesion between the axles and the agate edges, the needle will settle in such a position as to indicate the true magnetic dip. The positions of the two ends of the needle are observed by means of two microscopes *m*, *m′*, which are attached to, and move round with a cross-piece carrying the verniers, for reading off the position on the vertical circle V (Fig. 138). If we wish to determine the position of the upper end of the needle we move the upper microscope round until the cross-wire seen in its field of view lying along the line between the two microscopes

symmetrically cuts the upper extremity of the needle. The screw *c* is then clamped, and a perfectly accurate adjustment made by the tangent screw *tt'*. The upper vernier is then read by the help of the reading glass *g'*. The same process is repeated for the lower vernier, and the mean of the two readings is taken. The vertical circle is so graduated that this mean will accurately denote the magnetic dip, provided all the adjustments are accurate.

For the purpose of bringing the plane of the needle into the magnetic meridian the body of the instrument rotates about a vertical axis, and may therefore be turned in azimuth into any position which may then be read by the vernier (not seen in the figure) of the horizontal circle H.

Sources of Error.—With regard to the needle there may be (1.) a want of symmetry in mass, that is to say, the centre of gravity of the needle may not coincide with its axis of motion; (2.) a want of symmetry in magnetism, that is to say, the magnetic axis may not be coincident with the axis of figure; (3.) there may be friction or adhesion of the axles as they rest upon their agate supports. In the next place, with regard to the instrument; (4.) the axis about which the vertical circle moves may not be vertical; (5.) the plane in which the observation is made may not be the magnetic meridian; (6.) the vertical circle may be erroneously set so that the line between 90° above and 90° below is not strictly a vertical line; (7.) and lastly the axis of rotation of the needle may not pass through the centre of the vertical circle, thus causing an error due to this eccentricity.

Method of Observation—I. Preliminary Adjustment.—The instrument having been placed upon a stone pedestal of a suitable height, we proceed to (1.) make sure that the axis about which the upper part of the instrument moves is truly vertical. This adjustment is made by means of the level *l* attached to the instrument on the same principle as the corresponding adjustment in the cathetometer

(Vol. I. p. 33). When the axis has thus been made vertical, it may be taken for granted that the divided circle is vertical likewise. (2.) Next the plane in which the observation is made must be that of the magnetic meridian. To ensure this, set, in the first place, the *upper* vernier to 90°. Then take one of the two dip needles (say No. 1) and clean its axles by inserting them gently into pieces of soft cork, and clean also the agate knife-edges by rubbing them with cork. Having done this, let the needle rest on its knife-edges. Next turn the movable part of the instrument round in azimuth until the *face* of the instrument (that is to say, the part bearing the vertical circle) is towards the *south*, and the upper extremity of the needle is exactly bisected by the thread of the upper microscope. In order to do away with adhesion or friction, raise now the needle in its bearings by means of the head C, and then allow it gently to fall on the agate planes once more. If, when this is done, the thread of the microscope still nearly bisects the end of the needle, a second adjustment should be made, that is to say, the circle must be turned round in azimuth until the upper end of the needle shall be precisely bisected by the thread of the upper microscope, when the instrument is prevented from further turning by making use of the appropriate clamp screw. The vernier of the horizontal circle is then read. Call the reading A. (3.) Let us independently perform a similar operation as regards the lower extremity of the needle and its microscope. Call the reading of the horizontal circle A′. The above observations have been made, let us suppose, with the face of the needle turned to the face of the instrument. (4.) Let us now reverse the needle so that the back of the needle is turned to the face of the instrument, and make similar readings of the horizontal circle. Call these readings B, B′. (5.) Next turn the movable circle round in azimuth through 180°, until the face is towards the *north*, and re-

peat these sets of observations. Let the new readings be C, C′ and D, D′.

We have thus eight readings of the horizontal circle, the mean E of which will give the true azimuth for verticality of the dipping needle, and a position of the movable circle = 90 + E, will denote the magnetic meridian. The horizontal, like the vertical circle, is not graduated above 90°, so that the same reading of it which corresponds to verticality of the needle denotes in the next quadrant of the horizontal circle the true meridian.

II. Determination of the Dip.—Having determined by these means the plane in which we are to observe the needle, let us now proceed with our observation of dip.

(1.) Presuming that the axles of the needle have been properly cleansed by the method alluded to, let us turn the face of the instrument to magnetic *east*, and let the face of the needle be *towards* the face of the instrument. Now set the cross-piece carrying the microscope and vernier so that the thread in the upper microscope shall bisect the *upper* extremity of the needle. Then raise the needle by means of the lifter, and gently let it down again. If it comes very near its previous position we may adopt the reading given by the vernier as that of the upper extremity of the needle.[1] (2.) Next perform independently a precisely similar operation for the *lower* end of the needle, and read its position as given by the lower vernier. We have thus read both ends of the needle. (3.) Now turn the face of the instrument to the magnetic *west*, and repeat in this position the above observations. (4.) Then *reverse* the needle and repeat them, and then, keeping the needle reversed, turn the face to magnetic *east* and repeat the first set of observations once more, with the difference that the *back* of the needle is now turned to the face of the instrument.

Of the two extremities of the needle, which are marked

[1] The new position, by setting the vernier to it, will give us a second reading (see example).

α and β, let us suppose that α dips. We have thus made in all eight observations, as follows :—

		Upper Extremity.	Lower Extremity.
Face of instrument east,	Face of needle to face of instrument . .	A_α	A'_α
,, ,, west	,, ,,	B_α	B'_α
,, ,, ,,	Back of needle to face	C_α	C'_α
,, ,, east	,, ,,	D_α	D'_α

(5.) We must now reverse the polarity of the needle by the method of "double touch," so as to make β dip. To do this let us lay the needle, AB, Fig. 140, in the wooden frame FF′ made to receive it, and hold it tight by slipping over the centre the holder C, and by means of the two bar magnets provided for the purpose let us rub the upper side of the needle with the appropriate poles of the

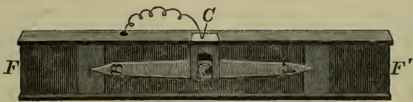


Fig. 140.—Reversal of Polarity.

bar magnets. The bar magnets should be held, one in each hand, nearly in a vertical position, and drawn along the grooves in the wooden frame from the centre of the needle towards its ends. Repeat this operation ten times. Next turn the needle back to front, and perform a similar series of operations upon its other side. (6.) We may now suppose the needle to be saturated with magnetism, the end β dipping. Having cleaned its axles with cork, let us now proceed to make with it a series of eight observations, precisely analogous to these already described. Call these

$$A_\beta, A'_\beta;\ B_\beta, B'_\beta;\ C_\beta, C'_\beta;\ D_\beta, D'_\beta.$$

The observation is now complete, and the mean of the sixteen readings will give us the true dip. (7.) For a

complete observation it is desirable that the whole of the processes should now be repeated with needle No. 2. If the circle and needles are good, and the observation be well made, the values for the dip given by the two needles will differ only very slightly from one another.

Theory of Adjustments.—(I.) First of all, with regard to the method of determining the magnetic meridian, it must be remembered that the needle is only free to move in a plane perpendicular to its axis. Now, should this plane of free motion be at right angles to the magnetic meridian, the resolved portion of the horizontal magnetic force acting in this plane will be zero, or, in other words, the needle will not be under the influence of the horizontal magnetic component at all. It will, however, continue to be influenced by the vertical component, and will therefore, if correctly constructed, place itself in a truly vertical position.

(II.) The various reversals made in the process of determining this vertical position are rendered necessary by the possibly faulty construction of the needle and the imperfect placing of the vertical circle, and their object, as well as that of the other reversals necessary to a complete observation, will now be described. A needle, assuming that its axle is truly cylindrical, may yet be imperfect in three ways:—

(α) Its centre of mass may not coincide with its centre of motion as regards the length of the needle.

(β) Its centre of mass may not coincide with its centre of motion as regards the breadth of the needle.

(γ) Its magnetic axis may not coincide with its axis of figure.

Again, the vertical circle may not be properly set in such a manner that the line between the upper and lower reading of 90° is truly vertical. Further, the axis of motion of the needle may not pass through the centre of this circle. This last error, or that caused by

eccentricity, is overcome (Vol. I. p. 50) by reading both ends of the needle.

When the needle is reversed in its bearings the action of the needle errors (β) and (γ) will be likewise reversed. The student may assure himself of this statement by making needles of tissue paper and denoting the magnetic axis by an ink line and the centre of gravity by a dot of ink. It will at once be seen that if the action of either error is (say) to increase the dip when the face of the needle is towards the observer, it will act so as to diminish the dip when the needle is reversed.

When the face of the circle is turned round through 180° the extremities of the needle are brought into different quadrants of the vertical circle. If, therefore, the points (90°) have been erroneously set, so as to make the needle read too low in the previous position, it will now read too high, and thus by taking a mean of the two the error caused by an erroneous setting of the circle is avoided. Another advantage of this reversal of the vertical circle is that new points of the steel axle are brought in contact with the agate plane.

The only error left uncompensated is (α), for it will be noticed that during all these changes its position with respect to the axis of motion remains unreversed.

This error is got rid of by reversing the poles of the needle. For if, when the first series was made, the centre of mass should have happened to be below the axis of motion, thus causing a moment tending to increase the dip, after the reversal, the same centre of mass will be above the axis and thus cause a moment tending to diminish the dip. The student may render this point obvious to himself by means of tissue-paper needles.

Having thus described the reason for the various steps of the process, it only remains to state that in the determination of the position of verticality it is obviously unnecessary to reverse the poles of the needle, inasmuch as any dis-

placement of the centre of mass of the needle with regard to its length could have no effect in altering its verticality. It is only when the needle assumes a non-vertical position that this can be influenced by the error in question.[1]

Example.[2]—

Kew Observatory, 24th November 1885.—Circle No. 33; setting of azimuth circle (determined by the method described above) corresponding to magnetic meridian 65° 44′; time 2 h. 18 m. to 2 h. 46 m. P.M.

Pole α Dipping.			Pole β Dipping.		
A_α	A'_α	mean	A_β	A'_β	mean
67 21	67 26		67 25	67 29	
23	32	67° 25′·50	28	28	67° 27′·50
B_α	B'_α		B_β	B'_β	
68 10	67 56		67 34	67 25	
10	54	68° 2′·50	33	30	67° 30′·50
C_α	C'_α		C_β	C'_β	
67 51	67 46		67 54	67 38	
50	47	67° 48′·50	51	38	67° 45′·25
D_α	D'_α		D_β	D'_β	
67 32	67 34		67 17	67 28	
33	34	67° 33′·25	21	28	67° 23′·50
Mean of means		. 67° 42′·44	Mean of means		. 67° 31′·69
Mean of all the observations, 67° 37′·06.					

111. The horizontal magnetic intensity and the declination may be accurately determined by the portable unifilar

[1] For the mathematical theory of the dip needle, see a *Treatise on Magnetism* by Sir George Airy, pp. 83-90.

[2] For the examples of observations of the three magnetic elements we are indebted to Mr. George Whipple, director of the Kew Observatory; and to Mr. T. W. Baker, magnetical observer there.

magnetometer (Kew pattern), which is suitable not only for fixed observatories, but also for magnetic surveys.

The student having been made familiar in Chap. II. with the general principles of the determination of the horizontal intensity with simple apparatus, we shall describe at once the details relating to the Kew instrument. It should, however, first be remarked that many instruments have been graduated for the Foot-Grain-Second system. To convert the value of H so obtained to the C. G. S. system, it is necessary to multiply by ·04611.

Lesson LI.—Determination of Time of Vibration.

112. *Apparatus.*—The Kew portable magnetometer, a chronometer, and a blank schedule, as used at Kew. A general view of the instrument fitted for the lesson is seen in Fig. 141.

The vibration magnet M_1 (Fig. 141) is suspended by means of a thread (freed from torsion as much as possible) from the torsion head A, and so placed in the centre of a wooden box provided for the purpose, light being thrown upon its scale by means of the transit mirror C, and the divisions of this scale being read by the telescope T_1 of the instrument. The portion of the instrument supporting the telescope and wooden box is capable of rotation about an axis fixed to the base E, which is provided with a circular scale.

At B, Fig. 142, is seen an enlarged view of the vibration magnet. It consists of a hollow steel cylinder, having a glass scale at one end and a lens at the other, the scale being at the chief focus of the lens. It is held within a short brass tube, within which it just fits. A second brass tube *b* is fixed above B for holding the brass inertia bar A, used for determining the moment of inertia (see Vol. I., Lesson LIV.)

Arrangement of Apparatus.—(1.) The instrument should be placed upon the top of a small stone pedestal that is

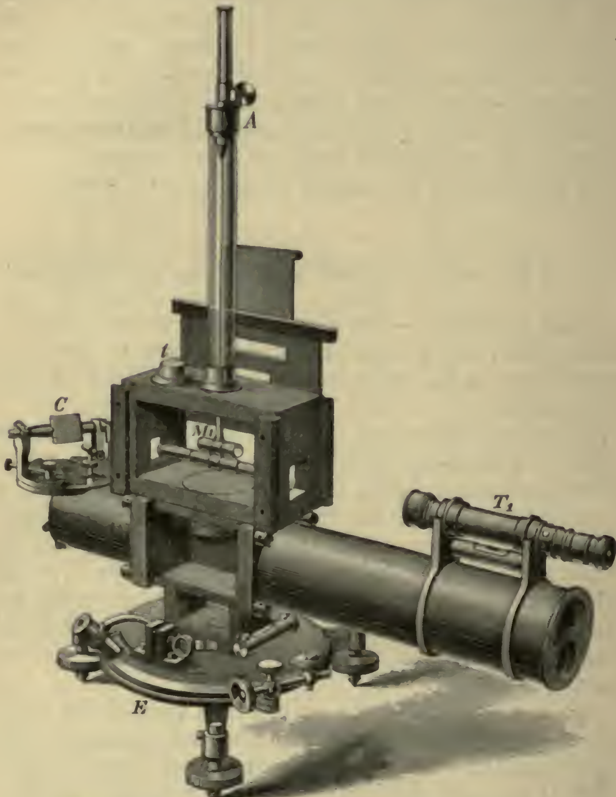


Fig. 141.—Magnetometer fitted for Vibration Observation.

about 3 feet high, so that when the observer is in a sitting position the telescope may be about the level of his

eye. (2.) Fix the wooden box in position. (3.) Focus the eye-piece of the telescope so that the cross-wire is seen distinctly, then adjust the instrument for viewing distant objects. Place the telescope in its Y supports, and turn the body of the magnetometer until the axis of collimation of the telescope lies approximately in the magnetic meridian, the telescope pointing to the north. (4.) Level by means of the cross level on the stand, and that on the telescope. (5.) Turn the mirror C until the field of the telescope is well illuminated. (6.) Screw the glass tube with its torsion head into its place, the silk fibres being meanwhile made stationary inside the tube by a cork thrust in its open end. (7.) Suspend a plummet having a cross bar to the ends of the fibres, allow it to hang for some time, and turn the torsion head until the cross bar lies in the magnetic meridian. (8.) Carefully replace the plummet by the vibration magnet, remembering that the end of the magnet marked N must be placed northwards. (9.) Adjust the magnet to the right height, so that its axis lies in that of the telescope. Bring the magnet to rest, when its scale should be distinctly seen in the telescope. The scale of the magnet is exhibited in Fig. 143.

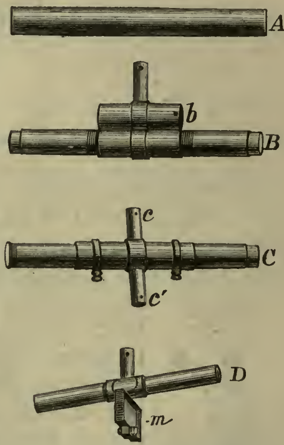


Fig. 142.—Magnetometer Magnets and Fittings.

Special Adjustments and Corrections—Setting Axis Horizontal.—It will be noticed that we have here two scales—a comparatively long scale and a short one. In the meantime let us consider the latter. The object of this is to enable us to make the magnetic axis horizontal.

It is clear that if this axis be not horizontal the vibration will not take place under the full value of the horizontal component of the earth's magnetic force, and we shall obtain an erroneous result. Further, it is clear that in

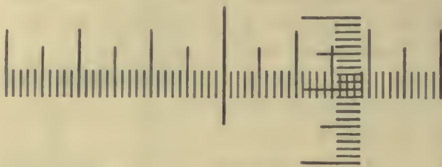


Fig. 143.—Scale of Magnet B.

order to render this axis horizontal the needle must be differently balanced in its stirrup in places for which the horizontal magnetic component is widely different. On this account it is extremely desirable that the observer should not trust to a ready-made adjustment, but should learn how to make it himself. For this purpose, having placed the divisions of the short scale *vertical*, he should notice the reading of this scale which corresponds to the *vertical* wire of the telescope. He should then turn the magnet round in its stirrup through 180°, and take the above reading once more. The mean of these readings will be that of the axis. Now let him turn the magnet round until the short scale divisions are *horizontal*, and if that point of it determined as above be now coincident with the *horizontal* wire, the axis may be regarded as horizontal.

Fig. 144. Apparatus for moving Magnet in Stirrup.

If not, there is an arrangement (Fig. 144) by which the tube, within which the magnet fits, may be grasped by *a*, *b*, and *c* when the ring *r* is pushed down, and the magnet's position in the stirrup altered by a slow downward movement of *s*′, produced by turning *s*. This should be done

until the short scale has the point which represents the axis coincident with the horizontal wire.

Moment of Inertia.—When this adjustment has been made it is desirable that the observer should determine for himself the moment of inertia of the magnet as it rests in its stirrup. This is done precisely after the manner described in Lesson LIV. of our first volume, so that the method need not here be repeated. It must not be forgotten that the moment of inertia so determined has a slight temperature variation.

Temperature and Induction Coefficients.—These are generally determined at some central magnetic station, such as the Kew Observatory; and the methods by which these determinations are made will be given in Appendix E. In the text we shall confine ourselves to a description of the part which they play. We have already (Chap. II.) mentioned that an increase of temperature diminishes to some extent the magnetism of a magnet, and that it recovers the loss, if not wholly yet to a very great extent, when it is brought back to its previous temperature. Now let t_0 denote a standard temperature, and let t be the temperature of observation; then we may assume the following expression for the magnetic moment m' at t—

$$m' = m\{1 - q(t - t_0) - q'(t - t_0)^2\} \quad . \quad . \quad . \quad (1)$$

where q and q' are coefficients which must be determined.

In the next place, with respect to induction, let V denote the vertical component of the magnetic force at the place of observation. If now we set the magnet on the table with the marked pole downwards, we shall have, besides the permanent magnetism, an induced effect produced in our magnet by the earth's vertical component. Let μ be the increase in the magnetic moment of the magnet that would be produced by an inducing influence equal to unity, then, under the above circumstances, an

effect $= V\mu$ would be produced, and it would act in the same direction as the permanent magnetism, so that the whole magnetic moment would be $m' + V\mu$. If now we place the magnet with the marked pole upwards its moment will be $m' - V\mu$, so that it is distinctly weaker in the latter position than in the former. Here μ denotes the induction coefficient; and in the case of the vibrating magnet oscillating under the action of a force mH, it will tend to augment this force, which will become $(m' + H\mu)H$.

Torsion.—We have thus defined these two coefficients which relate strictly to the magnetism of the bar. Now, could we suspend our magnet by a thread without torsion, the way would be clear for making a vibration observation. But this we cannot do; for, even if we suppose that the magnet rests at the zero of torsion, any alteration in its position will be resisted by the force of torsion of the thread, which must virtually be reckoned as a force to be added to that of the earth's magnetic action. It is clear then that we shall have to find the value of this force of torsion. To do this we must first ascertain the value of the long scale of our vibration magnet.

Let the horizontal circle be turned until the extreme left of this scale coincides with the vertical wire of the telescope, and in this position take the reading of the vernier of the horizontal circle. Turn the circle now until the extreme right of the scale coincides with the telescope wire, and take once more the reading of the vernier. On the assumption that the true position of the needle is virtually the same in both these observations, and knowing the number of divisions embraced in the horizontal scale, we can at once determine the value of a division. Thus, if D denote the difference between the two readings of the horizontal circle, and n the number of divisions of the scale, then $\frac{D}{n}$ will be the angular value of one division.

Example.—

		Reading on Circular Scale.	Reading on Magnet Scale.
1st Position.—	Vernier A	180° 0′ 0″	0
	Vernier B	0′ 40″	
	Mean	180° 0′ 20″	
2d Position.—	Vernier A	182° 19′ 0″	60
	Vernier B	19′ 20″	
	Mean	182° 19′ 10″	
		180° 0′ 20″	
	Difference	2° 18′ 50″	

Hence one division on magnet scale $= \dfrac{2^\circ\ 18'\ 50''}{60} = 2'\ 18''\cdot 8$ $= 2'\cdot 32$ nearly.

We are now prepared to measure the torsion by means of the torsion-circle, to which the suspension-thread is attached. The mode of observation will be seen from the following example:—The magnet having been brought to rest, the position of the telescope was exactly adjusted by means of the tangent screw until the cross-wire of the telescope stood at the middle division of the magnet scale. The torsion head was then turned through + 180° (clock-wise), and the scale reading was now 6·5. The torsion head was now turned back to its original position, and the reading – 0·6 taken; it was further turned in the same direction (counter clock-wise) to – 180°, which gave a scale reading of – 7·4. Finally, on coming back to 0°, the scale reading was – 0·2. Tabulating these results we have :—

OBSERVATION OF TORSION. (1 Scale Division $=2'\cdot32$.)

Torsion Circle. Scale.	Mean.	Difference.
$0° = \;\; 0\cdot0\ (a)$		
$+180° = +6\cdot5\ (b)$	(of a and c) $-0\cdot3=f$	(between b and f) $6\cdot8=h$
$0° = -0\cdot6\ (c)$		
$-180° = -7\cdot4\ (d)$	(of c and e) $-0\cdot4=g$	(between d and g) $7\cdot0=i$
$0° = -0\cdot2\ (e)$		
(Adding h and i and dividing sum by 4)		$13\cdot8$
Effect of 90° of torsion[1] in scale divisions $=3\cdot45$.		

Hence we find the effect of 90° of torsion on the position of the magnet to be, in angular measure,

$$3\cdot45\times2'\cdot32=8'\cdot004.$$

Now let Fd denote the force of torsion for a small angle d, and let Hmd be likewise the magnetic force called into play, then if d denote the angular deflection produced by 90° of twist, it is clear that

$$\frac{\mathrm{F}}{\mathrm{H}m}=\frac{d}{90°-d} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Application of Corrections.—Finally (Vol. I. p. 251), the following expression gives us the relation between the observed time of vibration of the magnet and the force in operation in accordance with the various corrections now indicated :—

$$\begin{aligned}\mathrm{T}_1{}^2&=\frac{\pi^2\mathrm{I}}{\{m\{1-q(t-t_0)-q'(t-t_0)^2\}+\mathrm{H}\mu\}\,\mathrm{H}+\mathrm{F}},\\ &=\frac{\pi^2\mathrm{I}}{m\mathrm{H}\left\{1-\mathrm{Q}(t-t_0)+\dfrac{\mathrm{H}\mu}{m}+\dfrac{\mathrm{F}}{\mathrm{H}m}\right\}} \quad . \quad . \quad . \quad (3)\end{aligned}$$

[1] This determination was made at Manchester, not at Kew.

if we denote by $Q(t-t_0)$ the whole temperature correction.

But the correct value of T corresponding to t_0 will be expressed as follows:—

$$T^2=\frac{\pi^2 I}{mH},$$

hence

$$T^2=T_1^2\left\{1+\frac{F}{Hm}-Q(t-t_0)+\frac{H\mu}{m}\right\} \quad . \quad . \quad . \quad (4)$$

where T_1 is the true observed time of vibration. In order to facilitate calculation of this formula the following table has been drawn up of the value of $1+\frac{F}{Hm}$ by the aid of equation (2):—

Table M.

Value of $1+\frac{F}{Hm}$ for Different Values of d.

Effect of 90° of Torsion $=d$.	$1+\frac{F}{Hm}$.	Effect of 90° of Torsion $=d$.	$1+\frac{F}{Hm}$.	Effect of 90° of Torsion $=d$.	$1+\frac{F}{Hm}$.
1′	1·00019	6′	1·00111	11′	1·00204
2′	·00037	7′	·00130	12′	·00223
3′	·00056	8′	·00148	13′	·00241
4′	·00074	9′	·00167	14′	·00260
5′	·00093	10′	·00185	15′	·00278

The values of the correction $Q(t-t_0)$, where Q is the coefficient which must be determined for the particular magnet in use, t is the temperature of observation, and t_0 the standard temperature, should also be embodied in a table.

Error of Chronometer and Arc of Vibration.—The time T_1 is liable to an error on account of chronometer rate, and also to one on account of the magnitude of the arc of vibration. Let s denote the daily chronometer rate in seconds, + when gaining, − when losing, then, since there are 86,400 seconds in a day, we have

$$\text{True time of vibration}=\text{observed chronometer time}\left(1-\frac{s}{86400}\right) \quad (5)$$

In order to facilitate calculation the value of $1 - \frac{s}{86400}$ for different rates of the chronometer is exhibited in the following table :—

TABLE N.

CHRONOMETER FACTORS.

Daily Rate, Seconds $= \pm S$.	Chronometer Gaining.	Chronometer Losing.
5	·99994	1·00006
10	·99988	1·00012
15	·99983	1·00017
20	·99977	1·00023
25	·99971	1·00029
30	·99965	1·00035
35	·99959	1·00041
40	·99954	1·00046
45	·99948	1·00052
50	·99942	1·00058

The next correction is on account of circular arc. We deduce from Vol. I. Art. 151 that the observed time is increased above the true time for infinitely small arc, thus

$$\text{True time} = \text{observed time} \left(1 - \frac{A^2}{16}\right) \text{ nearly,}$$

where A is the mean angle denoting the semi-arc of vibration expressed in circular measure. Let the angular value of this semi-arc be at the commencement of the vibration a and at the end a', then

$$\frac{\left(\frac{a+a'}{2}\right)^2}{16} = \frac{A^2}{16}.$$

But we see from Vol. I. p. 88 that in such a case as this the geometric and arithmetical mean are as nearly as possible identical, so that for the square of the one

we may substitute the square of the other. We may therefore say

$$\text{True time} = \text{observed time} \left\{1 - \frac{aa'}{16}\right\} \quad . \quad . \quad . \quad (6)$$

This correction is embodied for different values in the following table.

TABLE O.

VALUE OF $\frac{aa'}{16}$.

Semi-arc at Commence-ment in Minutes.	Semi-arc at End of Observation.					
	80′	70′	60′	50′	40′	30′
100	·00004	·00004	·00003	·00003	·00002	·00002
90	·00004	·00003	·00003	·00002	·00002	·00001
80	·00003	·00003	·00003	·00002	·00002	·00001
70		·00003	·00002	·00002	·00001	·00001
60			·00002	·00002	·00001	·00001
50				·00001	·00001	·00001

Embodying both corrections therefore, calling T_1 the true time and T_0 the apparent time of observation, we have

$$T_1 = T_0 \left\{1 - \frac{s}{86400} - \frac{aa'}{16}\right\} \text{nearly} \quad . \quad . \quad . \quad (7)$$

In general, however, these corrections may be neglected, and the observed time of oscillation reckoned as the true time corresponding to an infinitely small arc.

Observation of Time of Vibration.—The method of observing the time of vibration is very similar to that described in Vol. I. pp. 187-190. The observation should be recorded in the schedule, as shown in the next example. Having caused the magnet to oscillate through an arc of about 60′ on each side of the middle line of the scale, the times of every 5th passage are recorded until we arrive at the 55th, then the 0th is subtracted from the 50th, and the difference

added to the latter. This gives us the time at which the 100th transit will take place. In a similar manner the 5th is subtracted from the 55th, and the difference added to the latter gives the time of the 105th transit. The magnet is now allowed to continue swinging until the observer notes that the 100th transit is near, he then observes the actual time, and proceeds, then observing every 5th until the 155th is reached. Subtracting in succession the times of the 0th from 100th, 10th from 110th, 5th from 105th, etc., we obtain a series of numbers, each giving the time of 100 vibrations. The mean of the number of the right-hand transits is obtained, this is called "Mean (1);" also the mean of the left-hand transits, "Mean (2)." These means being in minutes, we convert them to seconds and divide by 100 to obtain the time of a single vibration in seconds. The mean of the two is T_0.

The student will at once perceive that in determining the time of vibration it is essential to make use of oscillations going from left to right, as well as of oscillations going from right to left. For, owing to the diurnal variation, and possibly other causes, the position of rest of the vibrating magnet may be gradually altered. Let us, for instance, imagine that this has gone to the right between the beginning and the end of our observation. Then the moment of crossing the wire for oscillations proceeding from left to right will be sooner than it ought to be; while, on the other hand, the moment of crossing for oscillations going the other way will be later than it ought to be. There may thus be a difference in the value of T_0 as derived from these two sets, but the mean of the whole will represent the truth. The following is an actual observation made at the Kew Observatory:—

Example.—

12th August 1884—Station, Kew Observatory.—Latitude, 51° 28′ 6″ N.; longitude, 1 m. 15 sec. W.; chronometer, A 86; error at station, + 5 m. 30 sec.; daily rate ($s.$) = − 10 sec.; magnet (62 A)

suspended, Q = 0·000304; log μ = 0·692643; effect of 90° of torsion = 2·7 div. = 5′·0; one division of scale = 1′·85; mean time at commencement, 5 h. 50 m.; mean time at end, 6 h. 2 m. Moment of inertia (I) at 25° C. = 286·27.

Semi-arc of vibration { 70′ 50′ }	{ Temp. of magnet } 25°·2 25°·6	{ Mean temperature Cor. for thermometer }	25°·4 C. − 0°·4 C.
			25°·0 C.

Scale moving apparently to Right.					Scale moving apparently to Left.				
No. of Vibrations. (a)	Time of centre passing wire. (b)	No. of Vibrations. (c)	Time of centre passing wire. (d)	Time of 100 Vibrations. (e)	No. of Vibrations. (f)	Time of centre passing wire. (g)	No. of Vibrations. (h)	Time of centre passing wire. (k)	Time of 100 Vibrations. (l)
	h. m. s.		m. s.	(1) m. s.		h. m. s.		m. s.	(2) m. s.
0	5 49 45·1	100	57 14·2	7 29·1	5	5 50 7·6	105	57 36·6	7 29·0
10	„ 50 30·0	110	57 59·1	7 29·1	15	„ 50 52·5	115	58 21·5	7 29·0
20	„ 51 15·0	120	58 44·0	7 29·0	25	„ 51 37·4	125	59 6·4	7 29·0
30	„ 52 0·0	130	59 28·9	7 28·9	35	„ 52 22·4	135	59 51·3	7 28·9
40	„ 52 44·8	140	0 13·7	7 28·9	45	„ 53 7·2	145	0 36·2	7 29·0
50	„ 53 29·7	150	0 58·7	7 29·0	55	„ 53 52·1	155	1 21·0	7 28·9
60	„ 54 14·6	160	1 43·6	7 29·0	65	„ 54 37·0	165	2 6·0	7 29·0
			h. m. s.					h. m. s.	
Diff. for 40 vibs. .			0 2 59·7		Diff. for 40 vibs. .			0 2 59·6	
100 at . .			5 57 14·3		105 at . .			5 57 36·6	
Mean of 1 vibration (1) = 4·4900 sec. Mean of 1 vibration (2) = 4·4897 sec. T_0 = mean of (1) and (2) = 4·4898 sec.									

Observer, T. W. BAKER.

Explanation of the Example.—Here the observer has proceeded somewhat differently from the order of the observations described above. The tabulation of the transits has been continued until the 65th was reached. Then subtracting the 0th from the 40th he obtained the difference for 40 vibrations = 2 m. 59·7 sec. This is added to the 60th vibration to obtain the time of the 100th vibration, which number is recorded at the bottom of the column. In a similar manner the time of the 105th is obtained.

Calculations of the Results.—

$$T_1 = T_0\left\{1 - \frac{s}{86400} - \frac{aa_1}{16}\right\},\quad T^2 = T_1^2\left\{1 + \frac{F}{Hm} - Q(t - t_0) + \mu\frac{H_0}{m_0}\right\},$$

$$mH = \frac{\pi^2 I}{T^2}.$$

$$1 - \frac{s}{86400} = \quad 1\cdot00012\ (a)$$

$$-\frac{aa'}{16} = -0\cdot00002\ (b) \qquad T_0 = 4\cdot4898\ \log = 0\cdot65223\ (h)$$

$$1 - \frac{s}{86400} - \frac{aa'}{16} = \quad 1\cdot00010\ (c) \qquad .\quad .\ \log = 0\cdot00004\ (k)$$

$$1 + \frac{F}{Hm} = \quad 1\cdot00093\ (d) \qquad T_1 \log = 0\cdot65227\ (l)$$

$$2$$

$$-Q(t - t_0) = -\ \cdot00760\ (e)$$

$$+\mu \div \frac{m_0}{H_0} = +\ \cdot00115\ (f) \qquad T_1^2 \log = 1\cdot30454\ (m)$$

$$1 + \frac{F}{Hm} - Q(t - t_0) + \mu \div \frac{m_0}{H_0} = \quad 0\cdot99448\ (g) \qquad .\quad .\ \log = 9\cdot99760\ (n)$$

$$T^2 \log = 1\cdot30214\ (o)$$

$$\pi^2 I \log = 3\cdot45107\ (p)$$

$$mH = \frac{\pi^2 I}{T^2}\log = 2\cdot14893\ (q)$$

$$\left.\begin{array}{r} \mu \log = 0\cdot69264\ (w) \\ \dfrac{m_0}{H_0}\log = 3\cdot63284\ (x) \\ \mu \div \dfrac{m_0}{H_0} = \cdot00115\ \log = 7\cdot05980\ (y) \end{array}\right\} \qquad \frac{m}{H},\ \text{see deflection example,}\ \log = 3\cdot63574\ (r)$$

$$mH \div \frac{m}{H} = H^2 \log = 8\cdot51319\ (s)$$

$$H = 0\cdot18055\ \log = 9\cdot25660\ (t)$$

$$m^2 \log = 5\cdot78467\ (u)$$

$$m = 780\cdot42\ \log = 2\cdot89233\ (v)$$

Explanation of the Calculations.—(a) By making $s = -10$, or taking the value directly from the Table N. (b) Here $a = 70'$ and $a' = 50'$, therefore

$$a = \frac{70 \times \pi}{180 \times 60}, \text{ and } a' = \frac{50 \times \pi}{180 \times 60} \text{ in circular measure ;}$$

hence
$$(b) = \frac{70 \times \pi \times 50 \times \pi}{180 \times 60 \times 180 \times 60 \times 16} = \cdot00002 \text{ nearly.}$$

This value agrees with that given in Table O. (c) Subtracting b from a. (d) From $\frac{5}{(90 \times 60) - 5}$, or from Table M. ($e$) From $25 \times \cdot000304 = \cdot0076$. ($f$) This is obtained from (w) and (x). (x) is obtained by taking the mean of logs of $\frac{m_0}{H_0}$ from the deflection table. On sub-

tracting (x) from (w) we obtain (y), of which the anti-logarithm is the value of (f). Note that 10 has been added to (y) to avoid negative characteristic. (g) is $d-e+f$; (h) is log T_0; (k) is log (c); (l) is $h+k$; (m) is $2l$; (n) is log g with 10 added; (o) is $m+n$; (p) is log $\pi^2 I$, I being 286·27 at 25° C.; (q) is (p) − (o). (r) See deflection example, when $\frac{m}{H}$ is the closest approximation obtainable by the deflection experiment; (s) is (q) − (r) + (w); $t=\frac{s+10}{2}$; (u) is (q) + (r); (v) is $\frac{(u)}{2}$.

Lesson LII.—Observation of Deflections.

113. *Arrangement and Description of Apparatus.*—Having now determined accurately the time of vibration of the collimator magnet, we have next to make use of it to deflect another magnet, but it will be necessary first to alter the arrangement of the apparatus as follows:—(1.) Remove M_1, and having secured the fibres within the tube by means of a cork, remove the glass tube with its torsion head A. (2.) Unscrew the wooden box and remove telescope and transit mirror. (3.) Fix the long graduated bar SS′ (Fig. 145) in its place by means of the pins provided, place upon it the carriage C for holding the vibration magnet M_1 and the thermometer shown. (4.) Replace the torsion head A and fix the deflection magnet M_2. (5.) Screw in its place the telescope T. (6.) It is desirable to surround the deflecting magnet with a wooden box containing a thermometer, by which the temperature of this magnet as well as that of the deflection rod may be accurately ascertained. The deflected magnet, seen at D, Fig. 142, is a hollow cylinder of hard steel about 0·8 cm. in diameter and 7·5 cm. in length, and it has underneath it, when suspended, a vertical mirror m at right angles to its length. It is suspended in a metallic box, the metal of which acts inductively as a damper, tending to bring the magnet to rest when it is in a state of vibration.

A reference to Fig. 145 will show the telescope T with a scale s above it. The telescope is attached to the lower part

of a hollow cylinder, the upper portion of which is either open or covered with glass. The light from the scale

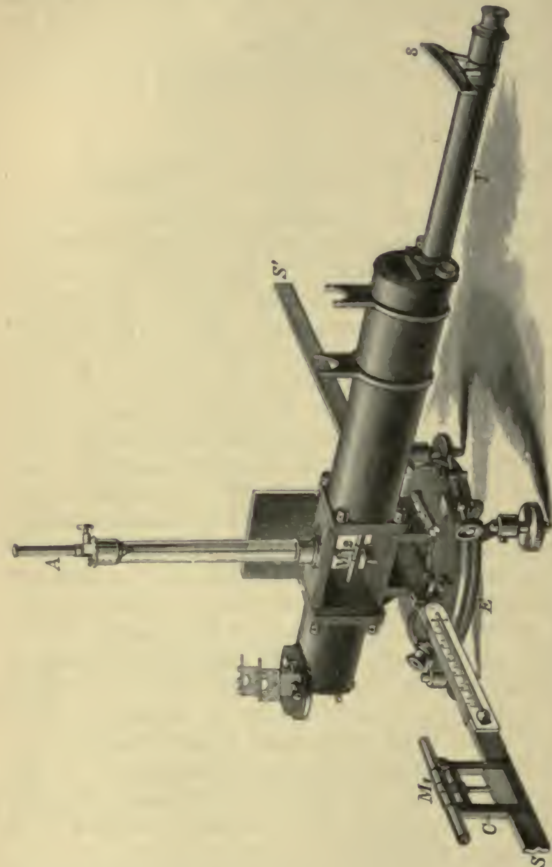


Fig. 143.—MAGNETOMETER FITTED FOR DEFLECTION OBSERVATIONS.

passes through this open space, and likewise through a

glass window in the metallic chamber which contains the suspended magnet, and is reflected back from the mirror attached to the magnet and through the telescope. This latter is so adjusted that an image of the scale is seen by the observer in the field of view of the telescope, which contains likewise a vertical reference wire.

Whenever the image of the central division of the scale is reflected upon this wire so as to coincide with it, we may assume that the magnetic axis of the magnet is either parallel to the visual axis of the telescope, or at least that the positions of these two axes bear a definite relation to each other, which is not far removed from that implied in parallelism.

When M_1 is placed on the carriage C, which slides upon the bar, by raising or lowering A, M_2 may be placed at the same level as the magnet M_1. Whenever, therefore, the deflected magnet is in this position, and at the same time the central division of the scale is seen to coincide with the cross-wire of the telescope, we may take it for granted (α) that the magnetic axis of the deflected magnet is parallel to the visual axis of the telescope, or nearly so; (β) that the magnetic axis of the deflecting magnet is perpendicular to that of the deflected magnet, or nearly so; and (γ) that a line drawn through the axis of the deflecting magnet will pass through the centre of the axis of the deflected magnet, or nearly so. By means of a small vernier on the carriage we may place the deflecting magnet in such a position that its centre precisely corresponds with a division of the scale of the deflection bar.

In every observation the movable part of the instrument has to be brought into such a position that the central division of the scale coincides with the wire of the telescope.[1] From this it is obvious that we have nothing to do with the torsion of the fibres, since in every observation their upper and lower positions remain unaltered as

[1] Or, in other words, the sine method is used (see p. 35).

regards the box; in fact the whole arrangement is carried round together without any twist of its separate parts. The position of this movable system in the various observations is ascertained by reading the verniers of the horizontal circle, assuming that the central division of the scale coincides with the telescopic wire. If, however, this central division does not exactly so coincide, it is easy to ascertain the angular value corresponding to one division of the reflected scale in the telescope by means similar to that which was employed to determine the scale of the collimator magnet (p. 290). When this value has been ascertained we may correct for non-coincidence between the middle division of the scale and the telescope wire.

Adjustments.—(1.) Turn the magnetometer in azimuth until the axis of the telescope lies in the axis of the suspended magnet. (2.) Focus the eye-piece of the telescope until the cross-wire is distinctly seen. (3.) See that the scale *s* is well illuminated either by means of a candle or, better still, by concentrating light upon it by means of a large lens from a distant lamp. (4.) The body of the telescope should now be focused in order to see the reflected image of the scale *s*, the magnet M_2 being meanwhile at rest. If the observer is not successful in seeing the image it is probably owing to the mirror of M_2 being tilted either too high or too low. To remedy this two screws are provided at the bottom of the mirror in order to give the requisite inclination. The adjustment is somewhat tedious, but once having been made should remain undisturbed. (5.) Remove M_1 and replace it by the *sighting tube*, consisting of a cylinder of the same diameter as M_1, with an axial aperture through which the suspended magnet M_2 may be observed, when the latter should have its height adjusted by lowering or raising the rackwork of A until the axis of the sighting tube and of M_2 are in the same plane. The magnet M_1 may now be replaced. (6.) If the reflected image should be dull it will probably be due to dirt or

moisture on the mirror or window. For cleaning the window at the end of the barrel a camel-hair brush with a handle sufficiently long to reach to the end of the barrel is used. (7.) The windows of the magnetometer box are closed with shutters of wood, one of which is seen in a raised position (Fig. 145).

Corrections—Length of Bar.—The deflection bar is compared at the central observatory with a standard bar in order to ascertain the graduation error, and, being of brass, its coefficient of expansion is known, so that the true distances between the graduations of the bar at different temperatures may be calculated for the instrument. If r_0 be the apparent distance between the centres of the deflecting and suspended magnets, and r the distance corrected for error of graduation and temperature, we shall have

$$r = r_0\{1 + \alpha(t - t_0)\} + \text{correction for scale error},$$

where α is the coefficient of expansion of brass, t_0 is the standard, and t the observed temperature. A table is compiled by the aid of this formula for a range of temperature from about $-5°$ C. to $+35°$ C.

Constant of Distribution.—The true formula for deflection is

$$\frac{m}{\text{H}} = \tfrac{1}{2}r^3 \sin u \left(\frac{1}{1 + \frac{\text{P}}{r^2} + \text{etc.}}\right).$$

Now it will be unnecessary in the above expression to make use of any constant beyond P, so that the following modified formula is generally adopted :—

$$\frac{m}{\text{H}} = \tfrac{1}{2}r^3 \sin u \left(1 - \frac{\text{P}}{r^2}\right),$$

where P is a constant depending on the distribution of magnetism in the two magnets employed. Since these magnets always remain the same, this constant may be determined by comparing together a series of values of $\frac{m}{\text{H}}$

made at different distances. It may be shown that the distances should, to obtain the best results, have the ratio of 1 to 1·3.[1] To find the value of P we proceed as follows :—

Let

$$A = \tfrac{1}{2} r^3 \sin u,$$

and

$$A' = \tfrac{1}{2} r_1^3 \sin u_1,$$

then

$$A\left(1 - \frac{P}{r^2}\right) = A'\left(1 - \frac{P}{r_1^2}\right);$$

hence

$$P = \frac{A - A'}{\dfrac{A}{r^2} - \dfrac{A'}{r_1^2}}.$$

Temperature Coefficient.—The vibration magnet being employed in this observation to deflect a suspended magnet, the amount of the deflection produced will of course depend upon the strength of the deflecting magnet, and hence upon its temperature. If the temperature of observation t be above the adopted zero t_0, the observed deflection will be too small. This may be rectified as follows : Let m_0 denote the observed moment of the magnet, and let m be its true moment at the adopted zero of temperature ; then, if

$$\frac{m_0}{H_0} = \tfrac{1}{2} r^3 \sin u_0,$$

$$\frac{m}{H} = \frac{m_0}{H_0} \left\{ 1 + Q(t - t_0) \right\}.$$

Induction Coefficient.—The axis of the deflecting magnet is in this observation at right angles to that of the deflected magnet. Now if the angle of deflection were zero, the axis of the deflecting magnet would be in a vertical plane at right angles to that which passes through the magnetic meridian, and hence there would be no inductive

[1] See Airy's *Magnetism*, Art. 30.

effect upon this magnet. If, however, u be the angle of deflection, the inductive effect upon the deflecting magnet will be

$$\mu\mathrm{H} \sin u = \mu\mathrm{H}\left\{\frac{2m}{\mathrm{H}r^3}\right\} \text{nearly} = \frac{2m\mu}{r^3} \text{ nearly},$$

in a direction tending to weaken the magnet. Hence, instead of the observed moment m_0, we must substitute

$$m_0\left\{1+\frac{2\mu}{r^3}\right\},$$

in order to obtain the true moment corrected for induction.

Order of Observations for a single Series.—(1.) Place M_1 upon its carriage at the distance of 30 cm. to the *east* of the suspended magnet, and with its *north* end towards the *east*. Turn the telescope in azimuth until the cross-wire is exactly or nearly exactly at the middle of the reflected scale, clamp the circle, bring the magnet to rest, and read the scale, verniers, and thermometers. (2.) Reverse M_1 with its carriage, and place it at the same distance *east* as before, but with the *north* end now to *west*. Make the same observations and adjustments as before. (3.) Place M_1 to *west* of the instrument, at 30 cm. distance from suspended magnet. *North* end to *west*. Read, etc. (4.) Everything as before, but *north* to *east*.

Method of Calculation.—This will be seen by a study of the following observation, made at Kew Observatory:—

12th August 1885.—Mean time commencing 4 h. 14 m. P.M., ending 5 h. 30 m. P.M. Magnet deflecting (62 A); suspended (62 C); one division of scale = 60·5 sec.

Distance.	Deflecting Magnet. North end.	Deflecting Magnet. Temperature cent.	Verniers.	Scale reading Divisions.	Correction to middle of Scale.	Mean of Verniers.	Corrected circle reading.	Means and Differences.
		°	° ′ ″		′ ″	° ′ ″	° ′ ″	° ′ ″
East. cm. 30	E	24·7	155 30 20 28 30	199·5	−0 30	155 29 25	155 28 55	155 20 5 118 14 2
40	W	24·6	129 6 50 5 0	200·3	+0 18	129 5 55	129 6 13	37 6 3 Diff.
40	E	24·5	144 30 20 28 10	200·0	0 0	144 29 15	144 29 15	18 33 1 = u_0
30	W	24·5	118 20 10 18 0	198·3	−1 43	118 19 5	118 17 22	
West. 30	W	24·6	118 10 0 8 0	201·7	+1 43	118 9 0	118 10 43	144 26 22 129 3 40
40	E	24·5	144 24 40 22 20	200·0	0 0	144 23 30	144 23 30	15 22 42 Diff.
40	W	24·5	129 2 30 0 20	199·7	−0 18	129 1 25	129 1 7	7 41 21 = u'_0
30	E	24·5	155 11 30 10 0	200·5	+0 30	155 10 45	155 11 15	

Mean Temp.

	°
	24·5
Correction to therm.	−0·4
	24·1

	° ′ ″
Observed angle of deflection at 30 cm.	18 33 1 = u_0
,, ,, 40 cm.	7 41 21 = u'_0

It will be noticed that the order adopted in this complete observation is such that if we name the above epochs (1), (2), (3), (4), (5), (6), (7), (8), and presume that each occupies the same time in its performance, then the mean epoch of u_0 is the same as that of u'_0. For the former is $\frac{(1)+(8)}{2} - \frac{(4)+(5)}{2}$, and the latter $\frac{(3)+(6)}{2} - \frac{(2)+(7)}{2}$, the numerators being thus the same.

$$\frac{m_0}{\mathrm{H}_0} = \tfrac{1}{2}r^3 \sin u_0, \quad \frac{m'}{\mathrm{H}'} = \frac{m_0}{\mathrm{H}_0}\left\{1 + \frac{2\mu}{r^3} + \mathrm{Q}(t - t_0)\right\}, \quad \frac{m}{\mathrm{H}} = \frac{m'}{\mathrm{H}'}\left(1 - \frac{\mathrm{P}}{r_0^2}\right).$$

	$r_0 = 30$ cm.	$r'_0 = 40$ cm.
$1 + \frac{2\mu}{r_0^3} =$	$1{\cdot}00037$ (a)	$1{\cdot}00015$ (a')
$(t - t_0)\mathrm{Q} =$	$\underline{{\cdot}00733}$ (b)	$\underline{{\cdot}00733}$ (b')
$1 + \frac{2\mu}{r_0^3} + (t - t_0)\mathrm{Q} =$	$1{\cdot}00770$ (c)	$1{\cdot}00748$ (c')

	$r_0 = 30$ cm.	$r'_0 = 40$ cm.
$\tfrac{1}{2}r^3$ log $=$	$4{\cdot}13090$ (d)	$4{\cdot}50572$ (d')
$\sin u_0$ log $=$	$\underline{9{\cdot}50261}$ (e)	$\underline{9{\cdot}12645}$ (e')
$\frac{m_0}{\mathrm{H}_0}$ log $=$	$3{\cdot}63351$ (f)	$3{\cdot}63217$ (f')
$1 + \frac{2\mu}{r_0^3} + (t - t_0)\mathrm{Q}$ . . log $=$	$\underline{0{\cdot}00333}$ (g)	$\underline{0{\cdot}00324}$ (g')
$\frac{m'}{\mathrm{H}'}$ log $=$	$3{\cdot}63684$ (h)	$3{\cdot}63541$ (h')
$1 - \frac{\mathrm{P}}{r_0^2}$ . . log $=$	$\underline{9{\cdot}99695}$ (i)	$\underline{9{\cdot}99830}$ (i')
$\frac{m}{\mathrm{H}}$ log $=$	$3{\cdot}63636$ (k)	$3{\cdot}63514$ (k')
mH, see vibration example, log $=$	$\underline{2{\cdot}14893}$ (l)	$\underline{2{\cdot}14893}$ (l')
$m\mathrm{H} \div \frac{m}{\mathrm{H}} = \mathrm{H}^2$ log $=$	$8{\cdot}51257$ (m)	$8{\cdot}51379$ (m')
H log $=$	$9{\cdot}25628$ (n)	$9{\cdot}25690$ (n')

Explanation of the Calculations.—Arranged in parallel columns are the details of the corrections for the distances 30 and 40 cm. (d) is log $\frac{1}{2}r^3$, and (d') is log $\frac{1}{2}r'^3$. These values are usually directly taken from tables giving the values for different temperatures. (a) log $\mu = \cdot 692643$ (see vibration example), hence $1 + \frac{2\mu}{30^3} = 1 \cdot 00037$. Similarly $(a') = 1 + \frac{2\mu}{40^3}$, ($b$) and $(b') = \cdot 000304 \times 24 \cdot 1$ (see vibration example; $(c) = (a) + (b)$, and $(c') = (a') + (b')$; (e) is tabular log of sin 18° 33′ 1″, and (e') is the same of 7° 14′ 21″; $(f) = (d) + (e) - 10$, and $(f') = (d') + (e') - 10$; (g) is log (c), and (g') is log (c'); $(h) = (f) + (g)$, and $(h') = (f') + (g')$; (i) and (i') are the logs of $1 - \frac{P}{r_0^2}$ and $1 - \frac{P}{r_0'^2}$. P is found by the formula given above. 10 is added to logs to prevent negative sign; $(k) = (h) + (i) - 10$, and $(k') = (h') + (i') - 10$; (l) or (l') is log mH; $(m) = (l) - (k) + 10$, and $(m') = (l') - (k') + 10$; $(n) = \frac{1}{2}(m - 10) + 10$, and $(n') = \frac{1}{2}(m' - 10) + 10$.

Lesson LIII.—Observation of Declination.

114. *Apparatus.*—The unifilar magnetometer as fitted for vibration will be required, but in place of the magnet then used we employ one in other respects similar, except that it may easily be suspended in a reversed position. C, Fig. 142, shows the declination magnet, which may be suspended either from c or c'. This magnet has a lens, and a scale at the chief focus of the lens.

Method.—(1.) As in the case of the vibration magnet, we must obtain a clear image of this scale through the observing telescope. The scale has a zero line, and the telescope has a cross-wire, and the instrument must be turned in azimuth until the zero line of the scale coincides with the cross-wire of the telescope. (2.) The horizontal circle must now be read. It is clear that if the line of optical collimation of the hollow steel magnet were to coincide with its magnetic axis, then the reading of the horizontal circle obtained as above would be that which represented the magnetic meridian. This, however, is rarely the case, and we must now (3.) suspend the magnet in a reversed position, and repeat the observation. The mean of the two sets of observations will denote the true reading

of the horizontal circle, which corresponds to the magnetic meridian. (4.) Great care must be taken in this observation to free the suspension thread from torsion. We may so arrange that in the beginning of the observation the thread shall be without torsion when the suspended plummet or magnet is in the magnetic meridian. This is done by removing the magnet and substituting a plummet with a cross bar of the same weight as the magnet, and allowing the bar to hang for some time and assume a position of rest. The torsion head of the suspension tube should now be turned until the bar of the plummet lies in the axis of the telescope. The magnet may now be replaced, but care must be taken that torsion is not given to the thread during the operation. Nevertheless we may find at the end of the observation that there is a sensible amount of torsion for the position. Hence we adopt the following plan. Begin with a zero of torsion, and having determined the torsion at the end, assume that half this represents the mean value of the torsion during the observation. (5.) We have thus, let us suppose, found the true reading of the horizontal circle of our instrument which corresponds to the magnetic meridian. But this is not enough, we must have a distant mark whose azimuth viewed from the instrument is known, and we must find the value of the circle reading when the telescope is pointed to this mark. This will be seen from the following example:—

		° ′ ″
Circle reading of zero of magnetic scale (scale erect)		297 0 57
,, ,, (scale inverted)		297 3 7
		297 2 2
Correction for torsion		0 0 26
		297 2 28
	° ′ ″	
Circle reading of mark . . .	318 10 13	
Azimuth of mark	2 48 40E	
Reading for north		315 21 33
Magnetic declination .		18 19 5W

LESSON LIV.—Determination of Geographical Meridian.

115. It thus appears that at a fixed station, in order to obtain the absolute magnetic declination, we must know the azimuth from the station of some sufficiently distant mark. If we are making a magnetic survey we cannot of course have such a mark, and in this case what we do is to take by means of a chronometer the exact time when the centre of the sun's disc crosses the line of collimation of the telescope, which is reflected into the instrument by means of the transit mirror, taking at the same moment the corresponding reading of the horizontal circle.

By calculation we can obtain from this the exact azimuth of the vertical plane passing through the sun and through the instrument, and knowing the reading on the horizontal circle which corresponds to it, we thus obtain the equivalent of a mark.

Let us suppose that we are engaged in such a magnetic survey, and that, furnished with a good chronometer of which the error is known and with a declinometer, we have, in addition to the purely magnetic observation, to find the geographical azimuth corresponding to some position of our telescope. Our first point is to see that the transit mirror is in accurate adjustment. Three things are necessary in order that this may be perfect. In the first place the axis of the mirror must be horizontal. This adjustment is made by means of a riding level. Again, the normal to the plane of the mirror must be perpendicular to the axis of revolution of mirror. There is a small screw at the back of the mirror by which its plane may be altered and this adjustment made in the following manner: Take some object at a sufficient elevation and reflect it into the telescope, getting the object bisected by the wire of the telescope. Then reverse the mirror in its

bearings. If the object still remains bisected by the wire, no correction is necessary, but if not, the screw at the back of the mirror must be moved so as to displace the image through a distance equal to half the difference between the two positions. This operation must be repeated if necessary, and continued until the object is in precisely the same position in both observations.

In the third place it is necessary that the line of collimation or visual axis of the telescope must be perpendicular to the plane of the mirror when this is vertical.

To secure this adjustment there is an arrangement through which the sun's light may be made to illuminate the cross-wires of the telescope. These illuminated wires will be reflected from the transit mirror when this mirror is vertical, and the transit mirror must be turned in azimuth until the illuminated wires and their reflections coincide. The line of collimation will now be perpendicular to the plane of the mirror, and we must note the reading of a small vernier which moves with the mirror, and take care that, when the instrument is in use, this vernier shall always have the same reading which it had when we were making this adjustment for collimation. The transit mirror may now be considered to be in perfect adjustment.

It is presumed that, by a method similar to that described in the Appendix to our first volume, p. 284, the centre of the cross-wires of the telescope has previously been made to coincide with the optical axis, and that when the telescope is properly placed in position one of its cross-wires is vertical while the other is horizontal. We have thus a vertical cross-wire passing through the optical axis of the telescope. The mirror and telescope being now in perfect adjustment, and the observer being furnished with a good chronometer, of which the error is accurately known, an observation of the sun may be made, a dark glass being used in order to prevent the light of the sun from too strongly affecting the eye of the observer.

The following is an example of such an observation:—

3d May 1886.—Made an observation at magnetic house at Kew. Latitude, 51° 28′ 6″ N.; longitude, 1 m. 15 sec. W. Error of chronometer at station on local time, +1 m. 46 sec.

The exact time at which the centre of the sun (mean of two limbs) crossed the central wire was 3 h. 37 m. 23 sec., as given by the chronometer. Hence

	h.	m.	s.	
Time observed . .	3	37	23	
Add equation of time as given by Nautical Almanac . . .	0	3	17	
	3	40	40	
Error of chronometer (fast)	0	1	46	
	3	38	54	= true local apparent time.

Now in Fig. 146 let P represent the pole, Z the zenith, and S the sun at the moment of observation. ZP denotes a portion of a great circle passing through the pole and the

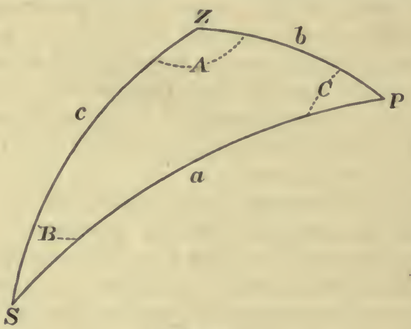


Fig. 146.

zenith, on a continuation of which the sun's centre would have been found at apparent noon. PS is a great circle passing through the pole and the centre of the sun at the moment of observation. Hence the angle C is denoted by

the difference in time between apparent noon and the moment of observation, allowing 15° for each hour, and is therefore = 54° 43′ 30″. Also the side b = co-latitude = 38° 31′ 54″, and the side a = north polar distance of the sun = 74° 13′ 56″. But by Napier's formulæ for spherical triangles we have

$$\operatorname{Tan}\frac{A+B}{2}=\frac{\cos\frac{a-b}{2}}{\cos\frac{a+b}{2}}\cot\frac{C}{2},$$

$$\operatorname{Tan}\frac{A-B}{2}=\frac{\sin\frac{a-b}{2}}{\sin\frac{a+b}{2}}\cot\frac{C}{2},$$

from which it follows that A, or azimuth of sun from north, = 108° 40′ 11″.

Again the reading of the horizontal divided circle of the instrument at the observation was 288° 19′ 44″, which is therefore the reading in azimuth of the great circle ZS passing through the zenith and through the sun at the moment of observation, and hence this great circle differs by 108° 40′ 11″ from that passing through the zenith and through the north pole, so that 108° 40′ 11″ + 288° 19′ 44″, or 36° 59′ 55″, will denote the circle reading for true north.

Lastly, a fixed and easily distinguishable mark was also observed, and the reading of the circle for this mark was 39° 48′ 35″. Hence the azimuth of this mark was 2° 48′ 40″ to the east of north. It is this azimuth that we have made use of in the example of the method of finding the absolute declination which has just been given.

CHAPTER VII.

ELECTRO-MAGNETISM AND ELECTRO-MAGNETIC INDUCTION.

PART I.—ELECTRO-MAGNETISM.

116. IN this part the student will have brought before him certain quantitative relations between electricity and magnetism of great practical importance. We shall first consider the following cases:—

(1.) Action of a linear current on a magnet.
(2.) Action of a circular ,, ,,
(3.) Action of a helical ,, ,,

LESSON LV.—Action of a Linear Current on a Magnet (Biot and Savart's law).

117. *Exercise.*—To verify the law that the intensity of magnetic action of a linear current of indefinite length on the pole of a magnet varies inversely as the perpendicular distance of the pole from the straight wire which conveys the current.

Apparatus.—A vertical wire at least three mètres long mounted on a board is arranged so as to form the movable side of a large rectangular circuit whose plane is nearly at right angles to the magnetic meridian. In this circuit is placed a battery provided with a commutator. An ordinary galvanometer scale has a rod fixed horizontally at

right angles to it, which rod supports at its extremity a small magnetometer *m* (Fig. 147), consisting of a suspended mirror with a magnet at its back enclosed in a small

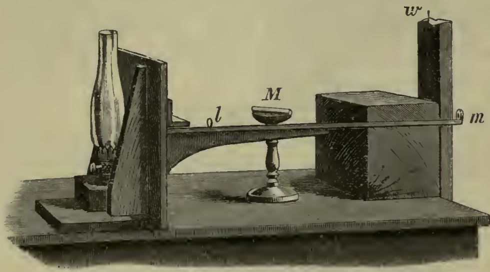


Fig. 147.—APPARATUS FOR PROVING LAW OF LINEAR CURRENTS.

brass box. In the figure *l* is a lens for focusing, and M a directing magnet intended to bring the magnet of *m* to zero without sensibly affecting the field. A millimètre scale will also be required.

Theory.—We shall *assume* that, other things remaining the same, *the action of a small portion of the wire conveying a current acts upon a magnetic pole with a force which varies directly as the length of the small portion or element, and inversely as the square of its distance from the pole.* From this assumption we shall prove, in an elementary manner, that Biot and Savart's law must directly follow.

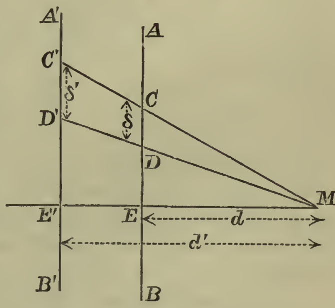


Fig. 148.

Let M (Fig. 148) be the magnetic pole which is acted upon by a current in the wire AB at a perpendicular dis-

tance $ME = d$ from the pole, or by the same current $A'B'$ at a different perpendicular distance $ME' = d'$ Draw two lines, MCC' and MDD', cutting off from AB and $A'B'$ two *very small* portions of current $CD = \delta$ and $C'D' = \delta'$. Let the forces exerted by δ and δ' on M acting, the one at the distance CM or $DM = p$, and the other at the distance C′M or $D'M = p'$, be f and f'.

Then, by the above assumptions,

$$f : f' = \frac{\delta}{p^2} : \frac{\delta'}{p'^2},$$

or

$$\frac{f}{f'} = \frac{\delta}{\delta'} \cdot \frac{p'^2}{p^2}.$$

But, by geometry,

$$\frac{\delta}{\delta'} = \frac{p}{p'} = \frac{d}{d'},$$

hence

$$\frac{f}{f'} = \frac{d'}{d},$$

that is to say, the force varies inversely as the simple distance. Now what is true of the elements δ and δ' is equally true of all other elements. Hence the whole forces F and F′ exerted by AB and A′B′ obey the same law, so that

$$\frac{F}{F'} = \frac{d'}{d}.$$

We know from our previous experiments that the direction of the force is such as to twist round the magnet to which the pole belongs until its axis shall be at right angles to the plane passing through its centre and the current.

Experimental Proof.—This important conclusion might be verified after the manner of Biot and Savart by determining the time of vibration of a magnetic needle placed at different distances from the wire conveying the current. It will, however, be easier to use the method

of deflection. For this purpose let a small magnet, free to move horizontally, be arranged so that the plane which passes through its centre and the wire conveying the current shall be that of the magnetic meridian. Let NS (Fig. 149) denote the horizontal line in which the needle points, W denoting the horizontal projection of the vertical wire, while O is the centre of the magnet. This magnet NS we shall suppose to be deflected by a current going down the wire until it assumes the position in Fig. 149, making an angle α with the magnetic meridian.

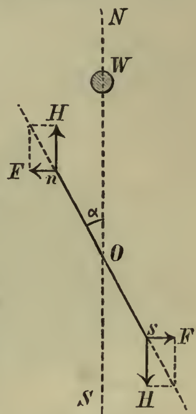


Fig. 149.

Now, when there is equilibrium, we shall have two couples acting upon the magnet. In the first place, there will be the couple consisting of the attraction of the earth for the north pole n, represented in magnitude by nH, and the equal and opposite repulsion for the south pole s, represented by sH. In the next place, there will be the couple consisting of the forces exerted by the current on the two poles, which forces are represented in magnitude and direction by nF and sF. Now when the resultant of the forces at each pole acts through the axis of the magnet, they will together constitute a *couple of no moment*, representing two equal and opposite pulls upon the magnet in the direction of its axis, and hence the magnet will remain at rest in this position. We find that in a position of equilibrium, in which the resolved portion of the force H perpendicular to the needle must balance that of the force F, we must have

$$F \cos \alpha = H \sin \alpha,$$

an expression which is independent of the moment of the magnet, as was the case with the expression of p. 31;

hence $$F = H \tan \alpha.$$

If the distance between the centre of the magnet and the current ($OW = d$) be changed to d', then

$$F' = H \tan \alpha'$$

where α' is the new deflection. But if

$$\frac{F}{F'} = \frac{d'}{d},$$

it follows that

$$\frac{\tan \alpha}{\tan \alpha'} = \frac{d'}{d},$$

or

$$d \tan \alpha = d' \tan \alpha' = \text{a constant quantity.}$$

It thus appears that what we have got to do in this experimental investigation is to move the wire to different distances from the magnet, and observe in each position the tangents of the angle of deviation, with the view of ascertaining if the relation expressed by the last equation be correct.

Example.—

Distance of Wire from centre of magnet in Millimètres (d).	Tangent of Mean Deflection ($\tan \alpha$).	$d \tan \alpha$.
83	177	14,691
105	135	14,175
146	99	14,454
183	78	14,274

In which the observed scale readings have been taken as proportional to $\tan \alpha$. The value of $d \tan \alpha$ is thus found within the limits of possible experimental error to be a constant.

Lesson LVI.—Action of a Circular Current on a Magnet.

118. *Exercise.*—To prove experimentally the law relating

to the action at different distances of a circular current upon a magnet.

Apparatus.—A compass box mounted so as to slide on a graduated platform which is mounted centrically and at right angles to the plane of a vertical circle conveying the current. That is to say, the compass box slides, having its centre in a line which is perpendicular to the plane of the circle, and which passes through the centre of the circle.

Fig. 150 shows a convenient form of the instrument, which is really a galvanometer with a movable compass box.

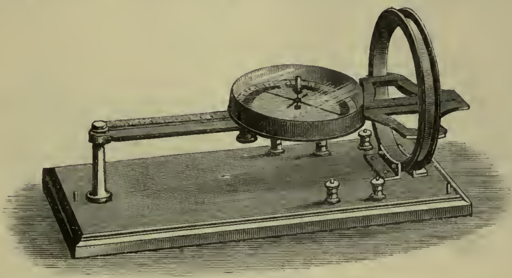


Fig. 150.—TANGENT GALVANOMETER WITH SLIDING COMPASS BOX.

A constant battery and a commutator will likewise be required.

Theory.—Let AB A′B′ (Fig. 151) be the circular wire of radius a conveying a current of strength C. Consider the action of an element ds of the current upon a magnetic pole of unit strength placed at M, which is at a distance x from the centre of the circle. The whole force df' on the magnetic pole due to this element will be

$$df' = \frac{\mathrm{C}ds}{r^2} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where $r = \mathrm{AM}$. Now this force will act in a direction at

right angles to the plane AMB. Let this force be therefore represented by ME, the line ME being a normal to the plane AMB. Now resolve ME into two components, one MF acting in the line of x, and another FE acting at right angles to x, and let the force along $MF = df$. Now, from the similar triangles EFM and AOM we have

$$\frac{df'}{df}=\frac{r}{a} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Hence from (1) $df = \frac{Cads}{r^3}$. But since for every other element a similar expression will be obtained, we may by

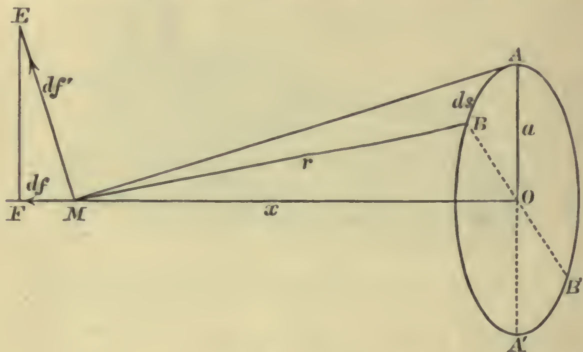


Fig. 151.

summation obtain the whole force F acting along MF, that is to say, in a normal to the plane of the current. In fine

$$F=\frac{Ca}{r^3}\times 2\pi a \text{ or } F=\frac{2\pi a^2 C}{(a^2+x^2)^{\frac{3}{2}}} \quad . \quad . \quad . \quad (3)$$

It is easily seen, moreover, that this expression represents the *total force*, since the various components expressed by EF will balance one another all round the circle, thus EF will be balanced by the vertical component of A′B′.

If for the unit magnetic pole we substitute a *small* mag-

net of strength s, then the poles will be respectively acted on by two equal and opposite forces Fs, and on the other hand the poles will likewise be respectively acted on by a second pair of two equal and opposite forces Hs, where H is the horizontal component of the earth's magnetic force, assuming that the plane of the circle is in the magnetic meridian.

Hence we shall have, as on p. 318,

$$F = H \tan \alpha \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Hence also, from (3),

$$\frac{2\pi a^2 C}{(a^2+x^2)^{\frac{3}{2}}} = H \tan \alpha \quad . \quad . \quad . \quad . \quad (5)$$

Note.—(If there were n circular currents so near each other that the distance of each one from the magnet might be regarded as identical, then the expression in the left-hand member of (5) must be multiplied by n. In this case a would signify the *mean radius* of the coils.)

We may write (5) thus,

$$K \frac{a^2}{(a^2+x^2)^{\frac{3}{2}}} = \tan \alpha \quad . \quad . \quad . \quad . \quad (6)$$

where K is some constant, as long as C and H are constant. This form will be most convenient for use in the experiment about to be described.

Experimental Proof.—Place the circular coil in the magnetic meridian, and connect it with a constant battery provided with a commutator. Next take deflections at different distances of the compass box from the coil. Read both ends of the needle and reverse the commutator at each position. Measure the radius of the coil with callipers. Finally compare the tangents of the mean deflections with values derived from calculation, as shown in the following example :—

Example.—$a = 3{\cdot}75$ inches.

(1.) Distance from centre of compass needle to centre of coil or x.	(2.) Mean deflection.	(3.) Tangent of deflection.	(4.) Value of $\frac{a^2}{(a^2+x^2)^{\frac{3}{2}}}$	(5.) 3÷4	(6.) Adopted value of $K \frac{a^2}{(a^2+x^2)^{\frac{3}{2}}}$
0	40°	·8391	·2667	3·147	·8391
1	37	·7536	·2406	3·132	·7570
1·5	34	·6745	·2135	3·160	·6716
2	30·75	·5949	·1832	3·247	·5766
2·5	26·75	·5040	·1536	3·281	·4834
3	22·25	·4091	·1270	3·221	·3995
3·5	18·5	·3346	·1042	3·210	·3278
4	15·375	·2750	·08532	3·222	·2685
4·5	12·75	·2263	·06996	3·234	·2192
5	10·5	·1853	·05760	3·218	·1812
5·5	8·875	·1561	·04767	3·275	·1500
6	7·5	1317	·03970	3·242	·1249

On dividing the numbers in the third column by the numbers in the fourth the quotient should be constant, as we see it is from (5), at least within the errors of observation. In experiments of this kind, where the degree of accuracy is not high, the law is best tested by the use of the graphical method—that is, by plotting two curves and comparing their form. The continuous line of Fig. 152 shows the result of taking the numbers in the first column as abscissæ and those in the third column as ordinates, thereby giving a curve showing the relation obtained by experiment. In order to make a curve showing the theoretical relation comparable with that due to experiment, some value must be given to K which will bring the numbers in the fourth column near those in the third column. If we wished the two curves to fall upon each other, the best value to give to K would be the mean of the constants in the fifth column. We have selected the value 3·1467, which will enable us to distinguish without confusion the two curves, the theoretical one being a dotted line. On multiplying the numbers of the fourth column

by 3·1467 the numbers of the sixth column are obtained,

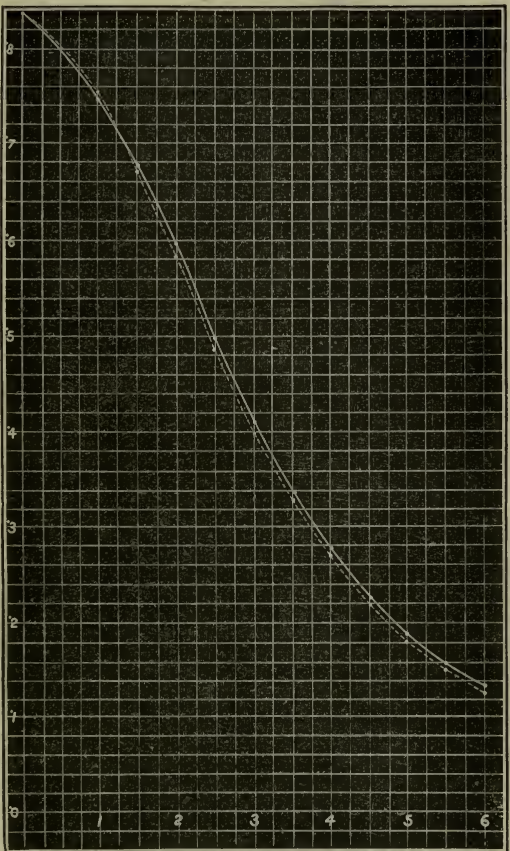


Fig. 152.—Graphical Proof of Law.

which give the required ordinates of the theoretical curve.

It will be noticed that the two curves are very like each other, thereby giving us reason to conclude that the theoretical formula is right.

The most useful case of (3) is when $x=0$, then from (3) and the note to (5) we obtain

$$F=\frac{2\pi anC}{a^2}=\frac{\begin{array}{c}\text{length of wire}\\ \text{in a turn}\end{array}\times\text{number of turns}\times\text{strength of current}}{\text{square of radius}}.$$

We shall now proceed to verify this formula independently by the zero methods of Poynting and Thury.[1]

Lesson LVII.—Proof of Electro-magnetic Laws.

119. *Apparatus.*—On a vertical board are arranged

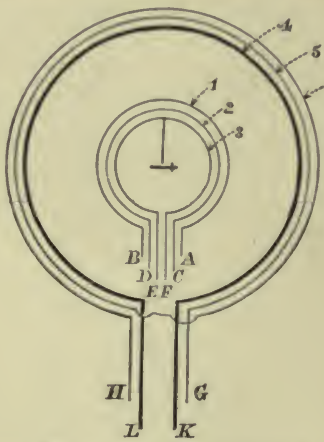


Fig. 153.—Poynting's Apparatus.

three equal circles of wire (Fig. 153), (1), (2), and (3), and

[1] Poynting, *Pro. of Manchester Lit. and Phil. Society,* vol. xviii. p. 85; Thury, *Mousson's Physik,* p. 582.

at a distance from the centre equal to twice that of these circles there are likewise arranged three other circles, (4), (5), and (6), concentric with the former. All these circles are formed of the same wire with the exception of (4), which has a cross-section equal to four times that of the others; (5) and (6) are in the same circuit. The ends of the circles are provided with mercury cups arranged so that by means of copper rods of appropriate length the connections to be described below may be conveniently made. The respective terminals are as follows .—

Coil (1)	Terminals A and B.
,, (2)	,, C and D.
,, (3)	,, E and F.
,, (4)	,, K and L.
,, (5) and (6)	,, G and H.

At the centre of these circles there is a small magnetic needle suitably pivoted. A battery and key will be required. Having placed the plane of the coils in the magnetic meridian, the following experiments may be made:—

Experiment—(1.) To prove that *If the current is reversed the force is reversed.*—Let the current enter at A, join B and D, and let the current leave at C. The current must therefore go round (1) in one direction and round (2) in the opposite direction. Hence the needle should be unaffected.

(2.) To prove that *The force is proportional to the strength of the current and to the length of circuit.*—Let the current enter at A, connect A with C, B with D, and D with E, and let the current leave at F. By means of this arrangement two half-currents go round by (1) and (2) in one direction, and the whole current returns by (3) in the opposite direction. Now if the effect of a current of half-strength passing twice round in one direction is equal to a unit current passing once round in the opposite direction, there should be

no deflection of the needle. This will be found to be the case, and hence the force is proportional to the strength of the current and to the length of circuit.

(3.) To prove that *The force is inversely proportional to the square of the distance.*—Let the current enter at A, join B with H, and let the current leave at G. Now coils (1), (5), and (6) are placed in series, so that the same current, say of strength C, passes in one direction round (5) and (6), and in the opposite direction round (1). We have thus for coil (1) $a = 1$, $n = 1$, hence

$$F = \frac{2\pi a n C}{a^2} = 2\pi C;$$

also for (5) and (6) $a = 2$, $n = 2$, hence

$$F = \frac{2\pi a n C}{a^2} = 2\pi C;$$

in other words, the forces are equal, and being opposite the needle should remain unaffected.

The apparatus is also adapted to prove two other laws relating to resistance.

(1.) *The resistance is proportional to the length of the circuit.*—Let the current enter at A, join B and C, D and F, A and E, and let the current leave at F. Here the current splits between (1) and (3) ((2) being in series with (1)), and goes round (3) in an opposite direction to that in which it circulates in (1) and (2). Hence if doubling the length doubles the resistance, the current should be halved, and hence the effect of the two turns of (1) and (2) should be equal and opposite to that of the one turn of (3), which will be found to be the case.

(2.) *The resistance is inversely proportional to the cross-*

section.—Introduce the current at A, connect A and L, B and C, D and K, and let the current leave at K.

Here we have two circuits connecting A with K. The first of these consists of the two coils (1) and (2) in series, and the second of a coil (4) of the same length as (1) and (2) together, but at a double distance from the needle and of quadruple cross-section—going round likewise in a direction the opposite to that of (1) and (2). Now the coil (4) being at a double distance, must, in order to make up for this increased distance, have a current four times as strong as that of (1) and (2) if it is to balance these latter coils, and produce no deflection on the needle. But we find that this is the case, and hence it follows that the effect of increasing the cross-section four times is to diminish the resistance and increase the current four times; in other words, the resistance varies inversely as the cross-section.

120. *Solenoid.*—Let us now study experimentally the simplest kind of solenoid, consisting of a series of circular currents lying with their planes parallel to each other, their centres being in the same straight line perpendicular to these planes. It is not possible to arrange such a series, but a helix so nearly imitates the conditions that it may be taken to represent the same thing. The action of the connecting portions may be neutralised by running the current back through a horizontal wire parallel to the axis of the helix so as to oppose the currents in those connecting portions. Such an arrangement constitutes an **Experimental Solenoid.**

Lesson LVIII.—Magnetic Action of a Solenoid.

121. *Apparatus.*—The magnetometer and scale of Lesson LV. This magnetometer is sufficiently small to pass

inside a helix of diameter 37 mm. and length 542 mm. wound on a brass tube. The helix consists of a single layer of No. 20 B. W. G. covered wire well steeped in paraffin, so as to keep the coils together. The helix is mounted on a wooden slide provided with a scale (see Fig. 154). A constant battery and key will likewise be required.

Fig. 154.—MAGNETOMETER AND HELIX.

Theory.—Let QD $= a$ (Fig. 155) be the radius of the helix, and let B be the point at which a unit magnetic pole is placed. Here the axis of the helix is supposed to be in the

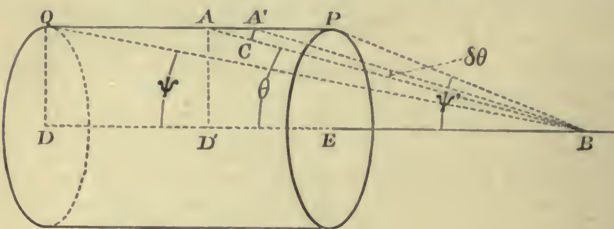


Fig. 155.

plane of the paper, and its circular cross-section to be perpendicular to the plane of the paper. Also let ABD $= \theta$, then clearly the angles made with the axis DB by lines drawn to B from any point in the circumference of the helical circle which passes through A will all be $= \theta$. In like

manner let A′BA = $\delta\theta$, and let C′ denote the whole current which flows round a slice of the helix of breadth = unity. This current may embrace a greater or smaller number of turnings, according to circumstances. Draw A′C perpendicular to ACB. Now A′C = A′B$\delta\theta$ = AB$\delta\theta$ when $\delta\theta$ is very small; and since the triangle AA′C is similar to ABD′, we have

$$\mathrm{AA'} = \mathrm{A'C} \times \frac{\mathrm{AB}}{\mathrm{AD'}} = \mathrm{A'C} \times \frac{\mathrm{AB}}{a}.$$

Hence

$$\mathrm{AA'} = \frac{(\mathrm{AB})^2}{a}\delta\theta.$$

Let F denote the force upon the pole of that element of the helix embraced between A and A′ all round. This will be (see p. 320)—

$$\mathrm{F} = \frac{2\pi\mathrm{C'}a^2 \times \mathrm{AA'}}{(a^2+x^2)^{\frac{3}{2}}} = \frac{2\pi\mathrm{C'}a(\mathrm{AB})^2\delta\theta}{(\mathrm{AB})^3} = 2\pi\mathrm{C'}\delta\theta \times \frac{\mathrm{AD'}}{\mathrm{AB}} = 2\pi\mathrm{C'}\sin\theta\delta\theta.$$

If we integrate between the limits ψ = QBD and ψ' = PBD, we have

$$\mathrm{F} = 2\pi\mathrm{C'}(\cos\psi - \cos\psi').$$

If l be the length of the helix, and n be the number of turns of the wire which conveys the current C, then

$$\mathrm{C'} = \frac{n\mathrm{C}}{l},$$

hence

$$\mathrm{F} = \frac{2\pi n\mathrm{C}}{l}(\cos\psi - \cos\psi') \quad . \quad . \quad . \quad . \quad (1)$$

If the unit pole be placed inside the helix, (1) becomes

$$\mathrm{F} = \frac{2\pi n\mathrm{C}}{l}(\cos\psi + \cos\psi') \quad . \quad . \quad . \quad . \quad (2)$$

and when, under these circumstances, the length of the solenoid is great in comparison with its diameter, cos ψ and cos ψ' become unity, and then

$$\mathrm{F} = \frac{4\pi n\mathrm{C}}{l} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

which is a constant quantity.

Experimental Proof.—The tangent of the angle of deflection of a small magnetic needle placed at m (Fig. 154) will be proportional to the magnetic strength at that point. But the divisions of the magnetometer scale may be taken to be proportional to such tangents, and hence all that it is necessary to do is to take the deflection at various distances from the helix. The base supporting the helix should be clamped down to the slab on which the magnetometer stands, so that the helix can only move in the direction of its axis.

Let BD (Fig. 155), the distance of the farthest end of the helix from centre of needle, $= d_1$; let BE, the distance of the nearest end of the helix from centre of needle, $= d_2$, so that $d_1 = d_2 + l$.

Then from (1) and (2) we may derive the following,

$$\text{K (constant)} = \left(\frac{d_1}{\sqrt{d_1^2 + a^2}} \pm \frac{d_2}{\sqrt{d_2^2 + a^2}}\right)\frac{1}{\text{D}}$$

where D is the deflection.

Example.—$l = 540$ mm., $a = 24$ mm.

A.

Expt.	Distance between Magnet and nearest end of Helix.	Mean Deflection.	Difference for 10 mm.	K.
1	0	188·5		
			69·5	
2	10	119		·00516
			50·5	
3	20	68·5		·00524
			29	
4	30	39·5		·00553

B. With a different current.

Expt.	Distance between Magnet and nearest end of Helix.	Mean Deflection.	Difference for 20 mm.	K.
1	100	226		·000119
			72·5	
2	120	153·5		·000122
			41·5	
3	140	112		·000122
			28	
4	160	84		·000125
			17	
5	180	67		·000122
			14	
6	200	53		·000125

C. With a different current—Magnet inside the Helix.

Expt.	Distance between Magnet and nearest end of Helix.	Mean Deflection.	Difference for 10 mm.	K.
1	0	137·5		
			43·5	
2	10	181		·00764
			26	
3	20	207		·00792
			14	
4	30	221		·00805

The calculation of K is facilitated by (1) finding, from tables, the angle whose tangent is $\frac{a}{d}$, and (2) obtaining the cosine of the angle found.

122. *The Electro-magnet.*—The introduction of an iron core within a helix increases very greatly the magnetic strength. This combination constitutes an electro-magnet. Here a quantity of magnetism will be separated across any section of the core equal to strength of the field within the helix × area of cross-section of the core × the *coefficient of magnetic induction.*

In order to discuss the theory of the electro-magnet it is desirable that we should now proceed to give to the action of the helix, already discussed, a magnetic interpretation. That is to say, we must find whether we cannot regard the helix as equivalent to a magnet, in which case we shall be enabled to express the joint action of the helix and of the iron core by a simple method of addition.

To do this let us begin by referring to Lesson LVI., in which we investigated the action of a circular current upon a magnetic pole of unit strength placed at a distance x from the centre of the circular current along the axis, a denoting the radius of the circle and C the strength of the current. We then obtained the following expression,

$$F=\frac{2\pi a^2 C}{(a^2+x^2)^{\frac{3}{2}}}.$$

If the distance x be very great with respect to the

radius of the circle, this expression will become approximately

$$F=\frac{2\pi a^2 C}{x^3}.$$

Suppose now that we replace this circular current by a magnet at the centre of the circle, the strength of whose pole is S and whose length is dx, the magnetic axis lying in the line x, and the poles being so placed that the magnet exercises the same sort of force upon the unit pole as the current does, that is to say, both are attractive or both are repulsive. It is clear that the moment of this magnet will be Sdx. Now (assuming that the action is attractive) we shall have the nearer pole of the magnet attracting the unit pole with a force $\frac{S}{x^2}$ or Sx^{-2}, while the further pole will repel the same with a force $\frac{S}{(x+dx)^2}=S\{x^{-2}-2x^{-3}dx\}$ nearly.

On the whole, therefore, there will be an excess of attraction represented by $2Sx^{-3}dx$. If this is to be equivalent to the action of the current or F we must have

$$2\pi a^2 Cx^{-3}=2Sx^{-3}dx,$$

and hence

$$\pi a^2 C=Sdx,$$

that is to say, *the moment of the magnet must be equal to the area of the circuit multiplied by the strength of the current.* Again the magnet must be so placed that looking at it from the south seeking pole, from which point the molecular currents of the magnet may be regarded as positive currents circulating in the direction of the hands of a watch, the direction shall coincide with that of the current in the circuit. Instead of a single magnet we may have a bundle of similarly placed small magnets of united moment $=\pi a^2 C$ filling up the area of the circle, all their south poles lying on one surface and all their north poles on another parallel to the former and contiguous to it.

These layers would constitute the *equivalent magnetic shell* of the circuit.

What we have proved applies to a very small circular current, but it will be seen from Fig. 156 that we may imagine a large circular current to be composed of an infinite number of very small circular currents, such as *abcd*, of the same strength as the large one, all having the same sort of rotation, and hence placed so that the adjacent elementary currents being in opposite directions cancel each other.

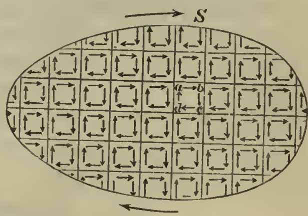


Fig. 156.

The only effective portion of this group of circular currents will therefore be the main current in the external circle.

Now each of these small hypothetical component circles may be replaced by means of an extension of the above demonstration (inasmuch as obliquity and distance act in the same way both on currents and magnets) by its equivalent small magnetic shell, and hence the main large circular current may be replaced by its large magnetic shell.

Let us now apply this mode of viewing things to the demonstration of Lesson LVIII., where we found

$$F = \frac{2\pi C' a^2 \times AA'}{(a^2+x^2)^{\frac{3}{2}}}.$$

Here for AA′ we may substitute dx, and if C′ denote the current for breadth unity, the current for breadth dx will evidently be $C'dx$. Hence substituting a magnetic shell for the current we shall have $\pi a^2 C' dx = S dx$. Hence

$$\text{S or the strength of the magnetic pole} = \pi a^2 C' \quad . \quad . \quad (1)$$

Now if we make the same substitution throughout every little distance dx we shall obtain a vast number of magnetic

shells joined in line, which represent the solenoid. But if we regard any interior shell we shall see that the south pole of one shell cancels the north pole of the one next it, so that the only uncompensated portions are the external members, *i.e.* the first and the last of the series. But this is only another way of expressing the fact that the solenoid is represented magnetically by a magnet, the strength of whose pole is S, the distance between its poles being equal to the length of the solenoid. Now, referring once more to Lesson LVIII., we find $C' = \frac{nC}{l}$. Making this substitution in (1) we obtain

$$S = \frac{\pi a^2 nC}{l} = \frac{AnC}{l} \quad . \quad . \quad . \quad . \quad (2)$$

where A denotes the area of the circuit, n the number of turns of the current of strength C, and l the total length of the solenoid.

This expression therefore forms a magnetic representation of the solenoid. With regard to the strength of the magnetic pole of the core this will be denoted by the magnetism separated, that is to say, by the strength of the field within the helix multiplied by the cross-section of the core and by the coefficient of magnetic induction. Now the strength of the field within the core is ((3) p. 329)

$$F = \frac{4\pi nC}{l} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and hence the magnetism separated will be denoted by Fak, a being the area of the cross-section, and k the coefficient of magnetic induction.

The quantity Fak will likewise therefore denote the strength of the magnetic pole of the core.

Hence taking helix and core together, the magnetic strength of the joint pole will be

$$\frac{AnC}{l} + Fak = \frac{AnC}{l} + \frac{4\pi nCak}{l} = \frac{nC}{l}\left\{A + 4\pi ak\right\} \quad . \quad . \quad (4)$$

and the magnetic moment will be

$$\tilde{n}C\{A + 4\pi ak\} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

This theoretical formula requires modification in practice on account of the action of the magnetic induction on one another of neighbouring portions of the iron core. The true law is much more complicated. Several empirical formulæ have been devised for expressing this law more accurately; the one that is now most in favour being that due to Frölich,[1] namely,

$$m = \frac{C}{a + bC},$$

where m is the magnetic moment and a and b two constants, b being the reciprocal of the maximum value of m.

We shall now proceed to investigate the laws of the electro-magnet experimentally.

Lesson LIX.—Laws of the Electro-magnet.

123. *Apparatus.*—Two electro-magnets of equal size will be required, each wound with the same kind of wire. Each should have twelve separate layers connected with binding screws, so that any one or any number of these layers may, if desirable, be used (see Fig. 157). The electro-magnets are provided with cores consisting of a bundle of wires of soft iron. The coils are of the same construction as those described in Part II. of this

Fig. 157.
MULTIPLE WOUND ELECTRO-MAGNET.

[1] See Dr. S. P. Thompson on the "Law of the Electro-magnet and the Law of the Dynamo," *Phil. Mag.*, January and September 1886, also the references there given.

chapter. For measuring the moment of the magnet a magnetometer will be necessary, while for measuring the current a tangent galvanometer may be used. It will be necessary to vary the strength of the current, for which purpose a box of resistances, containing 5, 2, 2, 1, ·5, ·2, ·2, and ·1 ohms made of thick wire, will be the best. A constant battery, consisting of two or three Grove's cells, had better be employed for magnetising.

Method.—Behind the magnetometer M (Fig. 158) fix in a rigid position the electro-magnet E_1, with its axis horizontal

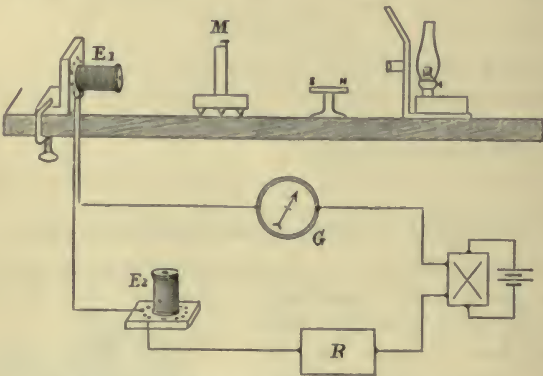


Fig. 158.—APPARATUS FOR INVESTIGATING MAGNET LAWS.

and at right angles to the suspended needle. The axis of the magnet should pass through the centre of suspension of the needle. Some distance away from E_1 place the other similar electro-magnet E_2 with its core removed. The two electro-magnets should be in series with the tangent galvanometer G, the battery (provided with a commutator), and the resistance coils R. The galvanometer should be sufficiently far away from the rest of the apparatus to prevent the possibility of indirect action on its needle.

In the first place, let us prove that the magnetic moment varies directly as the number of turns on the electro-magnet, the resistance of the circuit remaining unchanged. In order to fulfil this latter condition we must so arrange the connections that for every addition to the circuit of a coil of E_1 a similar coil of E_2 shall be removed from the circuit. Now if our proposition be true we should find that on dividing the magnetometer reading by the number of coils in circuit the result should be a constant quantity, provided that k may be regarded as constant.

Example. (A galvanometer in circuit showed the current to be uniform)—

n=number of turns,	11	10	9	8	7	6	5
m=mean deflection,	344	311	279	245	209	175	140
$\frac{m}{n}$=Constant,	31·2	31·1	31·0	30·6	29·9	29·2	28

The gradual decrease of the constant is due to alteration of k.

Secondly. Let us attempt to verify Frölich's formula. Here it will not be required to make use of E_2, but a fixed number of turns of E_1 having been placed in circuit the current must be varied by inserting resistances, and the readings of both G and M taken. The tangents of the deflections of G will denote the current strength, and the deflections of M will be proportional to the magnetic moments of E_1. From these values the constants of Frölich's formula should be obtained, and we may then see how the formula agrees with the observations. For convenience sake the formula may be written thus—

$$\frac{C}{m}=a+bC.$$

This form is used in the next example.

Example.—

Expt.	Deflection of G = d.	Tan d = C.	Deflection of M = m.	Log tan d.	Log m.	Log $\frac{C}{m}$.	Observed $\frac{C}{m}$	Calculated $\frac{C}{m}$
1	56°·5	1·5108	487°	10·1793	2·6875	7·4918	·00310	·00237
2	50·1	1·196	404	10·0778	2·6064	7·4714	·00296	·00286
3	44·5	·9827	295	9·9924	2·4698	7·5226	·00333	·00318
4	34·7	·6924	190	9·8403	2·2788	7·5615	·00364	·00364
5	30·2	·582	152	9·7649	2·1818	7·5831	·00383	·00382
6	26	·4877	123	9·6881	1·0899	7 5982	·00396	·00396
7	21	·3839	87	9·5842	1·9395	7·6447	·00441	·00413
8	17	·3057	71	9·4853	1·8513	7·6340	·00431	·00425

Here a and b are calculated from experiments 4 and 6, $a = +\cdot00472$, $b = -\cdot00156$. This example shows us that Frölich's law is only approximately fulfilled.

124. *Study of the Electro-magnetic behaviour of Iron.*—An arrangement of apparatus similar to that now described may be employed for studying the law of variation of the magnetic moment of the iron core produced by change of the magnetising current. The helix E_1 used for magnetising should be longer than the iron placed within it, its length being so great in comparison to that of the latter that the region where the iron is placed may be regarded as a field of uniform force. The experiment will consist of two parts: (1.) With no iron inside E_1 the deflection resulting from currents of different intensity is observed. These deflections will result from the magnetic effect of the coil alone. (2.) The iron is now placed inside E_1 and a second set of observations taken with the same series of currents previously used. This will give *core* and *coil*, and the difference between the readings now and those with the coil alone may be taken to represent the effect due to the *core alone*. A curve should now be plotted with the deflections produced by the *core alone* as ordinates, and

those by the *coil alone* as abscissæ. The latter give the currents and the former the magnetic moments produced by the currents. In experiments of this nature the student will find that when the current is broken, the iron being within the coil, the needle does not return to its position of rest. This is due to *residual magnetism.* It will be interesting to plot curves showing how the amount of the residual magnetism is connected with strength of current. The difference between the total and the residual magnetism is the *temporary magnetism*, and a curve should likewise be plotted showing how it varies with the current.[1]

125. *Intensity of Magnetisation.*—If we divide the moment of a magnet by its volume we obtain the **intensity of magnetisation per unit volume**, or if the moment be divided by the mass we obtain the **intensity of magnetisation per unit mass.** The units should be expressed in the C. G. S. system. To determine the moment M, the formula for the A position of Gauss (see p. 31) must be used, namely,

$$M = H\frac{(d^2-l^2)^2}{2d}\tan\alpha \quad . \quad . \quad . \quad . \quad (1)$$

where H is the horizontal intensity of the earth's magnetism at the place of the magnetometer, while

d = distance from the centre of the magnet to that of the magnetometer needle.
l = half the length of the magnet.
α = angle of deflection of the magnetometer needle.

Tan α must be obtained from the scale reading S. Now

$$\tan 2\alpha = \frac{S}{L}$$

[1] These curves should be compared with the theoretical ones deduced by Weber and Clerk Maxwell (see Maxwell's *Electricity and Magnetism*, vol. ii. chap. vi.)

(see Vol. I. p. 55), where L is the distance of the magnetometer needle from the scale. But for small angles

$$\tan \alpha = \tfrac{1}{2} \tan 2\alpha,$$

hence

$$\tan \alpha = \frac{S}{2L}.$$

126. *Value of Magnetising Force.*—To find the intensity of the field inside the helix, we use the equation (3), p. 334,

$$F = \frac{4\pi n C}{l} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The value of C must be obtained in C. G. S. units. When curves are plotted as suggested in Art. 124, the moments and magnetising forces should be expressed in C. G. S. units by means of the above formulæ (1) and (2).[1]

127. *Lifting Power of Electro-magnets.*—In many cases it is not so much the moment of a magnet that we desire to know as its lifting power. To determine this it will be necessary to have some kind of balance adapted for measuring the maximum weight which the magnet is capable of supporting. The balance must be arranged so that its *load* may gradually be increased.

In Fig. 159 a spring balance S adapted to this purpose is exhibited. The wheel *c* is mounted so that when it is turned the cord, passing over the pulleys *a* and *b*, is wound upon it, thus causing the armature A that is fastened to the bottom of the spring S to be detached from the electromagnet E when the force becomes sufficient. The reading of the index at the moment of separation from the balance is taken. An india-rubber cork *r* is fixed to an arm on the stand in order to receive and lessen the shock of the recoil caused by the separation.

[1] For an example, see J. W. Gemmell on the "Magnetisation of Steel, Cast-iron, and Soft-iron," *Proc. Roy. Soc.*, vol. xxxix. p. 374.

A balance on the lever principle gives a much greater range than a spring balance. The gradual increase of the load may be obtained by using a slow stream of water, or by adopting the arrangement of Fig. 160, where the weight W, a brass bucket containing shot hanging from the carriage C, is drawn along the lever by the slender

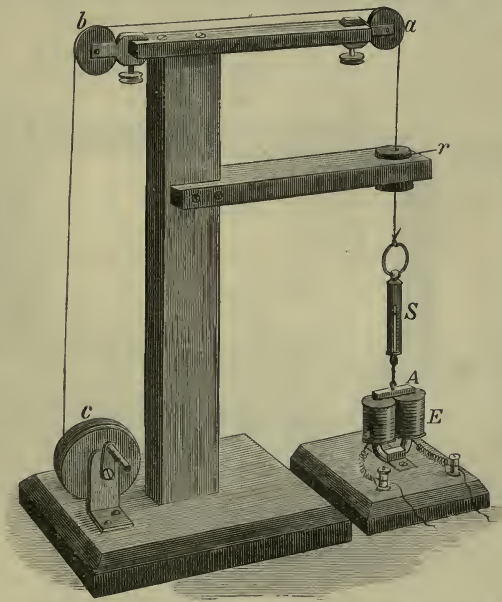


Fig. 159.—DETERMINATION OF LIFTING POWER BY SPRING BALANCE.

cord *c*, which passes over the pulley *a* and is wound on the pulley *b*. In using this balance the weight W is placed near O to begin with, and there balanced by adjusting the load contained in the bucket W′. To aid in doing this accurately a pointer *p* is attached to the end of the lever arm, and the end of this pointer oscillates in

front of a mirror m provided with graduation marks. The weight W is then moved several large divisions along the graduated lever arm, and the weight W′ increased until balance is again obtained. From the known weight added the value of a division in grammes may be ascertained. By changing the weight of W the value of a division may be altered at pleasure, and the balance may be thus made to suit itself to a considerable range of values. To the

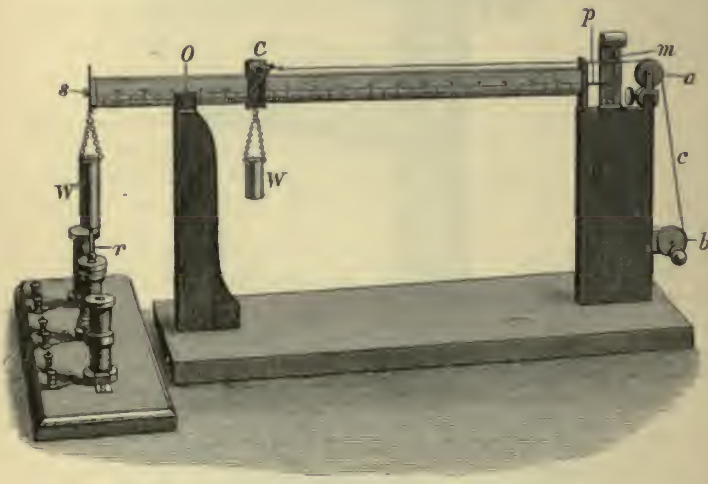


Fig. 160.—TESTING ELECTRO-MAGNETS BY LEVER BALANCE.

bottom of W′ an armature may be attached, or a small sphere r of iron, either of which is placed in contact with the end of the magnet *when both W and p are at their zero positions.* To secure the last condition the bucket W′ may be raised or lowered by aid of the screw s.

In Fig. 160 three electro-magnets are shown that have been wound with the same quantity of wire, but differently disposed in each. In (1) the wire is hooped towards the

middle; in (2), the one shown in Fig. 160 as being tested, it is hooped towards the ends; and in (3) it is wound regularly.

No great value can be attached to experiments relating to lifting power, the sources of variation being numerous. Much will depend upon the shape of the end of the magnet and of the armature, on the closeness with which their surfaces are brought into contact, on the time during which the current circulates, and on other causes. The measurements will further be complicated by the phenomenon of residual magnetism when the core has not been made of the very softest iron which can be procured.

Part II.—Electro-Magnetic Induction.

128. This section will fall under three heads:—

(A.) Fundamental Experiments relating to Electro-Magnetic Induction.
(B.) Application of to Various Measurements.
(C.) Measurements of Coefficients of Induction.

(A.) *Fundamental Experiments.*

Lesson LX.—Induction Experiments—Series I.

129. *Apparatus.*—(1.) A reel 2 inches long and 2 inches in diameter at its ends, has wound upon it continuously and in the same direction twelve layers of covered copper wire, No. 24 B. W. G. Forty turns of wire go to each layer. The ends of the wire are connected with two binding screws, and the coil is mounted on a base board (see S, Fig. 161). This is the *secondary coil.* There is also a smaller reel P, provided with a handle and two binding screws, and upon this are wound eight layers of No. 24

covered copper wire with forty turns to each layer. This forms the primary coil. Four narrow strips of brass are screwed on the outside of the primary coil, so as to be parallel to the axis. One of the strips is graduated in millimètres. The primary coil just fits within the secondary, the brass slips sliding in grooves, so that the former may be drawn out to any of the positions marked on its scale, the axes of the coils remaining meanwhile in the same straight line. The primary is hollow, so that iron wires may be

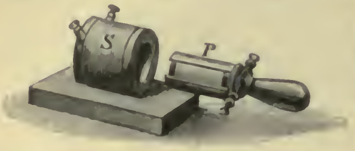


Fig. 161.—PRIMARY AND SECONDARY COIL.

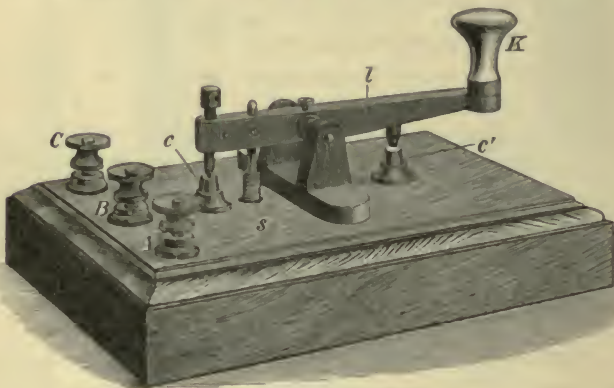


Fig. 162.—THE MORSE KEY.

The connections are B to c, A to c′, C to l. When K is depressed contact at c′ is made and at c is broken. The end of the screw which normally rests on c being held down by the spring s is adjustable in order to regulate the extent of the "play" of the lever.

placed within it. These iron wires should be of the same length as the coil forming the primary, and should be secured in a fixed position symmetrical with respect

to the primary by means of cork wedges. (2.) Covered German-silver wire, No. 28, and covered copper wire, No. 20 B. W. G., will also be required. (3.) A mirror galvanometer, of which the needle is not much damped, and a Morse key will likewise be necessary. As the Morse key will be frequently used in this section we give a figure of a suitable key (Fig. 162). (4.) A 6-inch permanent magnet. (5.) Finally, a battery, consisting of two of Fuller's bichromate cells, arranged so that either one or two cells may be used at pleasure. This may be done by means of a switch. As this arrangement is useful for many experiments, and may be applied to any number of cells, we give in the Appendix a diagram exhibiting the connections.

We shall now describe a series of experiments actually made with the above apparatus.

Experiment I.—The primary was placed within its secondary, no iron wire being within the former, and the

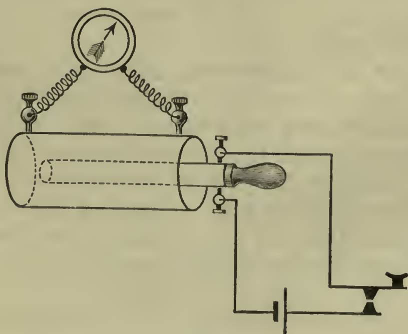


Fig. 163.

connections were made as in Fig. 163 with two cells, the binding screws A and C of the Morse key being used. On pressing the Morse key a sudden deflection was

observed in the galvanometer of forty-eight divisions to the right. When the spot of light had come to rest the key was released, when a deflection of forty-seven divisions in the opposite direction was observed. This experiment was repeated several times, and the conclusion arrived at was that the current induced in the secondary coil on making was equal and opposite to that produced on breaking the primary circuit. It was furthermore observed that the induced currents were but of short duration, no permanent deflection being noticeable, while the battery circuit continued permanently made.

Experiment II.—The connections being as before, a resistance box was placed in the circuit. The Morse key was pressed so as to complete the circuit, and it was found on suddenly varying the resistance that induced currents were obtained when the primary current was in this manner suddenly varied. The induced currents were equal and in opposite directions according as the primary was suddenly increased or diminished. It is thus shown that a variation of the primary current will produce an induced current in the secondary.

Experiment III.—Some soft iron wires were well *annealed*, and one of them was inserted in the primary coil, the connections being otherwise the same as before. It was now found that the induced current obtained on making or breaking the primary was increased so greatly as almost to send the spot of light off the scale of the galvanometer. One cell of the battery was switched off and the following observations were then taken :—

No. of Iron Wires.	Current Made.	Current Broken.
0	10	9
1	40	40
2	76	77
3	170	165

Experiment IV.—The battery current being made, it

was found that induced effects were obtained when the primary was suddenly withdrawn from the secondary and when it was suddenly placed within the secondary, the former corresponding with a breaking and the latter with a making of the circuit.

Experiment V.—The primary was placed within the secondary, and deflections were obtained as in the first experiment. The primary was then withdrawn a few millimètres at a time, and observations were taken in different positions. The following results were thus obtained:—

Position of Primary.	Break.	Make.
Completely in secondary . . .	50	50
10 mm. out of ,, . . .	40	38
20 ,, ,, ,, . . .	27	27
40 ,, ,, ,, . . .	17	...
Completely withdrawn . . .	5	...

Experiment VI.—The primary was placed so as to be on the outside of the secondary, with its axis parallel to that of the secondary. On making or breaking a deflection of four divisions was obtained.

The axis of the primary was then turned so as to be at right angles to that of the secondary. In this position no effect whatever could be observed.

Experiment VII.—A permanent magnet was substituted for the primary circuit, and motions of this were found to produce phenomena exactly similar to those described as resulting from similar motions of the primary current. The effect of moving the magnet in different ways was tried, and in all these the magnet was found to act just as if it were a primary coil conveying a positive current, which descends on the east side and ascends on the west, magnetically speaking. Finally, the magnet was dropped right through the secondary coil, and it was observed that the spot of light made a sudden motion to the left, which was immediately checked, and that it then moved for a moment to the right, thus finally showing an oscillation

about its previous position, without however recording a permanent change of place.

Lesson LXI.—Induction Experiments—Series II.

130. *Apparatus.*—(1.) In place of the primary and secondary coils of last lesson use two flat spirals of covered copper wire, No. 24 B. W. G., each having about seventy turns — the outside diameter being 4 inches, and with a hole in the middle. They must be mounted on thin boards provided with binding screws (see Fig. 164). (2.) Discs of cardboard, copper, iron, and zinc of the same thickness and size, say 5 inches in diameter and $\frac{1}{16}$ inch thick. (3.) An electro-magnet, copper and German-silver wire as in last lesson. (4.) Battery, galvanometer, and their accessories, as in last lesson.

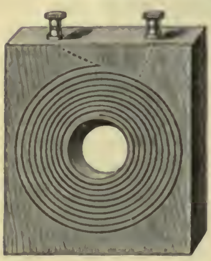


Fig. 164.
Flat Spiral.

Experiment I.—The flat spirals were laid face to face, the disc of cardboard being between them. One was connected with the battery, serving as the primary, and the other with the galvanometer, serving as the secondary circuit. An induced current was obtained after the manner already described. The cardboard was then replaced by other substances, and the amount of induced current noted in each case as shown below :—

Substance between Spirals.	Break.	Make.
Cardboard	175	174
Zinc	174	175
Copper	175	175
Iron	68	70

Experiment II.—Two flat spirals, each of the same number of turns, were made, one of No. 28 B. W. G. silk-

covered copper wire, and the other of No. 25 silk-covered German-silver wire. The number of turns in each coil was thirty-three. The spiral made of the thicker wire was 4 inches in diameter, and that made of the thinner wire $2\frac{1}{2}$ inches. Each had a central aperture of one inch. They were connected in series to the galvanometer, so that both formed part of the same circuit. The spirals were placed the one on the top of the other—the direction of winding and of the current being the same in both. A magnet was thrust through the central aperture of the spirals, and a large deflection was obtained. One of the spirals was then turned, so that the direction of winding in the one spiral was opposed to that in the other. No induced current could now be obtained. This shows that under these circumstances the currents induced by the two spirals in the compound circuit embracing both were equal and opposite, although the nature of the conducting material and the area of the spiral were different in each.

Experiment III.—The importance of this result justified a more searching trial after the manner described by Faraday in his *Experimental Researches* (First Series, p. 195). A German silver and a copper wire of the same length but of different thicknesses were fastened together at one end. The two wires were then twisted together and wound on a reel. The two free ends were connected with a galvanometer, made as delicate as possible. The reel was then placed on one of the poles of an electromagnet that was so far away as not to affect the galvanometer. On putting on the battery power, so as to make the pole strongly magnetic, no induced current whatever was obtained.

LESSON LXII.—Induction Experiments—Series III.

131. *Apparatus.*—In addition to the apparatus mentioned in the previous lessons, we require two other

similar secondary coils and two other *similar* primary coils. By similar we mean of exactly the same size and similarly wound with equal quantities of the same kind of wire.

Experiment I.—Let us now make use of several zero methods, such as those used by Professor Felici of Pisa.[1] Let us attempt to show *that the induction of a given current in the primary on the secondary is equal to that of the same current in the secondary on the primary.* Connections were made as in Fig. 165, where P_1, P_2 are two primaries, and S_1, S_2 two secondaries. The current is

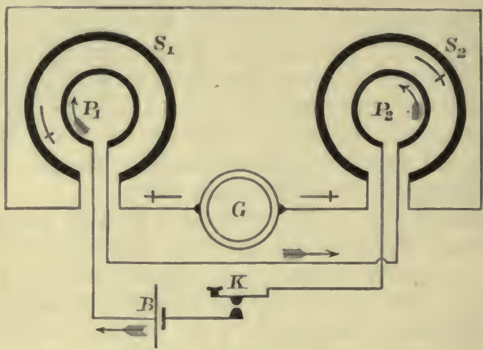


Fig. 165.

supposed to pass in opposite directions round the two primaries, so that the induction currents produced in the secondaries are equal and opposite, and hence the galvanometer will remain at rest. If on pressing the key a deflection should be obtained, one of the primaries must be pulled out a little distance until an exact balance is procured. Had the coils been exactly similar this adjustment would of course have been unnecessary.

[1] See Clerk Maxwell's *Electricity and Magnetism*, vol. ii. p. 169; also Jamin, *Cours de Physique*, iv. p. 153, and the references given there.

The direction of the currents in Fig. 165 being retained, the connections were next altered in such a manner that while the primary current circulated in P_1 and the secondary in S_1, as before, yet in the right-hand coil we had the primary circulating in S_2 and the secondary in P_2. Of course it was only the secondary current which passed through the galvanometer. No deflection was obtained on pressing the key.

This experiment shows us, therefore, that the action of P_1 on S_1 is equivalent to that of S_2 on P_2, these two actions producing currents which pass towards the galvanometer, but which exactly balance each other. Now the previous experiment showed us that the action of P_1 on S_1 is equivalent to that of P_2 on S_2, and hence it follows that the action of P_2 on S_2 is equivalent to that of S_2 on P_2 (since both are equivalent to that of P_1 on S_1). Thus the proposition is proved.

Experiment II.—Let us now attempt to show that *the electromotive force of induction is proportional to the inducing current.* For this purpose two additional coils, P_3 and S_3, exactly similar to the others in construction, were obtained. They were tested against P_1 and S_1 and also P_2 and S_2 by the method of Experiment I. It was thus shown that the action of P_1 on S_1 is equal to that of P_2 on S_2, as also to that of P_3 on S_3.

Connections were then made as in Fig. 166, and on closing the battery circuit no deflection was noticeable in the galvanometer.

Since the coils P_2 and P_3, S_2 and S_3 are alike, it follows that the resistance of P_2 is equal to that of P_3, and therefore the current C_1 circulating in P_1 will divide itself exactly equally between P_2 and P_3, giving a current $= \frac{1}{2}C_1$ in each. But since no deflection was obtained the effect of $\frac{1}{2}C_1$ in P_2 and P_3 must have been to produce an induced current in S_2 and S_3, each of only half the strength of that produced in S_1 by the current of whole strength in P_1.

In a similar manner it could be shown, by using a number of additional primaries and secondaries, that generally the inductive effect is proportional to the strength of the inducing current.

We could likewise show by a zero method that *the electromotive force induced in a coil of* n *windings by a current in a coil of* m *windings is proportional to the product* mn. We shall, however, employ instead a quantitative method.

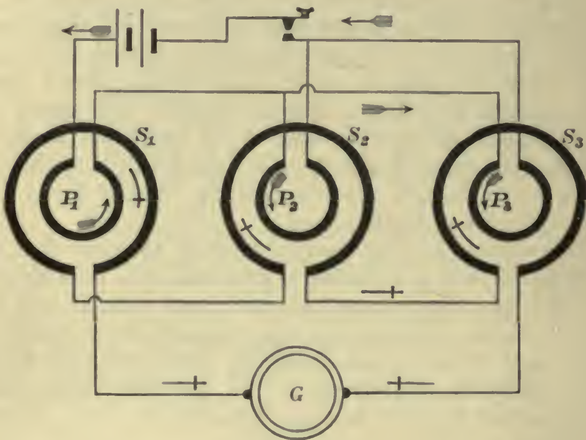


Fig. 166.

LESSON LXIII.—Induction Experiments—Series IV.

132. *Apparatus.*—We shall require two additional coils, each wound with twelve separate layers of wire. These coils must be identical in every respect. Fig. 157 exhibits one of these mounted on a base board, which is provided with thirteen binding screws. The ends of each of the separate layers are connected with the binding screws in the manner indicated in Fig. 167, where the binding screws

are numbered 1-13 and the separate layers I-XII. Here the ends of the first layer No. I. are connected with the binding screws 1 and 2, the ends of No. II. with 2 and 3, and so on. This arrangement enables any number of the layers to be put in circuit at the same time. Also these twelve layers may either be made to form primaries or secondaries at will. Battery, galvanometer, key, etc., will be likewise necessary.

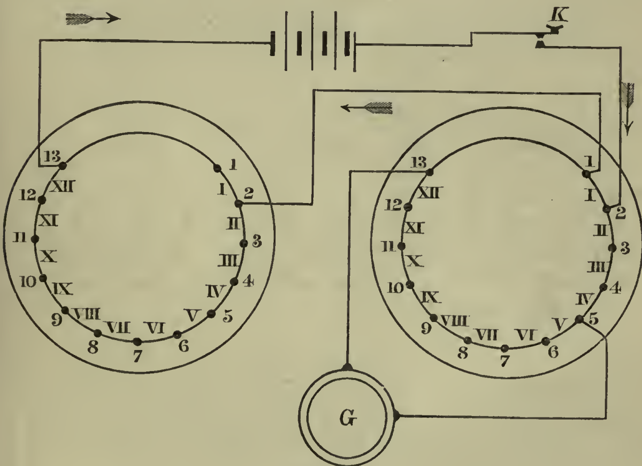


Fig. 167.

Method.—In the first place, connections must be completed as shown in Fig. 167. Here are exhibited plans of the two experimental coils—that on the right we shall call the *induction coil*, and that on the left the *compensating resistance coil.* The object of the latter is to ensure that the same resistance shall always be in the battery circuit quite independently of the number of primary coils that are in circuit. In the diagram eleven coils of the compensating coil and one coil of the induction coil are in

circuit with the battery, making twelve in all. Had there been two coils of the latter there would have been ten coils of the former, in order to make the total number twelve as before. Should this latter arrangement be desired the end of the wire passing to binding screw 2 of the induction coil must be moved to the binding screw 3, and the end of the wire at 2 of the compensating coil moved to 3. Twelve coils will be now, as before, in the battery circuit, for we have merely substituted the layer II. of one coil for the layer II. of the other, which have both the same resistance. If three coils of the induction coil are required, the ends of the wires at 3 in both coils are moved to 4, and so on. These layers of the induction coil form the primary coil. The secondary coil shown in the diagram consists of eight layers (V.-XII.), which are connected with the galvanometer.

The method consists simply in varying the number of layers in the primary and secondary, and noticing the deflections produced in the galvanometer needle when contact is made or broken.

Example.—

Primary.		Secondary.		Deflection = D.	$m \times n$.	$\frac{D}{m \times n}$
Layer.	No. of Layers.	Layers.	No. of Layers.			
I.	1	VII-XII.	6	47	6	7·833
I. and II. . .	2	,,	6	94	12	7·833
I., II., and III. .	3	,,	6	141	18	7·833
I., II., III., and IV.	4	,,	6	187	24	7·792

Here the secondary coil has been kept constant. Had we wished to vary the number of layers n forming the secondary, we should strictly have had to introduce a variable portion of the compensating resistance coil, so as to make the resistance of the galvanometer circuit always constant.

This, however, would be unnecessary if the resistance in the galvanometer were so high that but little variation in the total resistance would be produced by altering n.[1]

133. The arrangement of two primaries and two secondaries described in Fig. 165 constitutes an **Induction Balance**, and may be used in studying the inductive effect of various metals placed within P_1 or P_2.

It was first employed by Dove for this purpose.[2] Hughes, by the application of the telephone, has produced an instrument of this kind of extreme delicacy.

Lesson LXIV.—The Induction Balance of Hughes.

134. *Apparatus.*—An induction balance of the ordinary pattern. It consists of two primary coils fixed on a base-board 34 cm. apart. Above them are the two secondaries. An adjusting screw of ivory permits one of the secondaries to have its distance from its primary changed. The instrument is constructed as much as possible of wood and non-metallic materials. Two wooden cups fit within the secondaries for the reception of the metals to be tested, which are in the form of discs of the same diameter and thickness. One of the cups may be replaced by a wedge-shaped rod of zinc provided with a millimètre scale. By sliding the wedge more or less over the top of the secondary the inductive effect of the metal in the other cup may be balanced, and its value in divisions of the scale read off.

The primary coils are connected in series with a battery of three Daniell's cells and an interrupter. The latter consists of a piece of clockwork rotating a cogged wheel,

[1] The proposition contained in this lesson may also be proved for magneto-induction by substituting for the battery and primary coil a permanent magnet after the manner of Lenz (see Art. "Electricity," *Ency. Brit.*, p. 77), a compensating coil being used in the galvanometer circuit.

[2] See De La Rive, *Electricity*, vol. i. p. 425.

against the teeth of which there presses a metal tongue. The wheel and tongue being in circuit, as the former revolves, the current will be repeatedly interrupted. When the wooden cups are free from metal no noise should be heard in the telephone (which, in this instrument, is placed in the secondary circuit), provided the adjustment of the coils by the ivory screw is exact. Supposing that this adjustment has been secured, the following experiments should be made:—

Experiment I.—Test the sensitiveness of the arrangement by placing in one of the cups a very small piece of copper wire. A piece weighing not more than a milligramme ought to disturb the balance and cause a sound in the telephone. Coins of the same value placed in the cups should, as a rule, show a difference. If the arrangement is not sufficiently delicate the telephone should be re-adjusted or the battery power increased.

Experiment II.—Test discs of different metals against the zinc wedge, and note the reading on the scale attached to the wedge.

Experiment III.—Make a flat spiral of copper wire with its terminal wires unconnected, and show that there is no disturbance of the balance as long as the circuit of the spiral remains open, when the plane in which the spiral lies is parallel to the coils.

Experiment IV.—Show that discs of non-magnetic metals do not sensibly disturb the balance when placed with their plane at right angles to the coils. With magnetic metals the contrary will be found to be the case. The instrument thus furnishes a means of distinguishing between these two classes of conductors.

Experiment V.—Place the same disc at different depths within one of the secondary coils, and find the position of maximum effect.

Experiment VI.—Notice that the sounds in the telephone produced by the different materials are not of the

same character; thus iron gives out a "heavy, smothered tone," whilst hard steel has tones "exceedingly sharp."

135. The treatment of the theory of the induction balance is beyond the scope of this work. Further information respecting it will be found in the following memoirs:—"Induction Balance and Experimental Researches therewith," by Professor D. E. Hughes, *Pro. Phys. Soc., London*, vol. iii. p. 81; "On Intermittent Currents and the Theory of the Induction Balance," by Oliver J. Lodge, *Pro. Phys. Soc., London*, vol. iii. p. 187; "On the Graduation of the Sonometer," by J. H. Poynting, *Pro. Phys. Soc., London*, vol. iii. p. 169; "Molecular Electro-Magnetic Induction," by D. E. Hughes, *Pro. Roy. Soc.*, 1881, vol. xxxi. p. 525.

136. We proceed now to describe a series of experiments in illustration of the action of a current on itself, known as **Self-Induction.**

Lesson LXV.—Experiments on Self-Induction.

137. *Apparatus.*—An electro-magnet with removable core, such as that of Fig. 157; a galvanoscope or galvanometer differentially wound. Fig. 168 shows a suitable instrument. It has a pivoted needle with a pointer fixed at right angles to it, the end of which is seen in the figure. By means of a metal pin the movement of the pointer may be limited as desired. A key will likewise be necessary (a mercury-cup key being preferred), also a battery, such as Grove's, a box of coils, and Wheatstone's bridge apparatus.

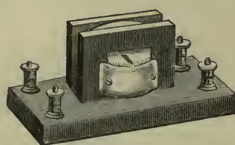


Fig. 168.—Differentially Wound Galvanoscope.

Experiment I.—Make a circuit (Fig. 169) containing in series the battery, the key K, and the coil of the electro-magnet CD without its core. Notice that when the circuit

is made there is scarcely any spark at the contact place of the key; but on the other hand, when the circuit is broken, there is a brilliant spark. Try coils wound with different numbers of turns of wire.

Experiment II. (Jenkin[1]).—Place a moistened finger of one hand on the bare wire at A, and a moistened finger of the other hand on the bare wire at B, that is to say, on opposite sides of the contact place. Break and make contact. On breaking contact a smart shock should be felt. For this experiment to succeed well the coil should consist of a large number of turns of wire.

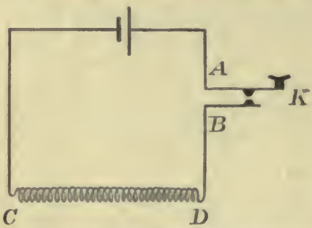


Fig. 169.—JENKIN'S EXPERIMENT.

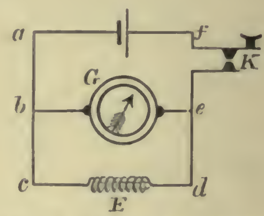


Fig. 170.—FARADAY'S EXPERIMENT.

Experiment III.—Repeat Experiments I. and II., but insert the core within the coil. The effects will now be much stronger.

Experiment IV. (Faraday[1]).—Arrange, as in Fig. 170, a cross-circuit *be*, in which the galvanometer G is placed. On making contact the current will split itself between the paths *eb* and *dc*, and in consequence the galvanometer needle will be deflected, say to the *right*. Now place a stop-pin to the right of the pointer, so as to keep it at zero, and prevent it from travelling to the right while the current is passing. Then raise the key, thus breaking contact, when there will be a sudden and momentary deflection to

[1] See Faraday's *Experimental Researches in Electricity*, Series ix. vol. i. The student is strongly recommended to read the Researches bearing on the induction of currents.

the *left*, due to the current of induction produced in E when the main current was broken. This induced current is in the same direction as the main current, and will travel in the lower part of the circuit in the same direction as the main current, but it will complete itself by passing through the galvanometer branch *eb* in a direction the opposite to that of the main current, and hence the galvanometer deflection will be the opposite of that which the original current produced.

The induced current formed when a circuit is broken is called the *direct extra current.*

Experiment V. (Faraday).—With the same connections as before, but with a current just strong enough to allow the permanent deflection to the right to be readable, place a stop-pin to prevent the pointer from returning to zero. Break and then remake contact, when the pointer will be suddenly deflected farther to the right. Here the current from *e* to *b* is inoperative, since the pointer is at the place of maximum deflection that would be produced by its means, hence the increased deflection can only be due to an induced current opposite in direction to the main current, and which, originating in the lower half of the circuit, travels through the galvanometer wire in the same direction as the main current.

Experiment VI. (Edlund).—The galvanometer, which we must now use as a differential instrument, permits the extra currents to be studied more easily than can be done by the preceding methods. Make connections as in Fig. 78, where R is a box of coils and x the electro-magnet. Balance R against x, when it will be found that if the key be raised or lowered the equilibrium will be momentarily disturbed, the needle being sharply deflected in the one case to the right and in the other to the left. The effect is due to the self-induction of x alone, for R is composed of coils wound double, which prevents the formation of induced currents. That this will do so

will be clearly seen if we reflect that the action of a coil in producing an extra current is due to the fact that it is composed of a great number of convolutions, in each of which the main current circulates in the same direction. If therefore we double the coil upon itself in such a manner that two neighbouring convolutions convey currents, which circulate in opposite directions, we destroy their power of producing induced currents.

Experiment VII.—By balancing the coil whose self-induction has to be investigated in a Wheatstone's bridge (see p. 136), and closing the galvanometer key B′b before the key A′a is closed or opened, the existence of the two induced currents may likewise be shown in a very convenient and obvious manner. It will here be remembered that in determining resistances by the bridge, attention was called to the necessity of closing the battery key before the galvanometer key, in order to avoid the action of induced currents on the galvanometer, which would otherwise lead the observer astray in his measurements.

(B.) *Application of Electro-Magnetic Induction to Measurements.*

138. *Measurement of Transient Currents.*—The currents produced by induction last but a very short time, and hence they must be measured in a special manner. In some of the preceding experiments we have tacitly assumed that the strength of the induced current is proportional to the amplitude of the first swing of the galvanometer. We must now proceed to justify this assumption, and explain more exactly the method of measuring currents of short duration.

(I.) Let a be the angle through which the galvanometer needle is deflected by a current of short duration, then (Fig. 171), at the instant when the needle is at the end of

its swing, its north pole (of strength m) has virtually moved against the force Hm (where H is the horizontal magnetic intensity) through a distance nn'; the south pole has likewise moved a similar distance, and hence the whole work done by the needle is

$$2\mathrm{H}m \times nn' = 2\mathrm{H}m \times l(1 - \cos \alpha) = \mathrm{HM}(1 - \cos \alpha),$$

where l is the half-length of the needle and M its magnetic moment. But if we refer to Vol. I. p. 243 we shall see that the work done may likewise be expressed as $= \frac{1}{2}\mathrm{I}\omega^2$, where I is the moment of inertia of the system with respect to the axis of rotation and ω is the angular velocity as it passes its point of rest. Hence we have

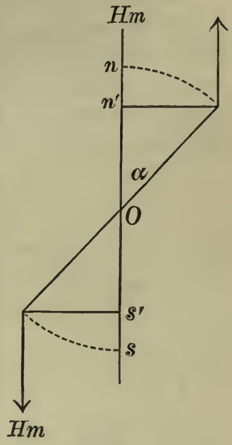


Fig. 171.

$$\tfrac{1}{2}\mathrm{I}\omega^2 = \mathrm{HM}(1 - \cos \alpha)$$

or

$$\omega = 2\sqrt{\frac{\mathrm{HM}}{\mathrm{I}}}\sin\frac{\alpha}{2} \quad . \quad . \quad . \quad . \quad (1)$$

(II.) Next, let C be the strength of the induced current, and τ the short time through which it lasts. This time may be regarded as so short that during the whole passage of the discharge the needle has not sensibly moved. If therefore the current acts upon it by means of a coil it will have its full effect without any deduction on account of the changed position of the needle, inasmuch as this change only begins to develop itself after the current has ceased to act. By the theory of the tangent galvanometer (see pp. 322-326), the moment of the couple acting upon the magnet will be

$$\mathrm{C} \times \frac{2\pi n}{a} \times \mathrm{M},$$

where a is the mean radius of the galvanometer coils and n the number of windings.

This will be equivalent to a force $= \mathrm{C} \times \frac{2\pi n}{a} \times \frac{\mathrm{M}}{2k}$ acting at the extremities of the radii of gyration $(=k)$ in each half of the needle, and lasting for the time τ. At these two extremities the mass of each of the two half needles may be supposed to be concentrated (Vol. I. p. 243), and hence the velocity produced will be

$$\mathrm{C} \times \frac{2\pi n}{a} \times \frac{\mathrm{M}}{2k} \times \frac{\tau}{\text{half mass}},$$

while the angular velocity will be

$$\omega = \mathrm{C} \times \frac{2\pi n}{a} \times \frac{\mathrm{M}}{2k^2} \times \frac{\tau}{\text{half mass}} = \mathrm{C} \times \frac{2\pi n}{a} \times \frac{\mathrm{M}\tau}{\text{mass} \times k^2} = \mathrm{C} \times \frac{2\pi n}{a} \times \frac{\mathrm{M}\tau}{\mathrm{I}},$$

or

$$\omega = \frac{\mathrm{MC}\tau}{\Gamma\mathrm{I}} = \frac{\mathrm{MQ}}{\Gamma\mathrm{I}} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where Q denotes the whole quantity of the induced current, while Γ is the true constant of the galvanometer (see p. 225).

(III.) From (1) and (2) we have

$$\frac{\mathrm{MQ}}{\Gamma\mathrm{I}} = 2\sqrt{\frac{\mathrm{HM}}{\mathrm{I}}} \sin\frac{a}{2};$$

hence

$$\mathrm{Q} = 2\Gamma \sin\frac{a}{2}\sqrt{\frac{\mathrm{HI}}{\mathrm{M}}} = 2\Gamma\mathrm{H}\sqrt{\frac{\mathrm{I}}{\mathrm{HM}}} \sin\frac{a}{2}.$$

Let T be the time of a single vibration of the magnet, then

$$\mathrm{T} = \pi\sqrt{\frac{\mathrm{I}}{\mathrm{HM}}}$$

(see Vol. I. p. 251); hence

$$\mathrm{Q} = \frac{2\Gamma\mathrm{HT}}{\pi} \sin\frac{a}{2} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

(IV.) If the deflections are taken by means of a mirror galvanometer, we have (Vol. I. p. 55) $\tan 2\alpha = \frac{s}{L}$, where s is the number of scale divisions, let us say in millimètres, and L the distance of the mirror from the scale, also in millimètres. But the deflections being small we may assume that

$$\tfrac{1}{2}\tan 2\alpha = \tan\alpha = \sin\alpha = \frac{s}{2L},$$

also

$$\tfrac{1}{2}\tan\alpha = \tan\frac{\alpha}{2} = \sin\frac{\alpha}{2} = \frac{s}{4L}.$$

Expression (3) will therefore become

$$Q = \frac{2\Gamma HT}{\pi}\,\frac{s}{4L} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

or the transient current is proportional to the amplitude of the first swing expressed in scale divisions.

139. *The value of T.*—It must be remembered that the value of T will depend upon the moment of inertia and the magnetic moment of the suspended magnet, as well as on the value of H. Now both the magnetic moment and H may vary—the former on account of the weakening of the suspended magnet, caused by the demagnetising influence of the transient currents; whilst the latter will depend, amongst other things, upon the position of the directing magnet of the galvanometer. Hence it is of importance to ascertain T at the commencement and likewise at the end of a series of experiments. The *method of passages* should be employed, as described in Vol. I. p. 186, and the amplitude of vibration both at the commencement and end of the passages should be noted in order that correction should be made to an infinitely small arc, as described in Chap. VI. p. 295. With the mirror galvanometer this correction is small, and may be best applied as in the next article.

140. The corrected value T_1 of T may, as in p. 294, be written thus—

$$T_1 = T\left(1 - \tfrac{1}{4}\sin^2\frac{\beta}{4}\right),$$

where β is the mean amplitude of the vibration. But with the mirror galvanometer

$$\tan 2\beta = \frac{s}{L},$$

where s and L have the same meaning as above, and with sufficient exactness for our purpose

$$\sin\frac{\beta}{4} = \tan\frac{\beta}{4} = \frac{s}{8L},$$

and hence

$$T_1 = T\left(1 - \frac{s^2}{256L^2}\right).$$

141. *Effect of Damping—The Ballistic Galvanometer.*—The preceding investigation is based upon the assumption that there are no causes tending to limit the extent of the needle's deflection other than that due to the horizontal magnetic couple. But there are three distinct sources of resistance that have not been considered, namely—

(1.) The resistance of the air.

(2.) The effect of the currents induced in the metal-work surrounding the needle, which, in accordance with Lenz's law, tend to oppose the motion of the body producing them.

(3.) The viscosity of the suspending fibre.

Accordingly, if we wish to measure accurately a transient current, these sources of error must be allowed for or made so small as to have a negligible effect. The **Ballistic Galvanometer** is an instrument adapted for measuring transient currents. This only differs from an ordinary galvanometer in possessing a heavy needle, made nearly spherical. The needle is made heavy in order to make its time of vibration large, and it is made spherical in order

that the air resistance may be a minimum. The first condition is clearly requisite, since we have assumed that the magnet remains practically at rest during the passage of the transient current.

141_a_. *Elementary Theory of Damping.*—In ordinary motions of the needle we may assume that the resisting forces are *simply proportional to the velocity*, so that our investigation is how to determine the motion of a body under an attraction varying as its distance from its point of rest, and a resisting force varying as its velocity. To simplify this problem we shall assume that the influence of damping is comparatively small.

Our first remark will be that a comparatively small damping force will not sensibly affect the *time of vibration.*[1] For, let us consider an oscillation beginning from an elongation at the left $= a$, and ending with one at the right $= a'$, which will be less than a. Had there been no retardation the two elongations would of course have been equal to each other. Let v denote the velocity as the point of rest was passed. Now divide each half vibration into a great number of parts, the principle of division being that a small change of velocity $= \frac{v}{n}$ has been produced between the beginning and end of each part.

If we now compare the damped vibration with another starting from the same point, which is undamped and subject to no retardation, we may easily perceive that the time of passing through each of these divisions will, in the *left-hand half*, be greater for the damped than for the undamped vibration, while in the *right-hand half* the opposite will hold. The result will be that the time of executing the first half will be greater, and that of executing the second half less for the damped than for the undamped vibration. On the whole, therefore, we may suppose the

[1] There will, however, be a very small increment in the time of vibration, but this will be of the second order.

one difference to counteract the other, and conclude that the whole time will not be sensibly affected.

Our next remark is that the effect of damping is to *diminish the amplitude* of any vibration in a *fixed proportion.* To prove this, let there be two undamped (that is to say unresisted) amplitudes a and b, and let each be divided into the same great number of equal parts, so that one division of a will bear to one division of b the ratio of a to b. Likewise imagine both vibrations to commence simultaneously from the extreme left elongation. Now the time of passing the first small division will be the same for both vibrations, inasmuch as the force (sensibly constant through one small division) of the first will be to that of the second as a is to b. But the spaces are likewise in the same proportion, and hence the time of describing the first division will be the same for both, while the velocities generated will be to one another as a is to b. Thus the vibrating bodies will enter their respective second divisions with these relative velocities, and the forces in this division are likewise to one another in the same proportion. So that the velocities with which the vibrating bodies enter their respective second divisions, the forces they experience in them, and the lengths of the divisions being all in the same proportion, the time of describing the second division will be the same for both. This result will hold for any other division, and hence, as is well known, the whole times will be the same for both.

Let us now consider the resisting forces (supposed small). It is clear that the velocity with which the first system enters any given division will bear to that with which the second system enters the corresponding division the ratio of a to b, the lengths of the divisions being likewise in this proportion. Now the energy taken away by the resistance during a division will be proportional to the resisting force × length of division, that is to say, to the velocity ×

length of division, since the force is proportional to velocity. Hence the energy taken from the first system in any division or number of divisions, in other words, the whole loss of energy, will be to that taken from the second system in the ratio of a^2 to b^2. Now the whole energy of a vibration is proportional to the square of the amplitude, so that we may represent the energy of the two vibrations by $\mathrm{K}a^2$ and $\mathrm{K}b^2$, while ma^2 and mb^2 may represent the loss of energy in each due to damping. Hence the energy left will be $(\mathrm{K}-m)a^2$ and $(\mathrm{K}-m)b^2$. But we may put $\mathrm{K}-m$ into the form $\frac{\mathrm{K}}{\rho^2}$, and hence the energy left will be $\mathrm{K}\frac{a^2}{\rho^2}$ and $\mathrm{K}\frac{b^2}{\rho^2}$.

This will correspond to a diminished amplitude equal to $\frac{a}{\rho}$ and $\frac{b}{\rho}$, and hence it follows that the successive amplitudes will always be diminished in the same proportion. Now let a_1 denote the *first* amplitude, then the *second* will be $a_2=\frac{a_1}{\rho}$, the *third* $a_3=\frac{a_1}{\rho^2}$, the nth $a_n=\frac{a_1}{\rho^{n-1}}$, and so on.

142. *Logarithmic Decrement.*—Let λ denote the logarithm corresponding to the base e of the proportional decrement ρ, then λ is termed the logarithmic decrement. We have already shown that $a_n=\frac{a_1}{\rho^{n-1}}$, and hence it follows that

$$\log_e a_n = \log_e a_1 - (n-1)\log_e \rho.$$

Similarly

$$\log_e a_m = \log_e a_1 - (m-1)\log_e \rho.$$

Hence

$$\log_e a_m - \log_e a_n = (n-m)\log_e \rho = (n-m)\lambda,$$

and hence

$$\lambda = \frac{1}{n-m}(\log_e a_m - \log_e a_n).$$

Theory shows that errors of observation have the least influence when $\frac{a_m}{a_n}=e=2{\cdot}718$, or 3 nearly.

We can now tell how much the value of s in the

equation $Q = \frac{2\Gamma HT}{\pi} \frac{s}{4L}$ has been diminished by damping. For suppose that by means of a series of experiments we have determined λ. Now s does not represent the true amplitude of the induction kick, but the amplitude after the damping has influenced it during *half a vibration.* Let s_o represent the true amplitude, hence

$$\lambda = \frac{1}{\frac{1}{2}}(\log_e s_o - \log_e s),$$

and hence

$$\log_e s_o = \tfrac{1}{2}\lambda + \log_e s \quad . \quad . \quad . \quad . \quad (1)$$

But, by the nature of logarithms,

$$s_o = e^{\log_e s_o} = e^{\frac{1}{2}\lambda + \log_e s} = e^{\frac{1}{2}\lambda} \times e^{\log_e s} = e^{\frac{1}{2}\lambda} \times s \quad . \quad . \quad (2)$$

Now $e^x = 1 + \frac{x}{1} + \frac{x^2}{1.2} + \text{etc.}$, and hence where x is small $e^x = 1 + x$, nearly. Hence

$$e^{\frac{1}{2}\lambda} = 1 + \tfrac{1}{2}\lambda, \text{ and finally } s_o = s(1 + \tfrac{1}{2}\lambda) \quad . \quad . \quad . \quad (3)$$

When great accuracy is required the observed values of the amplitudes must be reduced to numbers proportional to angular measure by subtracting $\frac{s^3}{3L^2}$ (see Vol. I. p. 58), where L is the distance in scale divisions of mirror from scale.

Example (Kohlrausch).—Middle division of scale = 500, L = 2600.

Observed Turning Points $= t$.	$s = t \sim 500$.	$\frac{s^3}{3 \times 2600^2}$.	Corrected Turning Points.	Arcs.
285	215	·5	285·5	$424·0 = a_0$
710	210	·5	709·5	$368·1 = a_1$
341·2	159	·2	341·4	$320·9 = a_2$
662·5	162	·2	662·3	$278·3 = a_3$
383·9	116	·1	384·0	$241·6 = a_4$
625·7	126	·1	625·6	$210·0 = a_5$
415·6	84	·0	415·6	

From a_0 and a_3	$\lambda = \frac{1}{3}$ ($\log_{10}$424	$- \log_{10}$ 278·3)	= ·0610
,, a_1 and a_4	,, ,,	368·1	,, 241·6	= ·0609
,, a_2 and a_5	,, ,,	320·9	,, 210·0	= ·0614
			Mean λ	= ·0611

This will be the value of the logarithmic decrement to the base 10. To find it to the base e we must multiply by the modulus of the Napierian system of logarithms. Hence

$$\lambda_e = \cdot0611 \times 2\cdot3026 = \cdot141.$$

143. *The Ballistic Galvanometer in Practice.*—Generally speaking, the less a galvanometer is damped the more tedious work with it becomes, and hence one with a very small decrement should, if possible, be avoided. For many purposes the high and low resistance galvanometers described in Chap. IV. are applicable when the directing magnet is placed so as to give a time of vibration of about five seconds. But for absolute measurements it would be necessary to employ a proper ballistic galvanometer, as already described. To bring its needle to rest a small coil may be employed. The coil is fixed at the back of the galvanometer case, and placed in the circuit of a Leclanché cell, provided with a *tapper commutator.* By pressing momentarily one or other of the keys of the commutator as the needle passes its middle point, it may, with a little practice, be quickly brought to rest.

A difficulty inseparable from the type of reflecting galvanometer employed in England is that two observers are required, the one to read the galvanometer and the other to manipulate some apparatus, which, owing to its magnetic effect on the galvanometer, must be kept at a distance. The German subjective or telescopic system must therefore be employed whenever only one observer is available, the observing telescope being placed sufficiently far away from the galvanometer (see Vol. I. p. 55).

Lesson LXVI.—Experiments on Damping.

144. *Apparatus.*—A wooden box (Fig. 172) with a glass sliding door has suspended within it, from the rod *r*, by means of a silk thread, a bar magnet *m*, 10 cm. long, 1 cm. thick, and 1 cm. broad. A hole is drilled in the centre of one of its faces for the reception of a hook. To this hook is attached a galvanometer mirror. Below the

Fig. 172.—Magnetometer and Dampers.

bottom of the box there is a recess, within which plates of different metals, all of the same superficial area, may be introduced. These plates are of different thicknesses, but in order to bring the surface of the metal to the same distance from the magnet the thinner plates are backed with wood of such a thickness that all the plates just fit

within the recess. The magnetometer (for the instrument is such in reality) is provided with the usual lamp and scale.

Method.—Set the magnet swinging without a plate beneath it, in order to ascertain the damping effect of the air above. Calculate the logarithmic decrement, then proceed likewise to ascertain its value when different plates are placed under the magnet.

Example.—Four plates of copper, whose thicknesses were in the ratio of 1 : 2 : 3 : 4, were employed, and a brass plate was likewise used equal in thickness to the thickest copper. The following results were obtained :—

Cause of Damping.	Logarithmic Decrement.	Time taken to reduce Amplitude from 470 to 250.
Air	·001483	22 min. 13 sec.
Copper (1) + air . .	·00359	7 ,, 10 ,,
Copper (2) + air . .	·00518	5 ,, 10 ,,
Copper (3) + air . .	·00667	3 ,, 55 ,,
Copper (4) + air . .	·00851	3 ,, 2 ,,
Brass + air . . .	·00620	4 ,, 20 ,,

The logarithmic decrements due to metal alone are clearly to be obtained by subtracting from the logarithmic decrement due to metal and air that due to air alone. This follows from the nature of "logarithmic decrement." Hence we have—

	Log. Dec.
Copper (1)	$\cdot 00211 = \cdot 00211 \times 1$
,, (2)	$\cdot 00370 = \cdot 00185 \times 2$
,, (3)	$\cdot 00519 = \cdot 00173 \times 3$
,, (4)	$\cdot 00703 = \cdot 00176 \times 4$
Brass	$\cdot 00472$

From this we perceive that copper is a better damper than brass, and that the logarithmic decrements of the four copper plates are (very roughly) proportional to their thickness, and hence to their conductivity.

LESSON LXVII.—Determination of Resistance by the Damping Method.

145. *Apparatus.*—A low resistance galvanometer whose needle is not much damped; the resistances to be compared.

Method.—Let

λ_α be the logarithmic decrement when the galvanometer terminals are disconnected. This will give the damping due to air alone.

λ_0 be the same when the terminals are connected by a thick wire of negligible resistance. Here the resistance R_0 is that of the galvanometer alone.

λ_1 be the same when the resistance R_1 is in circuit.

λ_2 be the same " " R_2 "

Then observing that the logarithmic decrement expressing the damping due to the induced currents will be proportional to the intensities of these currents, we shall have

$$\lambda_0 - \lambda_\alpha : \lambda_1 - \lambda_\alpha : \lambda_2 - \lambda_\alpha = \frac{1}{R_0} : \frac{1}{R_0 + R_1} : \frac{1}{R_0 + R_2},$$

hence

$$\frac{R_1}{R_2} = \frac{\lambda_0 - \lambda_1}{\lambda_0 - \lambda_2} \cdot \frac{\lambda_2 - \lambda_\alpha}{\lambda_1 - \lambda_\alpha}.$$

We have therefore to observe the four logarithmic decrements given above. If R_1 and R_2 are *very* large λ_1 and λ_2 would be nearly equal to λ_α, and the method would not then give accurate results.

Example.—$\lambda_\alpha = \cdot 0020$, $\lambda_0 = \cdot 1511$, $\lambda_1 = \cdot 1032$, $\lambda_2 = \cdot 0667$, also $R_2 = 1$ ohm. Hence

$$R_1 = 1 \times \frac{\cdot 1511 - \cdot 1032}{\cdot 1511 - \cdot 0667} \times \frac{\cdot 0667 - \cdot 0020}{\cdot 1032 - \cdot 0020} = \cdot 363.$$

146. The effect produced in a coil by the establishment of a magnetic field within it may be regarded as of the nature of a transient E. M. F. Such a force being subject to Ohm's law, will produce a current which will

be inversely proportional to the resistance of the circuit. On this principle there is based a method of comparing resistances which possesses the advantage that in the extremely short time during which the discharge lasts the alteration of the resistance due to heating will be very small. An instrument adapted for comparisons by this method is called a **Magneto-Inductor.**

Lesson LXVIII.—Comparison of Resistances by the Magneto-Inductor.

147. *Apparatus.*—In Fig. 173 we have a convenient form of this instrument suitable for use with a ballistic galvanometer of the type used in England. Two magnets are fastened together, with their poles in the same straight line, the two north poles being nearly in contact, and this double magnet is mounted in a framework that is suspended after the manner of a pendulum by a long metal strip from a wooden support. The double magnet passes without contact through the central hole of the bobbin (fixed to the wooden upright, and thus not moving with the pendulum) on which a coil is wound. The coil is half the length of the double magnet. At the bottom of the framework is a bob by which the pendulum may be

Fig. 173.—The Magneto-Inductor.

moved to and fro. A ballistic galvanometer will also be required.

Theory and Method.—It will be observed that the limits of the motion of the pendulum are defined by the fixed bobbin coming in contact with the framework of the pendulum at either end. The amount of oscillation is thus limited. Suppose now that the pendulum is suddenly pushed from the one extreme to the other. During the whole of its course the bobbin is leaving the one magnet and approaching the other. Now, since these magnets are placed so as to have reversed polarity, it follows that the induction current produced in the bobbin by leaving the one will be in the same direction with, and will supplement that produced in it by approaching the other. The induced current will thus have a perfectly definite value, and it will be measured by means of the ballistic galvanometer, with which the bobbin must be connected.

Now let R be the joint resistance of the bobbin and galvanometer, and let d denote the deflection when there is no other resistance in circuit.

Also let R_1 and R_2 be the resistances to be compared, and d_1 and d_2 the deflections produced when these resistances are respectively included in the circuit of the magneto-inductor and galvanometer.

Hence

$$\frac{R}{R+R_1}=\frac{d_1}{d},$$

and

$$\frac{R}{R+R_2}=\frac{d_2}{d};$$

and hence also

$$\frac{R_1}{R_2}=\frac{d-d_1}{d-d_2}\cdot\frac{d_2}{d_1}.$$

LESSON LXIX.—Investigation of a Uniform Field of Force.

148. *Apparatus.*—An electro-magnet provided with two

large cubical pole pieces. A hole should pass right through the centres of the cubes from face to back, as in Fig. 174. A hollow helix 20 cm. long and 5 cm. in diameter, a low resistance mirror galvanometer fixed some distance away from the magnet. Battery, keys, wires, etc. Cardboard, iron filings.

Method.—Fix the electro-magnet vertically, and place the pole pieces in position, as in Fig. 174. Arrange the circuit of the electro-magnet and run the connecting wires together to prevent disturbance.

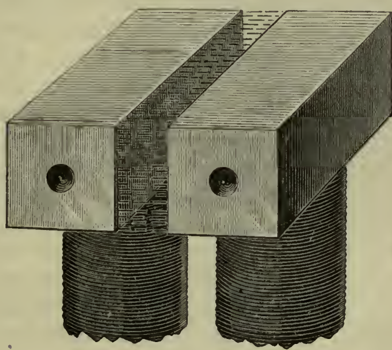


Fig. 174.

A sheet of cardboard must next be placed over the pole pieces, and by means of the iron filings magnetic curves may then be obtained. The lines of force will be found to be concentrated across the space between the pole pieces in a direction perpendicular to the opposing faces (see Fig. 174). This region may therefore be considered to be a uniform field of force.

Next connect a long straight wire with the circuit of the galvanometer (the wire of p. 61), and proceed with it to investigate the field of force after the manner of the following experiments :—

Experiment I.—The wire was placed between the pole pieces with its length horizontal, and at right angles to the lines of force. On moving the wire from the one pole to the other, keeping it always at right angles to the lines of force, and arranging so that the circuit should not cut lines of force when the wire moved, there was no effect upon the galvanometer.

Experiment II.—The position being as before, the wire was moved in the direction of its length without effect.

Experiment III.—The wire being as before, it was moved up and down so as to make the circuit cut lines of force, when a large deflection was produced.

Experiment IV.—The pole pieces being turned so that the holes faced each other, a wire was passed through the two holes so as to lie along lines of force. On moving the wire either in the direction of its length or in any other way, provided the circuit was not made to cut lines of force, there was no effect.

A ring was now made of ten turns of covered wire, the free ends being twisted together and connected with the galvanometer, and the action of the magnetic field upon it was ascertained by a second series of experiments as follows :—

Experiment I.—The ring was placed vertically with its plane parallel to the opposing polar surfaces, and hence at right angles to the lines of force. On moving it slightly in its own plane, or in the direction of its axis, so as to keep it always parallel to itself, there was no induced current; but when removed outside the region of uniform force, there was a large deflection in one direction, and when brought back to its previous position, there was an equal deflection in the opposite direction.

Experiment II.—The ring being still vertical, but with its plane parallel to the lines of force, no effect was produced by moving it in its own plane or parallel to its own

plane. There was likewise no effect when it was so moved to a position without the uniform field of force.

Experiment III.—The ring being now placed horizontally, there was no effect whether it was moved about within the uniform field or removed outside of it, provided it was always kept horizontal.

Experiment IV.—The ring being as in Experiment II., was rotated about a horizontal axis (this being parallel to the lines of force), but no effect was produced. It was next rotated about a vertical axis, when a large deflection resulted. Finally it was rotated about an axis passing through its centre and perpendicular to its plane, without any effect being produced.

Experiment V.—Lastly, the ring was placed vertically, in the position of Experiment I., and rotated about an axis at right angles to the lines of force, when a large deflection was produced.

The helix was now connected with the battery, and a piece of cardboard placed horizontally within it. By means of iron filings the lines of force were obtained, and found to be parallel with the axis of the helix, except at the ends of the coil, where they diverged. Assuming, in accordance with the principles of p. 329, that the field of force within the coil is uniform, a series of experiments similar to the above were made with the straight wire and ring within the helix. These and other experiments will be found to agree with the preceding in establishing the following rules :—

(1.) If the plane of the circuit be parallel to the lines of force, no E. M. F. results from the motion of a movable portion of the circuit, or of the whole circuit in any direction in the plane of the circuit, nor will any result if the circuit be translated into another plane parallel to its first.

(2.) If the plane of the circuit be perpendicular to the lines of force, so that a number of lines of force

pass through it, and if it be turned or carried into any position, or so altered that the number of these lines of force embraced by the circuit is increased or diminished, an induction current will be produced; the direction of this current caused by an increase being in the opposite direction to that caused by a diminution in the lines of force which pass through the circuit.

(3.) In a closed circuit no E. M. F. is generated when it receives a motion of translation (not rotation) in a uniform field. Here evidently the number of lines of force embraced by the circuit is the same at the end as at the beginning.

149. The best example of a uniform field of force is that due to the earth's magnetism, provided that we confine our attention to a region not too large. In order to see how the earth acts, suppose that we have a closed circle of wire lying horizontally on the table, and that we attach to it four paper labels N., E., S., and W., denoting magnetic north, east, south, and west. Next, keeping the point W. fixed, raise the circle into a vertical position, taking care that the line N.S. shall rise parallel to itself.

Now, in the first place, there are no lines of force due to the earth's *horizontal* component passing through the circular area in either position. Clearly, then, there will be no induced current due to the horizontal force.

On the other hand, in its horizontal position, a number of lines of force due to the earth's *vertical* component passed through the circle. This number will be denoted by VA, where V is the value of the vertical component and A the area of the circle. Again, in its vertical position, there are no lines of force due to this component passing through the circle.

Hence, according to the reasoning of Appendix B, we shall, during the time taken to perform this turning opera-

tion, have a quantity of electricity Q flowing through the wire such that $Q = \frac{VA}{R}$, in which R is the resistance of the circuit.

To find the direction of this current we have to remember that the earth's vertical component represents in the northern hemisphere a south pole, on which we are looking down, and hence that its molecular current may be supposed to be circulating clockwise. Also, the circle being withdrawn from the lines of force, the induced current in it will likewise circulate clockwise, that is to say, in the direction N.E.S.W. We have as yet only turned our circle through 90°, let us complete other 90° in the same direction, until the circle is brought once more flat upon the table.

It will, during this second revolution of 90°, have regained those lines of force which it had lost during the first, and hence there will be generated in it a current equal to that which was generated in it during the first 90°, but in an opposite direction as far as an observer looking at the circle from above is concerned. But the back of the circle is now above and the face containing the letters below, and hence, to an observer looking on the face, the direction of the current in the second quadrant will be the same as in the first.

If we continue the same treatment of the circle through another 180°, we shall produce currents that have the same reference to the back of the circle that those in the first 180° had to its face, and hence the currents in the second 180° will be, with reference to the face, the opposite of those in the first 180°, that is to say, they will circulate in the direction N.W.S.E. and not in the direction N.E.S.W.

A little reflection will at once show us that the result would have been the same had the horizontal circle been attached to an axis passing between N. and S., and then made to perform a complete revolution. We see likewise,

that were we to employ a vertical axis instead of a horizontal one, we should obtain precisely analogous results, only caused by the horizontal component of the earth's force and not by its vertical component. Here the total quantity Q for a half revolution will be

$$Q=\frac{2HA}{R}.$$

If the coil consisted of not a single turn but many windings, then every turn would produce a quantity Q; hence if a is the mean radius of the coil and n the number of windings,

$$Q=\frac{2Hn\pi a^2}{R}.$$

The quantity $n\pi a^2$ we shall call the **Area of Induction.**

150. It is thus clear that by turning the circle round through 180° about a horizontal axis parallel to the magnetic meridian, we get a current which we may call $d_1 = cV$, c being a constant belonging to the circle and V the earth's vertical force. On the other hand, by turning it through 180° round a vertical axis, we get a current $d_2 = cH$, H being the earth's horizontal force and c having the same value as before. Now

$$\text{Tangent dip} = \frac{V}{H}=\frac{d_1}{d_2},$$

hence by this observation the dip may be ascertained. This method was first suggested by Faraday (*Researches*, § 3214, vol. iv.), and Weber has shown that it is capable of good accuracy, and has devised an *earth inductor* for the purpose of the determination.[1]

[1] The method possesses several advantages for a fixed observatory over the dip circle, especially with regard to the short time necessary for a complete observation. For Weber's original account see Pogg. *Ann.*, Bd. xc. It may be mentioned that the mechanician Edelmann, of Munich, has devised an earth inductor adapted for observatory work.

Lesson LXX.—Use of Earth Inductor.

151. *Apparatus.*—Fig. 175 exhibits an ordinary Delezenne circle that may be adapted for the purpose of an earth inductor. The ends of the circular coil C should be connected with the binding screws directly by thin coiled

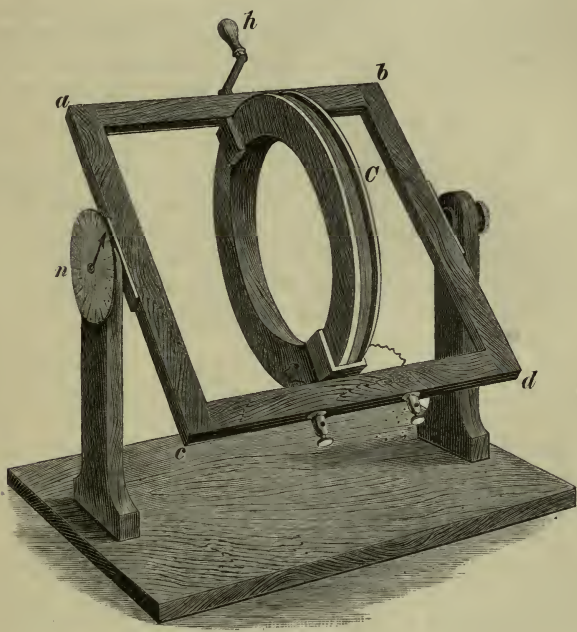


Fig. 175.—The Earth Inductor.

wires instead of through the split ring commutator, which is unsuitable. The coil C is mounted so as to revolve within the wooden framework by turning the handle *h*. The wooden framework is itself mounted so as to revolve.

We must, however, limit the play of the coil by screwing blocks of wood on the bar *ab*, so that the coil can only be rotated through 180°. The base should also be provided with levelling screws.

For the observation a low resistance mirror galvanometer with negligible logarithmic decrement and long time of vibration should be employed.

Method.—(1.) Turn *abcd* and C until both are horizontal, and by the aid of a compass needle place the axis of C in the magnetic meridian. (2.) Place a striding level on the frame *abcd* from axis to axis, and level this axis. (3.) Level the axis of C and clamp the framework by the milled-head screw seen at the top of the right-hand upright. (4.) Take a series of readings with the inductor turned through 180°, first in one direction and then in the other. The kicks of the current on the galvanometer will be proportional to V. (5.) Place *abcd* and the coil C vertical by turning through 90°, as observed by the index *n*. (6.) Make observation of the value of H by alternate semi-rotations as before.

Example.—Mean value of $d_1 = 112{\cdot}7$, mean value of $d_2 = 43$, hence the dip $= \frac{112{\cdot}7}{43} = 2{\cdot}621$, dip $= 69°\ 7'$.

152. *Rowland's application of the Earth Inductor.*[1]—The equation

$$Q = \frac{2\mathrm{H}n\pi a^2}{\mathrm{R}} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

suggests an exceedingly useful application of the earth inductor. By its means we can ascertain the strength in absolute measure of a magnetic field. For if the coil be connected with a ballistic galvanometer, then

$$\frac{2\mathrm{H}n\pi a^2}{\mathrm{R}} = k \sin\frac{\theta}{2} \quad . \quad . \quad . \quad . \quad (2)$$

where R is the total resistance in the circuit, θ the deflec-

[1] See *Phil. Mag.*, vol. xlvi., 1873.

tion produced by the rotation of the coil through 180°, and k the *ballistic constant* of the galvanometer. Suppose now that it is required to measure the strength of a particular field H′, between the poles of a magnet for instance. It would only be necessary to include in the circuit of the earth inductor a second or test inductor, placed between the poles, which latter in practice would be much smaller than the earth inductor. This second inductor would be placed in the proper position with regard to the field H′ and turned through 180°. We should similarly have

$$\frac{2H'n'\pi a'^2}{R} = k \sin\frac{\theta'}{2} \quad . \quad . \quad . \quad . \quad (3)$$

Where $n'\pi a'^2$ is the area of induction of the test coil and θ' the deflection. From (2) and (3), supposing the test inductor to have been in circuit when θ was observed,

$$\frac{Hn\pi a^2}{H'n'\pi a'^2} = \frac{\sin\frac{\theta}{2}}{\sin\frac{\theta'}{2}} = \frac{d}{d'} \text{ nearly,}$$

where d and d' are the observed deflections in scale divisions of the mirror galvanometer used, or

$$H' = H\frac{n\pi a^2}{n'\pi a'^2} \cdot \frac{d'}{d} \quad . \quad . \quad . \quad . \quad (4)$$

This equation allows us to compare directly the intensity of the field H′ with the horizontal intensity of the earth's magnetism.

153. *Use of Long Helix* (Thomson).—In the above article use has been made of the uniform field of the earth, but we have already pointed out, p. 329, that in the middle of a long helix conveying a current the field may be regarded as uniform, and having a value

$$\frac{4\pi nC}{l} = 4\pi NC,$$

where N is the number of turns per centimètre of length. Hence in the previous equation (4) we may substitute the value $4\pi NC$ for H. In this expression we assume that the value of C is known. This method is often more convenient than the last one, and it has further the advantage that instead of a movable test coil we may use a *fixed one wound outside the middle of the long helix.* For when the circuit of the helix is broken an induced current is generated due to $4\pi NC \times$ area of induction of the fixed coil, the lines of force being suddenly withdrawn from its circuit. In this latter case

$$H' = 4\pi NC \frac{n''\pi a''^2}{2n'\pi a'^2} \cdot \frac{d'}{d''} \quad . \quad . \quad . \quad . \quad (5)$$

where $n''\pi a''^2$ is the area of induction of the fixed coil and d'' the deflection produced when the circuit is broken. It must be noticed that owing to the lines of force being only once withdrawn from the helix, the right-hand member of the last equation has been divided by 2.

154. *The Trapeze Earth Inductor.*—In the absence of a proper earth inductor a simple form may be used. This consists of a wooden rod about 2 mètres long, having holes at each end. One end of a piece of thin copper wire, 6 mètres long, is passed through each hole, so that 2 mètres of the wire lie along the rod and 3 mètres are used at each end for the suspension of the rod, trapeze fashion, from the ceiling. The upper ends of the wire are connected with leading wires fixed so that they may not be disturbed by the swinging of the trapeze. Four uprights are fixed to the floor, two on either side of the trapeze, for the purpose of limiting its swing. The uprights on the one side are removed from the position of verticality exactly as far as those on the other. When in use the bar is lifted up against two of the stops and thrown towards the other stops, where it should be caught and held. During the movement the conductor fixed along the rod crosses lhV

lines of force, where l is the length of the horizontal wire, h the distance between the stops, and V the vertical component of the earth's magnetism. The value of the horizontal component does not enter into the question, for although the vertical wires cut the horizontal lines of force, they do so in such a manner that the currents generated on the one side of the vertical position are equal and opposite to those generated on the other, and hence destroy each other. It follows therefore that the rod may be placed in *any* azimuth. The equation for this inductor is

$$H' = V \frac{lh}{2n'\pi a'^2} \cdot \frac{d'}{d'''} \quad . \quad . \quad . \quad . \quad (6)$$

d''' being the deflection produced by the movement of the trapeze.

Lesson LXXI.—Comparison of Magnetic Fields.

155. *Exercise.*—To find the value in absolute measure of the field between the poles of an electro-magnet by the three above methods.

Apparatus.—(1.) An *Earth-Inductor.*—The one in use at Owens College consists of a large hoop, of circumference 206·2 cm., wound with a number of separate coils. During the winding of the coils the circumference of each layer was measured after the manner of Rayleigh and Schuster[1] by a steel tape measure. The mean of the measurements gives the *outside* of the mean circumference. To find the *axial* length of the mean circumference correction must be made for the thickness of the wire. From the corrected circumference the mean radius is obtained, which with the number of windings will give the necessary information regarding the inductor. (2.) *A helix for uniform field.*—This may consist of a stout glass tube about 50 cm. long and 4 cm. diameter. It is provided at its ends with

[1] See *Phil. Trans.*, 1882, part. ii. p. 672.

two large corks, so as to form a very large reel. Upon this is wound two layers of No. 20 covered wire, there being altogether about 500 turns. This will give about 10 turns per cm. Around its middle should be wound about 50 turns of No. 28 wire, having an area of induction of about 800 square cm. (3.) *Movable testing-coil.*—To make this an ebonite ring of 37 mm. internal diameter should be turned, and also a small reel of 30 mm. outside diameter, which will easily pass within the ring. Wind the reel with about 50 turns of No. 28 wire, and fasten two brass pivots to the reel. Make holes in the ring for the pivots, in order that the reel may be rotated within the ring. Fasten strings to the reel for the purpose of turning it through 180°, the semi-revolutions being determined by suitably placed stops. (4.) *The trapeze inductor.*—This will not require further explanation. (5.) *Electro-magnet.*—This should be provided with pole pieces. (6.) *Other apparatus.*—Tangent galvanometer of known constant, apparatus for ascertaining the dip and horizontal force, keys and batteries, low resistance galvanometer, low resistance coils.

Method.—On account of the nature of the operations of this lesson three students should work together. The necessary arrangements are as follows:—(1.) Place the electro-magnet some distance from the galvanometer, fix between its poles the test coil in the position previously explained. Arrange a circuit of Grove's cells with plug key for charging the magnet. (2.) Fix up the trapeze and coil earth inductors. (3.) Connect all the inductors in series with the galvanometer. (4.) Connect the long helix with a battery plug key, tangent galvanometer, and low resistance box. (5.) Test the inductive kicks and modify the conditions, such as the strength of helix current, the number of the coils of the earth inductor used, etc., so that the deflections are all readable on the galvanometer scale without alteration of the total resistance in the circuit of the latter.

The necessary observations are—(1.) The movable test coil should be turned several times through 180°, and the deflections observed. Let the mean deflection be d'. (2.) Repeated observations should be made with the coil earth inductor in order to obtain d. (3.) Break the circuit of the long helix in order to obtain d''. Observe also the deflection on the tangent galvanometer and calculate C. Remember that C must be in C. G. S. units. (4.) Obtain d''' by the trapeze. (5.) Ascertain the magnetic dip (δ) in the region of the trapeze, and also the value of H. Calculate V from the equation $V = H \tan \delta$. (6.) Find H in the region of the coil earth inductor. (7.) (*a*) Calculate from the known number of turns and radii the areas of induction of the coil inductors, and from V, l and h the area of induction of the trapeze. (*b*.) Calculate the value of the strength of the field in the helix from the expression $4\pi NC$. (*c*.) Find H' from equations (4), (5) and (6).

All these experiments, if properly conducted, will be found to give very nearly the same strength for the magnetic field.

156. *Application of Induced Currents to the Study of Magnetic Distribution.*—In Chap. II. several methods for the study of magnetic distribution have been given, and it was remarked that these methods are inexact. They have given place, therefore, to the method of induction currents. A thin coil is made to surround closely the axis of the magnetised bar under experiment. The coil, which is connected with a ballistic galvanometer, is moved by successive short steps along the bar. The inductive kicks produced by each movement must be noted. The exact meaning of these will be explained in the following lesson.

LESSON LXXII.—Study of Magnetic Distribution.

157. *Exercise.*—To apply the method of induced currents

to the case studied by Rowland,[1] of a long soft iron bar, magnetised by a magnetising helix, placed at the end of the bar.

Apparatus.—Fig. 176 shows a convenient arrangement. A bar, *cc′*, graduated into centimètres, is supported by passing it through a hole in each end of the brass uprights fixed to heavy metal bases. The bar is of the softest Lowmoor iron, 60 cm. long, and 6 mm. diameter, that has been well annealed by having been heated to redness, and then slowly cooled in a gas combustion furnace, such as is used for organic chemical analysis. At the ends (shown unshaded) are two short brass bars of the same diameter as the iron rod. These are for the purpose of allowing the

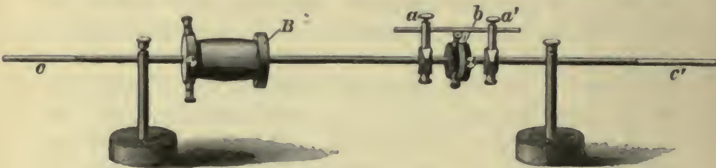


Fig. 176.—Apparatus for Studying Distribution of Magnetism.

whole length of the iron to be brought under experiment without using the ends as places of support. For magnetising purposes there is a bobbin B, wound with 400 turns of No. 20 wire. This may be clamped at any part of the bar. A smaller reel of ebonite *b*, wound with fine wire, is employed as the *induction coil.* This is allowed to play between two brass guides; the range of the play may be changed by sliding the guides more or less apart and then clamping them by means of the appropriate screws. When *b* is required to be fixed it may be secured by a clamp screw.

[1] See "Studies of Magnetic Distribution," by H. A. Rowland, *Phil. Mag.*, 1875, vol. l. pp. 257 and 348. The method of applying induced currents, though frequently called Rowland's method, was originally used by Van Rees in 1847.

The galvanometer should be an ordinary low resistance galvanometer (p. 146). A constant battery provided with a commutator and a box of coils will be necessary.

Method.—The apparatus being fixed some distance away from the galvanometer, and the iron bar having its length at right angles to the magnetic meridian, tests should be made to see whether the iron bar is free from residual magnetism. (It is supposed that the current is not passing through B.) To do this move the induction coil along the bar. If a deflection is produced, the bar must be demagnetised by hammering its ends. It will be difficult to remove the whole of the residual magnetism in this manner. A more effective process is as follows:—Move B to different parts of the bar, and send currents first in one direction and then in the other by means of the commutator, which currents should be made to decrease gradually in intensity by the gradual introduction of resistance into the battery circuit.

The helix B is now placed at one end of the bar, which should (when the circuit of B is complete) be normally magnetised. Commencing at either end of the bar, the induction coil is rapidly moved between the guides, and the inductive kicks observed for the several centimètre lengths. A curve should then be drawn, with the distances from the end of the bar as abscissæ, and the deflections as ordinates. This curve will show the required distribution.

A conception of the meaning of these measurements may be obtained by considering that *lines of magnetic induction* are entering the magnetised end of the bar. After passing down the bar a certain distance some of these lines will pass into the air. There will thus be two paths at every section of the rod open to the lines, either to pass farther down the rod or to pass into the air. Suppose now that the induction coil be fixed at any position x from the end of the bar, and that the current be suddenly stopped; a

very large deflection will be produced, proportional to the flow along the bar at the point x. If the induction coil be placed at a distance $x + dx$, and the experiment be repeated, the difference between the two deflections will be proportional to the number of lines that have escaped into the air through the distance dx. These air lines, for a distance equal to the depth of the coil, may be supposed to be perpendicular to the bar, hence when the coil is moved from the position x to $x + dx$, it will at the end of its path embrace fewer lines of force of the magnet, by a quantity equal to those that have escaped into the air, between the points x and $x + dx$. The current, therefore, produced by moving the induction coil from x to $x + dx$, will be the same as that obtained by taking the difference of the currents at the same two points in the previous demagnetisation experiments. Further, if instead of demagnetising the bar the coil were moved to a very distant position by sliding it off the bar, first from x and then from $x + dx$, the difference of the inductive kicks should be the same as before.

In making the demagnetisation experiment no good agreement will be practically found between it and the other methods. This will be owing to the fact that on the current being stopped in R the bar does not fall to a zero of magnetisation. To get rid of the effect of the residual magnetism, instead of breaking the current it should be reversed. This will give a deflection double of the required amount, but which will be free from this source of error.

(C.) *Measurements of Induction Coefficients.*

158. *Definition of Coefficient of Self-Induction.*—When a current is started in a coil a small time τ must elapse before it establishes a constant condition. It must not, however, be imagined that during this time τ there is a

uniform rate of increase of the current. We may, nevertheless, suppose the time τ to be divided into a great number n of equal parts, any one of which is so small that the rate of increase of the current is virtually constant throughout this interval. Now let r_1 represent the *rate of increase* of the current during the first of these intervals. Then we know that this will cause an electromotive force e_1, and that

$$e_1 = \mathrm{L}r_1.$$

Here L may be defined as the coefficient of self-induction. It follows from this expression that

$$\frac{e_1\frac{\tau}{n}}{\mathrm{R}} = \frac{\mathrm{L}r_1\frac{\tau}{n}}{\mathrm{R}}$$

denotes the quantity of induced electricity that passes, while $r_1\frac{\tau}{n}$ denotes the whole current generated during this small interval, R being the resistance of the circuit. Summing up for n such intervals, we have

$$\Sigma\left(\frac{e\tau}{n\mathrm{R}}\right) = \frac{\tau}{\mathrm{R}} \cdot \frac{\Sigma e}{n} = \frac{\mathrm{L}\tau}{\mathrm{R}} \cdot \frac{\Sigma r}{n}.$$

Here $\frac{\Sigma e}{n}$ may be regarded as the *average induced electromotive force*, and $\frac{\Sigma r}{n}$ as the average rate of increase of the current, while $\frac{\Sigma r}{n}\tau$ will denote the whole current established. If the current established be unity, then $\frac{\mathrm{L}}{\mathrm{R}}$ will be the whole quantity of induced electricity, while if the resistance be likewise unity this quantity will be denoted by L. The same reasoning will apply to the case when a current is broken. L *may therefore be defined as the quantity of induced or extra electricity which is made to circulate in the coil, having unit resistance in its circuit, by making or breaking unit current.*

LESSON LXXIII.—Determination of Coefficient of Self-Induction.[1]

159. *Exercise.*—To find L for a large coil of 60 cm. in diameter, which has been wound with all the necessary precautions, and whose constants are known.

Apparatus.—Ballistic galvanometer, Wheatstone's bridge apparatus, extra resistance coils.

Practice of the Method.—Let the coil whose coefficient is required be placed in the arm CD (Fig. 177) of the Wheatstone's bridge. When a balance is obtained by first making the battery circuit and then the galvanometer

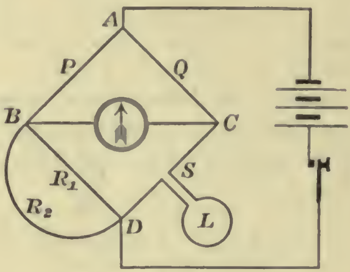


Fig. 177.

circuit, there is no current in the latter, provided that the usual relation between the arms is satisfied. But if the galvanometer circuit be first closed and then the battery circuit be closed or opened, the galvanometer needle will receive an impulse which will depend on L. If the current that becomes permanently established in the coil is x, then a *quantity of electricity* will be developed, of which a portion passes through the galvanometer.

To obtain a quantity with which to compare the im-

[1] This is Maxwell's method that has been modified by Rayleigh. See *Pro. Roy. Soc.*, vol. xxxii. (1881), p. 116, and *Phil. Trans.* (1882), part 2.

pulse due to L, the balance is made incorrect by altering the resistance S so as to disturb the balance for permanent currents. Let S be increased to $S + \delta S$.

Maxwell has shown that under these circumstances we may assume, when δS is small, that

$$L = \frac{T_1 \delta S}{\pi}\left(1 + \frac{\lambda}{2}\right)\frac{2 \sin \frac{1}{2}\alpha}{\tan \theta},$$

in which α is the kick produced by induction, and θ the steady deflection produced when the balance is disturbed.

In practice it is found better that (1) the current instead of being broken or made is *reversed*. This doubles the value of α, and hence the reading is more accurate. Moreover the battery is kept more constant than it would be if sometimes left in open circuit. (2.) The variation of δS should be secured by a multiple arc arrangement. When P is made equal to Q the disturbance of the balance may be made, if we choose, in the arm BD, by using two resistance boxes R_1 and R_2, as shown in the figure. (3.) The observations of α and θ should be alternated several times to eliminate any error due to variations of the battery.

Example (Rayleigh).—The ballistic galvanometer had a resistance of 80 ohms. The scale was divided into millimètres and placed at a distance of 218 cm. from the galvanometer mirror. Correction for damping = 1·0142. $P = Q = 10$. The coil whose self-induction was required was nearly 24 ohms. R_1 was made equal to 24 ohms, and between B and D a resistance R_2 of 753 ohms was placed in multiple arc with R_1, this gave a permanent balance. Hence the combined resistance between B and D is

$$\frac{1}{R} = \frac{1}{24} + \frac{1}{753} = \frac{1}{23 \cdot 259}$$

or

$$R = 23 \cdot 259.$$

The balance was then disturbed by substituting 853 ohms for the 753 ohms; hence

$$R + \delta R = 23{\cdot}343,$$

and

$$\delta R = \delta S = {\cdot}0845$$

$$\frac{2 \sin \frac{1}{2}\alpha}{\tan \theta}\text{, after all corrections,} = 1{\cdot}531$$

$$T = 11{\cdot}693 \text{ seconds.}$$

A small correction was made for the ratio of x' to x, that is to say, of the new current x' passing through CD when the balance is disturbed, and x the undisturbed value of the same, by supposing that

$$\frac{x'}{x} = \frac{10 + 23{\cdot}259}{10 + 23{\cdot}343}.$$

The above resistances are in BA units. To correct to true ohms the multiple ·987 was adopted.

δS	log $\cdot 08453 \times 10^9$	$= 7{\cdot}92701$
Correction to absolute units	log $\cdot 987$	$= \bar{1}{\cdot}99432$
$2 \sin \frac{1}{2}\alpha : \tan \theta$ . . .	log $1{\cdot}531$	$= \cdot 18498$
Correction for finite arcs .	log $\cdot 99925$	$= \bar{1}{\cdot}99967$
Correction for damping .	log $1{\cdot}0142$	$= \cdot 00612$
Time of vibration . .	log $11{\cdot}693$	$= 1{\cdot}06793$
Ratio of currents . .	log x'/x	$= \bar{1}{\cdot}99886$
		$9{\cdot}17889$
	log 2π . . .	$\cdot 79818$
	log L . . .	$8{\cdot}38071$

$$\text{or } L = 2{\cdot}4028 \times 10^8 \text{ centimètres.}$$

This value was found to agree with that obtained by direct calculation from Maxwell's formula.

Lesson LXXIV.—Comparison of Two Coefficients of Self-Induction.

160. *Exercise.*—To compare the self-induction of the standard coil of the last lesson with a second coil, such as that of the induction balance of Hughes.

Apparatus.—Instead of the ballistic galvanometer of the last lesson we shall require an ordinary galvanometer of high sensibility.

Practice of the Method.—Referring to Fig. 177, the coil of coefficient L′, which is to be compared with the standard coil, is placed in the arm BD.

The condition for no permanent current is

$$RQ = SP \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and Maxwell has shown (vol. ii. art. 757) that the condition for no transient current is

$$\frac{L}{L'} = \frac{Q}{P} = \frac{S}{R} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Suppose that S and Q are fixed, we have to find values of P and R which will satisfy both (1) and (2). To satisfy (1) any number of values may be found, but there is only one value of P and one value of R which will satisfy both.

The best procedure in practice will therefore be as follows:—(1.) With P and R large note the direction of induction kick. (2.) With P and R sufficiently small again note the direction of the kick, which ought now to be in the opposite direction. (3.) Having thus obtained limits between which the proper values lie, P and R are adjusted until the balance is obtained.

Example.—$S = 1000 +$ coil (of self-induction L) $= 1021{\cdot}4$. $Q = 1000$. These were kept fixed. P was put $= 1000$, and R adjusted until the balance was obtained for permanent currents. The induction kick was $+ 300$. P was now made 10, and R again adjusted; the induction kick was now $- 50$. A change of 990 in P thus caused a change of 350 in the deflection; but as we require a change of 300 to give no inductive kick, the true value of P should be about $10 + \frac{990}{7} = 150$ nearly. Several such adjustments showed that the true balancing value of P was 144 ohms.

R had then the value of 140 + coil of self-induction L′ = 147·35. Then

$$\frac{L}{L'}=\frac{1000}{144}=\frac{1}{\cdot 144}.$$

161. *Definition of Coefficient of Mutual Induction.*—Suppose that we have two adjacent coils, A and B, and that a current C be started in A, whilst the ends of B are connected. There will be a quantity of induced electricity passing in B during the establishment of the current in A equal to

$$\frac{MC}{R},$$

where M is the coefficient of mutual induction of A on B, or of B on A, and R is the resistance of the circuit of B.

Lesson LXXV.—Comparison of Self and Mutual Induction Coefficients.[1]

162. *Exercise.*—To find M for two coils such as those of an induction balance, the L of one of which has been previously determined.

Apparatus as in the last lesson.

Practice of the Method.—In the S arm (Fig. 178) of

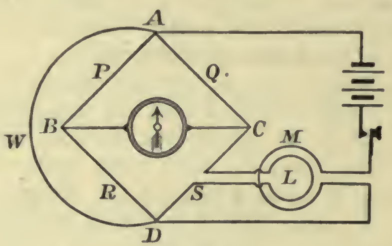


Fig. 178.

the bridge is connected the coil L, whose self-induction is

[1] Maxwell, vol. ii. art. 756.

known, whilst the adjacent coil M is connected with the points A and D. Neglecting at present the resistance W connecting A and D, we have, according to Maxwell,

$$L = -\left(1 + \frac{S}{R}\right)M \quad . \quad . \quad . \quad . \quad (1)$$

In practice we must have recourse to a double adjustment to obtain a balance both for permanent and transient currents, as in the previous example. But all this trouble will be avoided by using an extra branch W, whose resistance is adjusted until there is no transient current. The formula in this case is—

$$L = -\left\{1 + \frac{S}{R} + \frac{S+Q}{W}\right\}M \quad . \quad . \quad . \quad . \quad (2)$$

163. *Comparison of two Coefficients of Mutual Induction.*[1] —Suppose that coils A and B, whose coefficient of mutual induction is M, are to be compared with two coils A′ and B′ of coefficient M′. Place A and A′ in series with a battery. Place resistance boxes in the circuit of B and B′ respectively, and let the induced currents produced in B and B′ be sent in opposite directions through a galvanometer. The arrangement will be best understood by referring to Fig. 21, Appendix D, and substituting the coils L and L′ for the two cells. By adjusting the resistances in the circuit of B and B′ to R and R′, so that there is no current, then

$$\frac{M}{M'} = \frac{R}{R'}.$$

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[1] Maxwell, vol. ii. art. 755.

CHAPTER VIII.

THE CONDENSER.

164. A CONDENSER for the purpose of this chapter may be considered as consisting of a number of sheets of an insulating material, arranged alternately with sheets of a conducting material, the first, third, fifth, etc., sheets of the latter being connected together so as to form one coating or armature of the condenser, and the second, fourth, sixth, etc., sheets being also connected together to form the other coating or armature. When the armatures are connected with the terminals of a battery the condenser becomes *charged*, but may be *discharged* by short-circuiting its armatures, so as to bring them to the same potential.

If E be the E. M. F. of the battery, and Q the quantity of electricity the condenser retains, then

$$Q = EF \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where F is the **capacity** of the condenser.[1] In a condenser the one armature becomes charged with positive and the other with negative electricity, but what we regard in estimating Q is the positive current, which may be supposed to flow from the positive to the negative armature when a connection is established between them. In like

[1] C., E. and R. are the well recognised abbreviations for current, E. M. F. and resistance; but for capacity there is no special symbol. We shall, as a rule, follow Kempe in using F (the first letter of Farad).

manner, in a voltaic circuit, we always speak of the positive current. The relation (1) will enable us to define the **unit of capacity** as that of a condenser which, when charged with one *coulomb* of electricity, has a difference of potentials equivalent to an E. M. F. of one *volt* between its poles. The unit so defined is the **Farad.** Its theoretical value is, therefore,

$$\text{Farad} = \frac{\text{Coulomb}}{\text{Volt}} = \frac{10^{-1}}{10^{8}} = 10^{-9} \text{ C. G. S. units of capacity.}$$

The farad being far too large for practical purposes, it is customary to use instead its *millionth* part, or the **Microfarad.** Standard condensers having one-third this value, and approximately equal to the capacity of one knot length of cable, are used for cable testing, and are likewise very useful for laboratory purposes. These condensers, made of alternate sheets of tinfoil and mica coated with shellac, are mounted in brass cases with an ebonite top, and provided with binding screws in connection with the armatures of the condenser. By the insertion of a plug the armatures may be conveniently short-circuited. By the substitution of paper soaked in paraffin, much cheaper condensers may be made. It is a useful exercise for the student to make such a condenser, according to the details given in Appendix F.

Lesson LXXVI.—Work with the Condenser.

165. *Exercise.*—To compare two condensers by the methods of (1.) direct deflection; (2.) equal deflection. To prove the law of capacities by combining the condensers in different ways, and (3.) to test their rates of leakage and examine the influence of residual charge, also to estimate the time required for charging.

Apparatus.—(1.) A high resistance galvanometer, such as that of Lesson XXIII. ; (2.) a discharge key. One of the

most convenient forms of key is that of Kempe, shown in Fig. 179. Here an ebonite base supports three pillars, one of which bears a hinged lever *l*. This plays between the ends of two platinum stops, *s* and *s′*, that are supported by the other two pillars. When free to move, the lever *l* by means of the spring placed beneath its hinge will rise up and press against the upper stop. But when pressed down by the ebonite insulating stud C it will rest against the bottom stop, where it is held by the right-hand ebonite

Fig. 179.—Kempe's Discharge Key.

"trigger," on account of the action of a spring seen beneath the stud I. The lever is now in the **Charge** position, but when I is depressed the lever immediately springs up, but cannot reach the upper stop owing to its end being caught by a second or left-hand trigger attached to D, which is rather higher than the other trigger. The position that the lever is now in is called the **Insulate** position, for the reason that the lever touches neither the bottom nor the top stops, but rests between both. Should

now D be depressed, the lever will spring up against the upper stop and be in the **Discharge** position. Further, by pressing D without having previously touched I the lever will spring at once from the charge to the discharge position. (3.) A box of coils and a battery of from 1 to 12 Daniell's cells.

Theory of Method I.—If two condensers of capacity F_1 and F_2 be charged by means of the same E. M. F., then the quantities of electricity Q_1 and Q_2 which they hold will be

$$Q_1 = EF_1$$

and

$$Q_2 = EF_2,$$

or

$$\frac{Q_1}{Q_2} = \frac{F_1}{F_2}.$$

By discharging Q_1 and Q_2 through a galvanometer the amplitudes d_1 and d_2 of the first kick, as we have pointed out in the previous chapter, will be proportional to Q_1 and Q_2, hence

$$\frac{F_1}{F_2} = \frac{d_1}{d_2}.$$

This constitutes the simplest method of comparing two condensers.

Practice of Method I.—Fig. 180 shows the connections, but at present the shunt circuits S and S_1 will not be required. The condenser C, which is represented by two thick strokes, will become charged when the lever *hr* of the discharge key is depressed against the lower stop *c*, for now the left-hand pole of the condenser will receive a positive charge from the battery by the connection QOB, and the right-hand pole will receive a negative charge by the connection *hc*NB. The key being at this *charge* position, the galvanometer is set to zero, and when the observer is ready to take the reading, the *discharge* should be caused by depressing at once D (Fig. 179) without previously touching I.

When this is done the poles of the condenser are placed in metallic communication by the circuit QPGH*dh*, and the sudden rush of electricity through the galvanometer causes the kick, which has to be noted. Whilst the galvanometer is returning to rest the condenser should be short-circuited

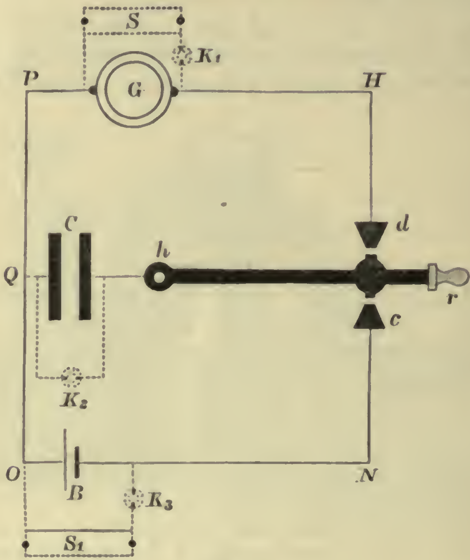

Fig. 180.

by the key K_2 in order to get rid of any *residual* charge that may remain Before again depressing the lever the short-circuit key K_2 should be opened. After the observation of the discharge has been repeated several times the condenser C is replaced by the second one, and the processes are repeated

Example.—

* Using a Clark's cell (1) with standard ⅓ microfarad (F_1), we got kicks 78, 79, 79, and 78, mean 78·5 ; (2) with paraffin paper condenser

(F_2), 169, 168, 169, 171, mean 169·25; (3) a third condenser (F_3) gave mean 27, hence

$$F_2 = F_1 \frac{169 \cdot 25}{78 \cdot 5} = \cdot 7186 \text{ microfarad},$$

and

$$F_3 = F_1 \frac{27}{78 \cdot 5} = \cdot 1146 \text{ microfarad}.$$

166. *Theory and Practice of Method II.*—When a condenser is much greater in capacity than the one with which it is to be compared, the shunt circuit S (Fig. 180) should be used with the galvanometer for the larger condenser, and the observed deflection multiplied by $\frac{G+S}{S}$, where G is the resistance of the galvanometer and S that of the shunt.[1] If the shunt be adjusted until the deflection is the same as with the smaller condenser without the shunt, the expression $\frac{G+S}{S}$ gives immediately the ratio of the two condensers. It should be noticed that although the fact of a shunt being inserted across the galvanometer circuit lowers the resistance external to the condenser, yet owing to the extremely high resistance of the latter this will not produce any perceptible difference in the *main* discharge current. In this respect the condenser resembles a battery of very high internal resistance.

Example.—

With F_3 we adjusted the sensibility of galvanometer so as to obtain deflection of 51. With F_1 we obtained with $S = 3120$ the same deflection. Hence, since G was 5612,

$$F_3 = \tfrac{1}{3} \cdot \frac{3120}{3120 + 5612} = \cdot 119.$$

---

[1] The student must be warned that this expression, although quite true for steady currents, requires a correction for those of short duration, due to the inductive action of the moving magnet. This acts like an extra resistance in the circuit of the latter. Hence for accurate work we must substitute $\frac{G+x+S}{S}$ for $\frac{G+S}{S}$, where x is a quantity to be determined by the use of condensers of known capacity. See Latimer Clark, in the Journal of *Telegraph Engineers*, vol. ii. p. 16.

167. *Laws for Combination of Condensers.*—Law I.—When a number of condensers are connected together in *multiple arc* (see Fig. 181), the capacity of the arrangement is equal to the *sum of the several capacities.* This law does

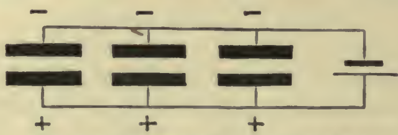

Fig. 181.—CONDENSERS IN MULTIPLE ARC.

not require any proof. Law II.—When the condensers are joined by *cascade* or *series* (see Fig. 182), to find the capacity of the arrangement we follow the same method as would be used in finding the resistance of a number of conductors in *multiple arc* (see Appendix A). In other words, the joint capacity would be equal to *the reciprocal of the sum of the reciprocals of the several capacities.* In order to prove this law let there be three condensers in series, as in Fig. 182. Let V represent the potential of the left-hand plate of the left-hand or first condenser, and let V_1

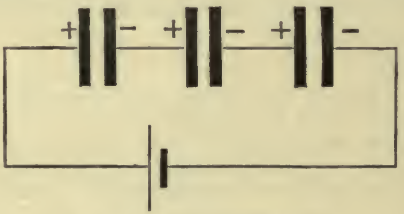

Fig. 182.—CONDENSERS IN SERIES.

be that of its right-hand plate. It is clear that V_1 will likewise be the potential of the left-hand plate of the middle or second condenser, this being metallically connected with the right-hand plate of the first. Again let V_2 denote the potential of the right-hand plate of the middle

condenser, and also of the left-hand plate of the third condenser. Finally, suppose that the right-hand plate of the third condenser is earth-connected and therefore at zero potential. Then if F_1, F_2 and F_3 denote the capacities of the first, second, and third condensers, it is evident that $F_1(V - V_1)$, $F_2(V_1 - V_2)$, and F_3V_2 will be the quantities of electricity separated at the various condensers. Again, since the *minus* charge of the first condenser must equal the *plus* charge of the second, we have $F_1(V - V_1) = F_2(V_1 - V_2)$. Likewise, since the *minus* charge of the second condenser must equal the *plus* charge of the third, we have $F_2(V_1 - V_2) = F_3V_2$. Hence

$$\frac{V - V_1}{\frac{1}{F_1}} = \frac{V_1 - V_2}{\frac{1}{F_2}} = \frac{V_2}{\frac{1}{F_3}} = \frac{V}{\frac{1}{F_1} + \frac{1}{F_2} + \frac{1}{F_3}} = \frac{V}{\frac{1}{F}}$$

suppose. Now $\frac{V}{\frac{1}{F}}$ or VF will be the charge produced by difference of potential $= V$ in a condenser of capacity F.

Again, it is evident that when the above series of condensers is discharged, $F_1(V - V_1)$ will denote the quantity of positive electricity which circulates through the galvanometer, and we have just shown that this is equal to VF when $\frac{1}{F} = \frac{1}{F_1} + \frac{1}{F_2} + \frac{1}{F_3}$. Hence the law. These laws should be experimentally verified.

Example.—

The capacity of $F_1 + F_3$ in multiple *arc* was measured and found to be ·4472, but $F_1 = {}$·3333 and $F_3 = {}$·1146 (see above example). Hence $F_1 + F_3 = {}$·4479, proving Law I. Again, F_1 and F_3 in cascade gave ·0872, now

$$\frac{1}{\frac{1}{F_1} + \frac{1}{F_3}} = \frac{1}{3 + \frac{1}{\cdot 1146}} = \cdot 0852.$$

Thus law II. is experimentally verified.

168. *Leakage of a Condenser.*—A charged condenser left to itself undergoes a slow discharge, chiefly through the insul-

ating sheets. To study the rate of the discharge it is only necessary after charging the condenser to press the button I of the discharge key, and after a given time to press the button D. The percentage of leakage per minute will depend upon the insulation resistance of the condenser.

Example.—Two Daniell's cells used for charging.

Kind of Condenser.	Immediate Discharge.	Discharge after ten minutes' Insulation.	Percentage Leakage per minute.
Mica . . .	166	148	1·08
Swiss Composition	281	260	·71
Paraffin (1 year old)	60	29	5·01
Paraffin (4 year old)	385	65	8·31

169. *Absorption and Residual Charge.*—The student, after he has removed the battery and taken a discharge from the condenser, will find that he will be able to obtain a second or residual discharge. This is due to the peculiarity of solid and liquid insulators[1] known as *electric absorption.* When a condenser is connected with a battery it does not receive its full charge immediately, but continues for some time to *absorb* or suck in the charge. Again, when the condenser is discharged it does not immediately give up this portion of the charge, but this will gradually ooze out and be available for producing a residual discharge. These phenomena should be observed as shown in the following example :—

Example.—(1.) Study of the influence of the length of time necessary to charge the condenser by twelve Daniell's cells.

TIME OF CHARGING.

	Immediate.	15 sec.	30 sec.	1 min.	5 min.
Mica condenser .	301	302	302	302	302
Paraffin condenser	329	330	332	334	336

[1] Calcite appears to be a remarkable exception, according to the experiments of Rowland and Nichols. See *Pro. Phys. Soc.*, vol. iv. p. 215.

(2.) Study of residual charge after the condensers had been charged by two cells for five minutes.

	Mica Condenser.	Paraffin Condenser.
First discharge	169	212
Residual after 1 minute insulation	3·5	24
,, ,, ,, more	1·0	16
,, ,, ,, ,,	0	13
,, ,, ,, ,,	...	10
,, ,, ,, ,,	...	9

From these experiments we see that the paraffin paper condenser exhibits the phenomenon of absorption in a very marked manner.

Lesson LXXVII.—Determination of the Absolute Capacity of a Condenser.

170. *Apparatus.*—As in Lesson LXXVI., with the addition of a means of measuring time.

Theory of the Method.—We have proved (Arts. 138-142) that when a quantity Q of electricity is discharged through a ballistic mirror galvanometer

$$Q=\frac{KT_1\left(1+\frac{\lambda}{2}\right)s}{2\pi L} \quad . \quad . \quad . \quad . \quad (1)$$

where $K=H\Gamma$ is the working constant of the galvanometer, T_1 is the corrected time of vibration of the needle, λ the logarithmic decrement, s the deflection in millimètre scale divisions, and L the distance of the scale from the mirror, also in millimètres. If the deflection had been produced by the discharge of a condenser through the galvanometer, then

$$Q=FE \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where F is the capacity of the condenser, and E the

E. M. F. of the battery used to charge it. Hence from (1) and (2)—

$$F = \frac{KT_1\left(1+\frac{\lambda}{2}\right)s}{2\pi LE} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

To apply this equation in practice we use the same battery that has been employed in charging the condenser to produce a constant deflection s_1 when connected with the galvanometer in series. If the total resistance in the circuit be now R, then, by the theory of the mirror galvanometer (Art. 34),

$$\frac{E}{R} = \frac{Ks_1}{2L} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Eliminating $\frac{E}{K}$ between (3) and (4) we obtain—

$$F = \frac{T_1\left(1+\frac{\lambda}{2}\right)s}{\pi Rs_1} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Practice of the Method.—We have three distinct operations to perform :—

(1.) *Determination of s.*—The connections and the operations are as in Method I., Lesson LXXVI.

(2.) *Determination of T and* λ.—The former must be done by the method of passages, and the latter by the method of Art. 142.

(3.) *Determination of R and* s_1.—The condenser must now be disconnected and a resistance box inserted in the battery circuit. Owing to the great sensitiveness of the mirror galvanometer it will probably be impossible to obtain a readable deflection with an ordinary resistance box, hence the galvanometer should be provided with the shunt S. Alter S and add resistances until s_1 is readable. Since a shunt is used, the resistance, consisting of that unplugged ($=R_1$), together with the battery resistance and the combined resistance of the galvanometer and shunt,

must be multiplied by $\frac{G+S}{S}$ in order to obtain R (see a similar case, p. 193).

Example.—

$$s=154,\ T_1=7\cdot667 \text{ seconds},\ \lambda=\cdot01.$$
$$s_1=95,\ R_1=11000,\ S=4,\ G=4292,\ B=4.$$
$$R=\left(11000+4+\frac{4292\times4}{4292+4}\right)\frac{4292+4}{4} \text{ in ohms}=11008\times1074.$$
$$F=\frac{7\cdot667\times154\times1\cdot005}{3\cdot1416\times11008\times1074\times95}=\frac{\cdot3363}{10^6} \text{ farad, or } =\cdot3363 \text{ microfarad.}$$

Lesson LXXVIII.—Comparison of Electromotive Forces, and Determination of Battery Resistance by the Condenser.

171. *Apparatus.*—As in Lesson LXXVI.

Theory of the Comparison of E. M. F.—Method of Law.—If the same condenser be charged first by means of a source of E. M. F. = E_1, and discharged through a galvanometer producing deflection d_1, and then by means of a source of E. M. F. = E_2, producing on discharge a deflection d_2, then

$$\frac{E_1}{E_2}=\frac{d_1}{d_2} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Should E_1 be very much greater than E_2, it will be necessary to shunt the galvanometer, when formula (1) becomes—

$$\frac{E_1}{E_2}=\frac{G+S}{S}\cdot\frac{d_1}{d_2} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Practice of Law's Method.—It is only necessary to use the arrangement of Fig. 180, and take the discharge first with one battery and then with the other. The condenser should be thoroughly freed from its residual charge after each battery has been tested, and it is important to take the discharges with the weaker batteries *first*, for otherwise the residual charge from the stronger battery

might be sufficient to cause the observation with the weaker one to be erroneous.

172. *Theory of the Battery Resistance Method* (Kempe).—Let us consider the effect on the discharge deflection of shunting the battery of internal resistance B and E. M. F. = E by the shunt S_1 (see Fig. 180). Let the current circulating through the battery and shunt be C, then, by Ohm's law,

$$E=C(B+S_1) \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Now when the shunt S_1 is removed by the plug key K_3, and the discharge deflection d_1 is taken, this deflection is proportional to the whole E. M. F. of the battery E, but when the shunt is in use the discharge deflection d_2 will be something less than d_1, for we are really measuring the difference of potentials represented by an E. M. F. of say e, which maintains a current C through S_1, hence

$$e=CS_1 \quad . \quad . \quad . \quad . \quad . \quad (2)$$

From (1) and (2)

$$\frac{e}{E}=\frac{S_1}{B+S_1} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

but

$$\frac{e}{E}=\frac{d_2}{d_1},$$

hence

$$B=\frac{d_1-d_2}{d_2}S_1 \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Practice of the Method.—When the first discharge is taken the battery is in open circuit, but in the second case it is in closed circuit. Hence the value of E may be greater in the first than in the second case, owing to polarisation setting in when the shunt is used. To lessen the error from this cause, it is important to insert the plug K_3 only for the short length of time necessary to charge the condenser.

173. *Defects of the Deflection Methods.*—

(1.) The theory of the application of a ballistic galvanometer demands that the whole duration of the impulsive current shall be a small fraction of the needle's time of vibration. Owing to the phenomenon of electrical absorption, there is some uncertainty concerning the period of time taken in discharging the condenser, which may become quite comparable with the period of the needle.[1]

(2.) Accurate work demands that the damping of the galvanometer shall be small, but in practice this means that the successive measurements cannot be quickly repeated, owing to the length of time required to stop the needle's vibration.

(3.) The successive discharges are apt to demagnetise the needle.

174. *The Zero Methods.*—By balancing one condenser against another, it becomes unnecessary to use a ballistic galvanometer, hence all the defects above mentioned may be avoided. We should use instead simply a galvanoscope (as delicate as may be), and adjust the charges of the condensers until no deflection is observable. The two best known zero methods are—

(1.) The Bridge Method of De Sauty. This is applicable in the laboratory to ordinary condensers, or short lengths of cable, but it cannot be employed for long lengths of cable on account of the influence of inductive retardation.

(2.) The Method of Mixtures of Sir William Thomson.[2] This is generally applicable, and is extensively used for cable testing.

Lesson LXXIX.—Comparison of Condensers by the Bridge Method.

175. *Apparatus.*—Two boxes of coils or a Post Office

[1] See F. Jenkin, *B. A. Report*, 1867.
[2] See *Journal of Telegraph Engineers*, vol. i. p. 394.

bridge, battery of twelve Daniell's cells, high resistance reflecting galvanometer, Morse key and condensers.

Method.—It is similar to that of the Wheatstone bridge. The condensers of capacity R and S are placed in the arms (Fig. 183) BD and CD, then, by the adjustment of the resistances P and Q in the other arms until the galvanometer is undeflected on pressing the key, we shall have

$$\frac{P}{Q}=\frac{S}{R}.$$

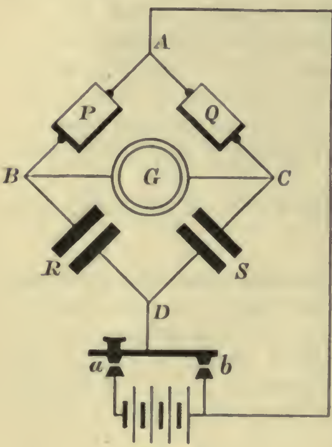

Fig. 183.

It should be noted that the right-hand member of this equation is the reciprocal of the one used when comparing resistances by the bridge.

During the test, whilst adjusting P and Q, the Morse key has the contact made at *b*, which will ensure that the condensers are kept discharged. When the adjustment is complete, whether contact be made at *a* or *b*, the galvanometer should be undeflected.

The resistance of P and Q should be high, and the number of cells of the charging battery should be increased until a sufficiently delicate adjustment can be made. According to Glazebrook[1] results within 1 per cent of the truth should be easily obtainable.

[1] See R. T. Glazebrook "On a Method of Comparing the Electrical Capacities of Two Condensers," *Pro. Phys. Soc.*, vol. iv. p. 207.

Lesson LXXX.—Comparison by the Method of Mixtures.

176. *Apparatus.*—As in the previous lesson, with the addition of a Pohl's commutator.

Method.—For cable testing a special key is used, for which an ordinary Pohl's commutator (see Fig. 25), with its horizontal wires removed, forms a good substitute. This is seen in the diagram (Fig. 184). By moving the switch

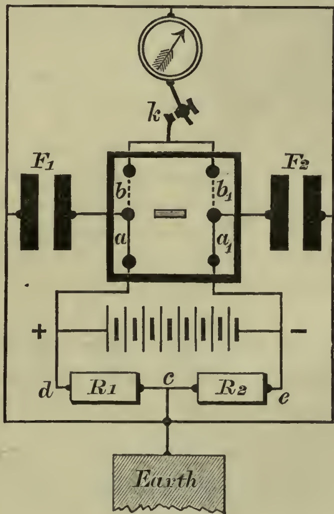

Fig. 184.

in one direction the mercury cups above and below a are placed in connection, and simultaneously, but quite independently, the cups above and below a_1 are connected. Call this position A. By moving the switch in the other direction the same will be the case for the cups above and

below b and b_1. Call this position B. One terminal of the galvanometer, two of the armatures of the condensers F_1 and F_2, and the wire c joining the resistance boxes R_1 and R_2 are connected together. They are also shown, as would be the case in cable testing, connected with the earth. Suppose that first the switch is in the A position, then the condensers will become simultaneously charged to different and opposite potentials, for their *inner* armatures are placed in contact with the opposite poles of a battery, whilst their *outer* armatures are earthed, as likewise is the point c. If the student will draw a diagram of the fall of potentials

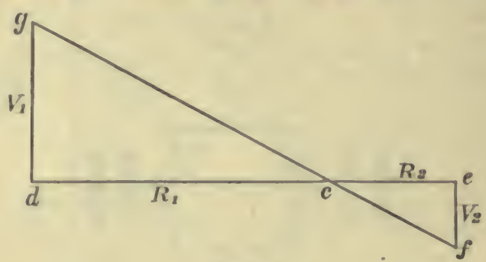

Fig. 185.

between d and e this will be at once clear (see Fig. 185). Here, if V_1 and V_2 be the potentials at d and e, then

$$\frac{V_1}{V_2}=\frac{R_1}{R_2}.$$

But the quantities Q_1 and Q_2, with which the condensers become charged, will be

$$Q_1=V_1F_1$$

and

$$Q_2=V_2F_2.$$

Now put the switch into the B position, the key k being open, when the charges in the two condensers will mix, and if equal will destroy each other, so that on closing

the key k there will be no deflection on the galvanometer. In this case

$$\frac{F_1}{F_2}=\frac{V_2}{V_1}=\frac{R_2}{R_1}.$$

In the application of the method to cable testing, in which the copper of the cable forms the one armature of a condenser and the outside metal and water the other, the switch is placed in position A for a time sufficient to charge the cable fully, and then in position B for a time sufficient for mixing. To avoid the effects of absorption it is desirable that the condensers should not differ much in capacity. R_1 and R_2 should have high values, and the battery power should be sufficiently high and the galvanometer sufficiently sensitive to allow accurate adjustment.

177. *Instantaneous Capacity.* — The student will now appreciate the difficulty of comparing condensers with any high degree of accuracy by any of the preceding methods. In fact a condenser cannot be said to have any true capacity, inasmuch as the charge that it will take depends upon the length of time that the E. M. F. is applied. If we could estimate the capacity from the charge produced by unit E. M. F. in a very brief interval of time, then the capacity so measured would be the *instantaneous capacity.* One of the best methods of determining the capacity so defined has been indicated by Maxwell (see *Electricity and Magnetism*, vol. ii. p. 375), and has been practically applied by J. J. Thomson (see *Phil. Trans.*, 1883, part iii.) Measurements by this method should be made by the advanced student, who should also consult *Phil. Mag.*, August 1884, for the practical details of the method described by Glazebrook.

CHAPTER IX.

THE ELECTROMETER.

178. An *electrometer* "is an instrument for measuring differences of electric potential between two conductors through the effects of *electrostatic* force, and is distinguished from the galvanometer, which, of whatever species, measures differences of electric potentials through the *electro-magnetic* effects of the currents produced by their differences" (Thomson).

The types of electrometers are :—

I. Repulsion Electrometers—
  (*a*) Gold-leaf electroscope provided with a means for measuring the divergence of leaves.
  (*b*) Peltier's electrometer.
  (*c*) Delmann's electrometer.

II. Symmetrical Electrometers—
  (*d*) Hankel's electrometer.
  (*e*) Quadrant electrometer.

III. Attracted Disc Electrometers—
  (*f*) Absolute electrometer.
  (*g*) Portable electrometer.

IV. Capillary Electrometers—
  (*h*) That of Lippmann.
  (*k*) That of Siemens and Dewar.

In this chapter we shall restrict ourselves to the types (*d*), (*e*) and (*g*).[1]

[1] For further information, see Sir William Thomson's *Electrostatics and Magnetism*, p. 260, which contains elaborate descriptions of types

179. *Precautions in Using Electrometers.*—It will be well at the outset to warn students that no successful work can be done in electrometry if attention be not paid to two important details, namely, *thorough insulation* and the avoidance of chance *electrification* due to friction of neighbouring insulators.

Lesson LXXXI.—The Electrometer of Hankel.

180. *Apparatus.*—We find this simple form of electrometer (Fig. 186) convenient. It consists of a shallow box 12 cm. high, 10 cm. broad, and 5 cm. deep, with a sliding door of glass. Through the sides pass two rods of brass, terminating at their inner ends in two discs of brass *a* and *b*, and at their outer ends in two binding screws. The rods are insulated from the box by ebonite (shown black in the figure), and are provided with collars of ebonite near the binding screws for the purpose of insulating

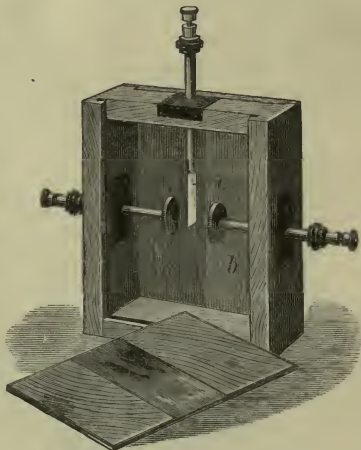

Fig. 186.—Hankel's Electrometer.

II. and III., many of them being the invention of Thomson. For the description of Delmann's electrometer in the form used by Kohlrausch, see Wiedemann I. p. 156. For Lippmann's capillary electrometer, see Jamin IV. (1) p. 238. Dewar's simple modification will be found in *Nature*, p. 210, 1877. For general descriptions of electrometers, see Chrystal, "Article on Electrometer" in *Ency. Brit.*; Bottomley in the South Kensington Science Lectures (vol. i.) (Macmillan); and Munro "On the Quadrant Electrometer," *Journal Soc. Tel. Eng.*, vol. ii. p. 339.

them from the fingers, when they have to be slided in or out. A third rod, the electrode, similar to the others, but supporting a gold leaf *c*, passes through the top of the box. The outside of the box, with the exception of the strips of ebonite and the middle of the sliding window, is coated with tinfoil for the purpose of protection from external electrification. For charging the plates, instead of using the dry pile of

Fig. 187.—A WATER BATTERY.

Zamboni that Bohnenberger applied to his electroscope, we shall make use of a simple form of battery, consisting of from 100 to 200 cells, with zinc and copper plates, and charged with water. The cells for this battery consist of small glass specimen tubes 5 cm. high and 1 cm. in diameter. The battery, which should be well insulated,

may conveniently be mounted as in Fig. 187, and provided with a number of terminals whereby 20, 40, 60, etc., cells may be employed as desired.

When the instrument is in use the plates are charged to equal and opposite potentials by connection with the two poles of the water battery, whose middle is put to earth. The gold leaf, which is earth-connected, should lie symmetrically between the two plates. When we wish to find the difference of potential of a source of electrification from the earth the gold leaf is connected with the source, and the movement of the leaf from the one plate to the other is measured. The measuring instrument consists of the cathetometer microscope of Quincke (see Vol. I. p. 42).

For use with electrometers we require a special form of reversing key. The requirements of such a key will be understood from the diagram (Fig. 188), in which *a*, *b*, *c* and *d* are four mercury cups. With *c* and *d* the two poles of a battery under test are connected, whilst *a* is connected with the electrode of the electrometer, and *b* is connected with the external case of the electrometer, which is connected with the earth, and may be regarded as the other electrode. The operations with the key would be as follows:—

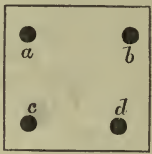

Fig. 188.

(1.) Connect *a* with *b*, this will earth both electrodes, and remove any charge from them.

(2.) Disconnect *a* and *b*, connect *a* and *c*, also *b* and *d*, *a* will receive, say, a + charge and *d* will be put into contact with the earth.

(3.) Disconnect *a* and *c*, also *b* and *d*. Connect *a* and *b* to discharge the electrometer.

(4.) Disconnect *a* and *b*, and connect *a* and *d*, also *b* and *c*, *a* will now receive a negative charge whilst *c* will be earthed.

These operations may be performed by the aid of a

suitably arranged switch key, such as the reversing key provided with Thomson's quadrant electrometer.[1]

Method of Using the Instrument for Comparing two E. M. Fs.—Connect the two plates to the poles of the water battery, which should be provided with an ordinary commutator. The plates must be at equal and opposite potentials. To ensure this the middle of the battery should be earthed. Connect *a* of the electrometer key with the gold leaf, and *b* with the outside of the electrometer, which should be earthed.

Focus the cathetometer microscope on some irregularity on the end of the gold leaf, which latter should be meanwhile earthed.

The battery whose E. M. F. is to be measured is connected with *c* and *d*, and then the operations with the key are gone through.

The number of scale divisions that the end of the gold leaf passes over when it is made + and then − will be proportional to the E. M. F. of the battery. The process should be repeated with a second and standard battery.

Example.—Fifty cells of a water battery gave a deflection of 95 micrometer scale divisions, and 20 cells of a Latimer Clark battery gave 60 divisions, hence the E. M. F. of the water battery per cell is

$$\frac{95 \times 20 \times 1{\cdot}457}{60 \times 50} = {\cdot}92 \text{ volt.}$$

Theory of the Instrument.—We shall presently, in a general manner, prove that in a symmetrical electrometer the resultant attraction F of the movable part at potential V_1, towards the plate of lower potential, is expressed by

[1] A really good electrometer key is a desideratum. Keys insulated by ebonite are apt to become electrified. Dr. Lodge has designed a simple key with low capacity that has many advantages above the keys commonly in use.

$$F = \text{constant}\left(V_1 - \frac{V_2 + V_3}{2}\right)(V_2 - V_3) \quad . \quad . \quad . \quad (1)$$

where V_2 and V_3 are the potentials of the symmetrically placed fixed conductors.

In Hankel's electrometer the fixed conductors are made of equal and opposite potentials, or $V_3 = -V_2$, hence in this case the above formula becomes

$$F = 2 \text{ constant} \times V_1 V_3.$$

Now the resultant attraction is measured by the number of divisions d that the gold leaf moves over from its zero position, hence as long as V_3 is constant we simply write

$$d = \text{some constant} \times V_1 \quad . \quad . \quad . \quad . \quad (2)$$

or the potential V_1 is simply proportional to d.

Further Experiments with the Electrometer.—(1.) The above simple result depends upon the condition $V_3 = -V_2$. To ensure the fulfilment of this condition, we have directed that the middle of the water battery, consisting say of 100 cells, should be earthed. If the cells were all of the same resistance and of equal E. M. F., it would be only necessary to earth the connection between the 50th and 51st cell. Since we have no right to make this supposition the middle point should be ascertained experimentally.

Referring to formula (1) we see that when we reverse V_2 and V_3, as occurs when we change the water battery commutator, F simply changes sign, but when we change the sign of V_1, F alters its magnitude unless $V_3 = -V_2$. This gives us therefore one method of ascertaining whether the middle point of the battery has been earthed. We further notice that if V_1 is the mean of V_2 and V_3, then F becomes zero. Now the potential of the true middle point of a battery is always the mean of the potentials at its two ends. Hence, if a wire be connected from the gold leaf

to various points of the battery, when we reach the true middle there will be no deflection. These experiments should be made.

(2.) Again referring to formula (1), if V_2 were made equal to V_1, a condition we obtain when only one battery is used, and one of its poles is both connected with the gold leaf and one of the plates, then

$$F = \frac{\text{constant}}{2}(V_1 - V_3)^2,$$

which has the convenient property that if V_1 and V_3 both be made to change places by using the commutator, F would still remain unchanged in sign and magnitude. Now this is precisely the condition that makes an instrument applicable for the measurement of alternate currents, and has been so applied by Joubert. This should be verified experimentally.

181. *Attraction between two parallel Plates.*—Consider two parallel plates A and B (Fig. 189) very close together, of which B is kept charged to a potential V, whilst A is earth-connected and at zero potential. Under these circumstances they will become equally and oppositely charged, so that if $+\rho$ be the quantity of electricity per unit area or *density* of B, $-\rho$ will be the density of A. Neglecting the consideration of the electrification at the back of the plates and the want of uniformity at their edges, the attraction of B for unit of area of A will be, since this unit area has ρ units of electricity (see Appendix B, Art. 4), $2\pi\rho \times \rho = 2\pi\rho^2$, or if the area of A be S, *the whole normal electrical attraction* F of B for A will be

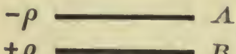

Fig. 189.

$$F = 2\pi\rho^2 S \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

But since the plates are close together, the resultant force R on an electrical unit placed between them will be com-

posed of an attraction $2\pi\rho$ (Appendix B) to the one plate, and of a repulsion $2\pi\rho$ from the other, so that this force will be

$$R = 4\pi\rho \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Another definition of this resultant force is the rate of fall of potential per unit length. Hence if D be the distance between the plates—

$$R = \frac{V}{D} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

From (2) and (3)

$$\rho = \frac{V}{4\pi D},$$

which value of ρ inserted in (1) gives

$$V = D\sqrt{\frac{8\pi F}{S}} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The last equation expresses V in quantities that may be determined by direct measurement. To find F, the attraction of B for A, the latter may be suspended from the arm of a balance, and the attraction actually measured by weights placed in the balance-pan. An electrometer of such a description is called an **Absolute Electrometer**, and the quantity that multiplies D is the constant of the instrument, which may be called α. If the potential of A had been V_1 instead of zero, we should have had

$$V - V_1 = D_1\alpha \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where D_1 is the distance that the plates must now be apart in order that the force of attraction may again be F. Hence from (4) and (5)

$$V_1 = (D - D_1)\alpha \quad . \quad . \quad . \quad . \quad . \quad (6)$$

an expression in which differences of distance are only concerned.

182. *Sir William Thomson's Attracted Disc Electrometer.*—

To make the above formula applicable in practice Sir William Thomson makes the attracted disc form the movable centre of a large plate, called the guard ring, whilst the attracting plate is of much larger size than the disc. Fig. 190 shows a diagrammatic section of an electrometer on this principle. The disc *ab*, shown supported by a spring, is surrounded on all sides by the guard ring *gr* and *r′g′*, which is accurately in the same plane as *ab* when the attracting plate C is a certain distance from the disc. This position is called the *sighted position.* To use the instrument the guard ring and disc are first charged to a high potential V, and then C having been earth-connected,

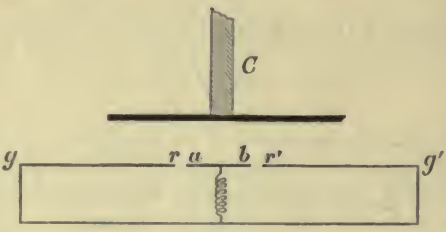

Fig. 190.

is moved up or down by means of a micrometer screw until *ab* is in the sighted position. Call the reading of the micrometer head r. Now repeat the experiment when C is at the potential V_1, then from the last article, supposing r_1 to be the new reading,

$$V_1 = (D - D_1)a = (r - r_1)a,$$

which would give us V_1 if we had a knowledge of the constant a. Failing this we can only make comparative measurements. Thus, if a new potential V_2 and a new reading r_2 were obtained, then

$$\frac{V_1}{V_2} = \frac{r - r_1}{r - r_2}.$$

An electrometer of the type of Fig. 190, in which the force of attraction is balanced against a spring, is called a Portable Electrometer.

Lesson LXXXII.—The Portable Electrometer.

183. *Apparatus.*—In Fig. 191 is shown the improved

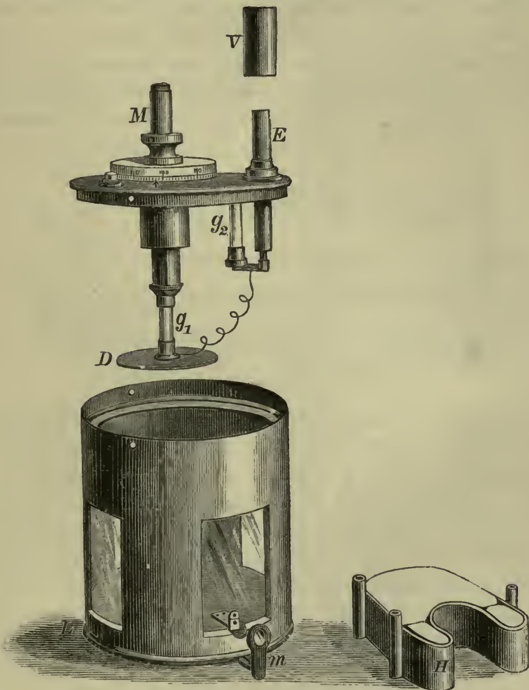

Fig. 191.—The Portable Electrometer.

portable electrometer of Thomson and Jenkin. It consists of a cylindrical brass box, 9 cm. in diameter and 10 cm. high,

within which fits a cylindrical glass jar. The brass casing has four windows, through which at the bottom of the jar may be seen the guard plate. The lid of the instrument bears the micrometer head M and the guard tube E of the electrode, over which fits the umbrella V. The micrometric arrangement adopted by Thomson, and afterwards improved by Jenkin, is an exceedingly ingenious piece of screw mechanism, which gives an accurate up and down movement of the disc D and prevents "back lash."

A section of the bottom of the jar is seen in Fig. 192, where *hh* is the guard plate and *f* the movable disc. The latter consists of a spade-shaped piece of sheet aluminium,

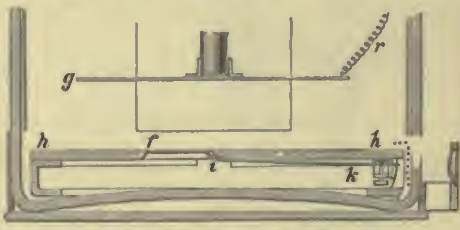

Fig. 192.—Section of Portable Electrometer.

the square end of which very nearly fills up the hole in the guard plate. It is supported by a platinum wire that passes tightly through two holes in the aluminium at the point *i*. In the section the wire cannot be seen, for it lies in a direction at right angles to the plane of the paper. The platinum wire is secured at its ends to two springs, so as always to remain tight. The attraction of the disc upwards by the charged plate *g* is resisted by the torsion of the wire, which is by means of certain adjustments so regulated that when the disc and guard plate are in the same plane the handle *k* of the aluminium lever rests midway between two stops, and is then in the *sighted position.* To know with accuracy that this is the case, the end *k* is forked and has stretched between the prongs of the fork a

fine opaque black hair, which passes in front of a plate of white enamel (see Fig. 191), having two black dots so : upon it. When the hair is seen to be between the dots, by the use of the simple lens m, the disc is in the required position.

The bottom of the instrument up to the level of the guard plate forms a shallow Leyden jar; the glass jar is coated with tinfoil both inside and outside, the latter being in connection with the brass casing. To see the hair a portion of the tinfoil covering of the glass under the guard plate must be removed. This will cause a disturbance of the uniformity of the electric distribution. To diminish this as much as possible, a screen of wire-fencing in connection with the inner coating is employed; the cut ends of the wires will be seen in Fig. 192.

A light spiral spring r (Fig. 192) connects D (Fig. 191) to the electrode, which is insulated by the glass stem g_2 (Fig. 191). The electrode passes freely through its guard tube without connection, but may be connected with it by placing V (the umbrella) over E. The umbrella, when somewhat raised, will leave the electrode insulated, and hence serves for the purpose of a wind guard when the electrometer is used for outdoor purposes.

The leaden box H may be fixed to the under side of the lid. It is intended to contain pumice that has been moistened with sulphuric acid for the purpose of drying the interior of the instrument. It is screwed into its position without touching the brass work.

To Prepare the Instrument for Use.—Boil some strong sulphuric acid with a few crystals of ammonium sulphate, in order that any nitrogen compounds that are often present in sulphuric acid, and which would injure the metalwork, may be destroyed.[1] Take out the pumice from the leaden box and heat it to redness. When cool replace it

[1] The acid should be boiled in a porcelain dish in a fume cupboard by means of a large Bunsen's burner. The boiled acid should be kept in a well-stoppered bottle labelled "Acid for Electrometers."

in the box, and by means of a pipette place a few drops of acid on different parts of the pumice. Excess of acid must be avoided, the quantity added should not make the surface moist.[1] The leaden box must now be screwed into its position underneath the lid.

Clean the inner surface of the glass jar, and remove any particles of dust, shreds, etc., especially from the guard plate and the aluminium and upper discs. Thoroughly warm the instrument and screw on the lid.

Method of Charging.—We next proceed to charge the Leyden jar. The charge may either be positive or negative, but the former is preferred, for a positive charge is found to dissipate less rapidly. For the purpose a small electrophorus is usually provided with the instrument, but a small Voss or Winhurst influence machine forms a more convenient source of electricity. The operations of charging are as follows :—(1.) Move the attracting plate by the micrometer near to its highest position, otherwise too strong a force of attraction may be exerted on the aluminium disc, and the jar may discharge itself. (2.) Next see that the umbrella is down, so that the upper plate is earthed. (3.) Uncover the hole in the cover of the instrument and pass down a wire, insulated from the case by a collar of ebonite, so that its bared end rests upon the guard plate. Give successive small sparks to the upper end of the wire until the hair is beneath the lower dot. Now remove the wire by means of its insulating covering, and close the aperture in the lid.

Testing the Instrument.—Place the instrument at a con-

[1] We find it far more convenient to use instead of pumice-stone threads of asbestos or asbestos-paper packed tightly at the bottom of the leaden jar. The asbestos should be just moistened with sulphuric acid. It will be well to add here that since the sulphuric acid is continually absorbing moisture, there may, if the acid is not changed periodically, be a destruction of the working parts of the instrument. On this account Sir William Thomson has caused to be engraved on the case the warning, "*Dangerous, if pumice not dried monthly.*"

venient height on a firm slab and in a good light. Turn the micrometer screw, and tap the instrument meanwhile—for the end of the lever is apt to stick against the stops—until the hair comes between the two spots on the enamel plate. Now proceed to make a careful setting of, say, the upper boundary of the hair, making it to coincide exactly with some easily recognisable irregularity on the lower edge of the upper dot. To avoid parallax the position of the eye must be such that the hair never appears convex, whether it be viewed from above or from below. Where the instrument is required for laboratory uses it is better to remove the simple lens and take the readings by means of a small compound microscope of low magnifying power that is placed on a separate stand.

After an accurate setting has been made the reading should be taken. Settings and readings should be made repeatedly until they are consistent to within $\frac{1}{10}$ of a division. If it is found that the plate has gradually to be brought nearer the disc the instrument is leaking, and the processes of drying and cleaning the instrument must be repeated.

Use of Instrument for Comparing E. M. F.—One pole of the battery under test should be connected through an electrometer key with the electrode by passing a wire through the hole in the top of the umbrella. The other pole is connected with the case of the instrument, which latter should be earthed. Readings should be taken in the three positions of the commutator.

Example.—Determination of the value of the constant by means of 20 Clark's cells of E. M. F. 29 volts.

POSITION OF COMMUTATOR.

	Up.		Earth.		Down.
Readings	10·588		10·728		10·870
Differences		·140		·142	
Sum of differences . .			·282		

Thus the difference of potential between the + and − poles is equal to ·282 large divisions, or each small division is equal to $\frac{29}{\cdot 282 \times 100} = 1{\cdot}06$ volt.

184. *Theory of a Symmetrical Electrometer.*—Let A denote the movable part of the electrometer, also let B denote one fixed conductor and C the other. We have thus three conductors, A, B and C. Further, let these conductors be kept respectively at the potentials V_1, V_2 and V_3.

Now when A is lying in its zero position it may be supposed to form two condensers, one with the conducting system B and another with the conducting system C. If A moves in either direction the capacity of the one condensing system will be increased and that of the other will be diminished. Further, if θ denote the angular change of position, and if this be comparatively small, we may imagine this change of capacity to be proportional to θ. Let us call it $c\theta$.

It follows from the definition of capacity that the amount of free electricity lost in the A and B system of condensers as A goes from B to C will be for A $c\theta(V_1 - V_2)$ and for B $c\theta(V_2 - V_1)$. Again, the amount of electricity gained in the A and C system on account of this motion of the needle will be for A $c\theta(V_1 - V_3)$ and for C $c\theta(V_3 - V_1)$.

Now the work expended in charging a conductor with the quantity Q up to the potential V will be $\frac{1}{2}QV$. This will be manifest if we reflect that when the first portions of the charge are communicated the potential is very low, its final value being V. We may therefore regard $\frac{1}{2}V$ as its average value, and hence from the definition of potential $\frac{1}{2}QV$ will denote the work expended in charging the conductor. Hence also if, while the potential remains the same the quantity is reduced to Q', then, since the energy of the system will now be $\frac{1}{2}Q'V$, it is evident that

a quantity of energy has been taken from the system $=\frac{1}{2}(Q-Q')V$.

Now when we have a condenser, one of whose plates is movable, the tendency of this is to lie as close as possible to the other fixed plate. Any effort to separate the plates is therefore made against electrical forces. It will thus imply work spent upon the movable plate, the amount of which is to be measured by the energy withdrawn from the system.

Applying these principles to the case before us, we see that the weakening of the A and B system on account of the motion of A implies the following work spent upon the needle :—

$$\tfrac{1}{2}c\theta\{V_1(V_1-V_2)+V_2(V_2-V_1)\} \quad . \quad . \quad . \quad (1)$$

On the other hand, the motion of A with respect to the condensing system A and C represents the following energy gained by the needle :—

$$\tfrac{1}{2}c\theta\{V_1(V_1-V_3)+V_3(V_3-V_1)\} \quad . \quad . \quad . \quad (2)$$

Hence the whole energy gained by the needle will be (2) – (1), or

$$c\theta(V_2-V_3)\left\{V_1-\frac{V_2+V_3}{2}\right\} \quad . \quad . \quad . \quad (3)$$

Now this would likewise be the energy produced by a similar motion under a couple whose moment is

$$F=c(V_2-V_3)\left\{V_1-\frac{V_2+V_3}{2}\right\} \quad . \quad . \quad . \quad (4)$$

The expression admits of simplification when V_1 is large compared with V_2 and V_3, for then we may write—

$$F=c(V_2-V_3)V_1 \,. \quad . \quad . \quad . \quad (5)$$

185. *The Quadrant Electrometer.*—The simplest type of a symmetrical electrometer is that of Hankel, but the one most in use is the quadrant electrometer. The fixed con-

ductors are four quadrantal metal boxes, I, II, I*a* and II*a* (see Fig. 193), supported horizontally by glass insulators. The edges of the quadrants nearly touch, so that they form a hollow cylinder, divided by two rectangular slits. Within them, supported by a bifilar suspension, is the needle. The opposite quadrants are connected together. When the needle and quadrants are at the same potential the needle is made to lie with its axes of symmetry parallel to the inner edges of the quadrants, with reference to which it is symmetrically placed. But when in use the needle is raised to a high potential V_1, whilst the two sets of quadrants are brought to the much lower potentials V_2 and V_3.

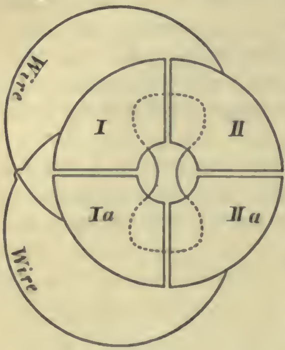

Fig. 193.

The needle will therefore be deflected until there is equilibrium due to the couple derived from equation 5, and that resulting from the resistance of the bifilar arrangement to turning. The deflection of the mirror needle is measured by the mirror and scale method. Hence the angular movement may be regarded as small, so that the following equation will be true as long as the charge of the needle remains the same—

$$d = \text{some constant} \times V,$$

where d is the deflection in scale divisions, and V the difference of potential between the two quadrants.

Lesson LXXXIII.—The Quadrant Electrometer.

186. *Apparatus.*—We shall restrict our description to the

most complete type of Sir William Thomson's instrument, known as the White Pattern. It is largely used for the purpose of cable testing. There are other simpler forms [1] of the instrument which are very convenient, indeed the student may, with little labour, construct an instrument that, in conjunction with a water battery, may be employed for the measurements of this chapter.

Figs. 194 and 195 show the instrument complete.[2]

The parts of the instrument are—

(1.) The outer brass framework, supported by ebonite levelling screws.

(2.) The Leyden jar, consisting of an inverted bell jar containing sulphuric acid. The jar is coated externally with strips of tinfoil in connection with the outer framework. The jar is not completely coated, but windows are left through which the inside may be seen.

(3.) The jar is covered by the *main cover*, on the top of which may be seen—

(*a*) *Micrometer head* M for adjusting one of the quadrants.

(*b*) The electrode of the *induction plate.*

(*c*) The head of the *replenisher.*

(*d*) The *electrodes* A, B, and C.

(*e*) The circular level.

(*f*) The *lantern.*

(4.) On the inside of the main cover are—

(*a*) The quadrants.

(*b*) The induction plate above one of the quadrants.

(*c*) The aluminium needle.

(5.) Within the lantern is seen the *glass stem* supporting the *attracting plate*, and above the lantern is the *gauge*, which is really a portable electrometer. The attracting plate

[1] Such as Elliott's Lecture Pattern, and the instruments of Kohlrausch and Mascart.

[2] We have reproduced these figures with some changes from Gordon's *Electricity and Magnetism*, which should be consulted.

supports, by a *bifilar suspension*, the *mirror* and needle. From the bottom of the last is suspended a fine platinum wire, having a *platinum weight* at its end. The weight

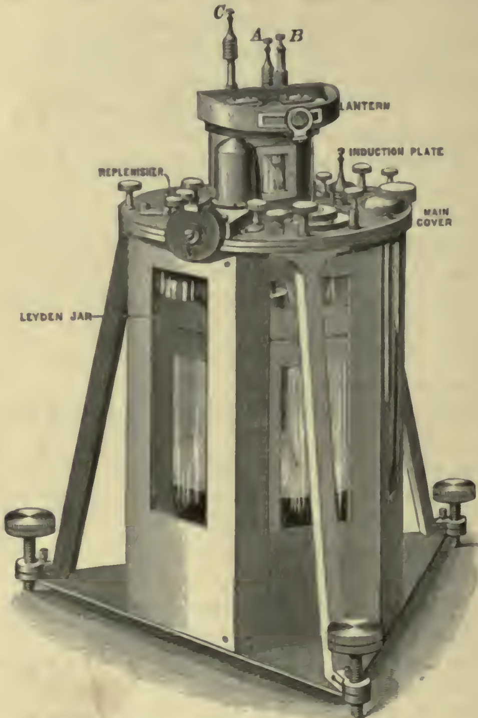

Fig. 194.—THE QUADRANT ELECTROMETER.

and the lower portion of the wire are immersed in the sulphuric acid. The upper portion of the wire passes through a *protecting tube*.

(6.) The method of suspension of the needle allows

either fibre to be raised or lowered, or the upper ends to be more or less separated, according to the degree of sensibility required. These adjustments are made by the

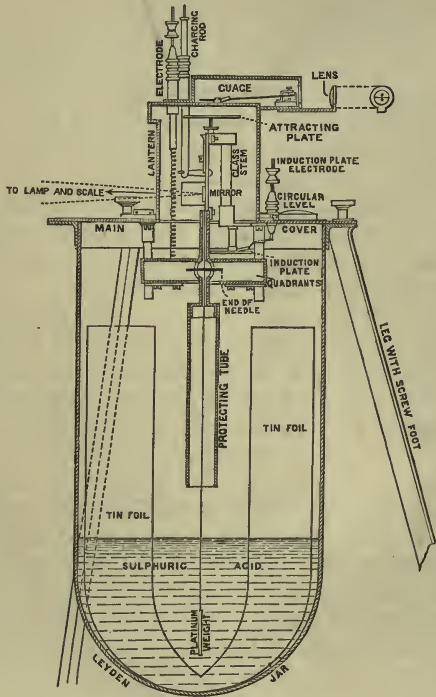

Fig. 195.—Section of Quadrant Electrometer.

help of two screw keys, which, when not in use, are kept near the place marked "main cover" (Fig. 194).

(7.) The needle is of the thinnest sheet aluminium that will give the necessary stiffness. It is cut in the form of a

flat dumb-bell or canoe-paddle. Its area is 4·2 sq. cm., and it weighs ·07 gm.

(8.) The gauge is really a secondary electrometer of the attracted disc type.

(9.) The quadrants. One of them may be moved by the screw M, the others by sliding. The quadrant having the induction plate above it is provided with a *disinsulator*, which is simply a brass arm mounted on a vertical axis, so that by turning a milled head on the main cover the brass arm may be caused to touch the quadrant and so remove any charge.

(10.) The Replenisher. This is a simple influence machine, by rotating which the charge of the needle may be increased or lessened. For a description of it the ordinary text books must be consulted.

Preparing the Instrument for Use.—The exact details of the adjustments are given in a pamphlet[1] issued with the instrument. We shall therefore suppose that the instrument is in the position for use, with its lamp and scale arranged at a distance of about a mètre from the mirror.

Charging the Instrument.—(1.) Twist a fine copper wire round the charging electrode C, fasten it by the binding screws to the electrodes A and B, to the induction plate electrode, and to one of the binding screws on the main cover. Raise C and turn it clock-wise. This will bring it into contact with the metal-work and in electrical communication through the acid with the needle. The reflected image should now be at the middle of the scale.

(2.) Disconnect the wire from C and give the needle a positive charge by a succession of small sparks from an electrophorus until the hair of the aluminium balance of the gauge rises. Make an exact adjustment of the hair by

[1] "Directions for the Adjustment and Use of the Quadrant Electrometer," drawn up by W. Leitch.

means of the replenisher, tapping meanwhile the gauge to free the end of the lever from the stops.

(3.) The effect of charging the jar will probably be that, owing to want of symmetry of the quadrants, the needle will be deflected. By means of the micrometer screw and sliding the quadrants in or out the reflection must be brought back to the middle of the scale.

Method of comparing E. M. Fs. not greater than 4 *Volts.*—Connect with the electrometer key (Fig. 188, p. 419) as follows:—*a* and *b* with electrodes A and B. A is also connected with one of the binding screws on the main cover, and B is connected with the electrode of the induction plate. The poles of the battery are connected with *c* and *d* of the key. Readings are taken in the three positions of the key. The deflection on both sides of the zero position should be equal.

Method of comparing E. M. Fs. not greater than 100 *Volts.*—Everything being as before, we raise the electrode B out of contact with its quadrant. The only charge that will be received by this quadrant will be due to what may be induced by the induction plate above it. Hence the sensibility of the instrument will be so diminished that E. M. Fs. as high as 100 volts will come within the range of the scale. Should the act of raising the electrode cause an induced charge, producing a deflection of the needle, it will be necessary to put the quadrant into connection for a short time with the case by turning the milled head of the disinsulator.

187. *Other Uses of the Electrometer.*—(1.) The electrometer may replace the galvanometer in many tests, such as the following:—

(*a*) *Internal Resistance of Battery.*—The deflection is observed with the battery in open circuit, and then with a shunt between its poles. The formula of Art. 172 is then applied.

(*b*) *Resistance of a Conductor.*—Replace the high resistance galvanometer of Lesson XXX. by the electrometer.

(*c*) *Measurement of Capacities.*—Replace the galvanometer of Lesson LXXX. by the electrometer.

(2.) For cable work the electrometer is convenient, especially for determining the insulation resistance of the cable. The method consists in observing the time T in seconds that the cable charged as a condenser of capacity F takes in falling from the potential V to v. The formula

$$R = \frac{T}{2{\cdot}303 \; F \log \frac{V}{v}}$$

gives the insulation resistance.[1]

[1] For details, see *Journal Tel. Eng.*, vol. ii. pp. 174 and 351. The method may also be applied to find the insulation resistance of ebonite, etc., insulators. See Gray, *Absolute Measurements*, p. 112.

APPENDIX.

A.

THE WHEATSTONE NET.

1. In describing the measurement of resistance by means of the Wheatstone's bridge nothing was said about the best values to give to the arms in order to determine the unknown resistance with the greatest possible exactness. A complete discussion of these conditions requires a perfect familiarity with certain corollaries of Ohm's law known as **Kirchhoff's Laws.** We shall in this Appendix endeavour to explain these laws, and apply them to a system of resistances arranged after the manner of the Wheatstone's bridge, and which may be called the **Wheatstone's Net,** a *net* or *network* being the general term for a system of interlaced resistances.

Let us consider two points, A and B (Fig. 1), in a conductor conveying a current of electricity, the resistance between the

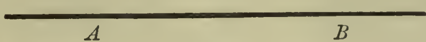

Fig. 1.

two points being R. If the potentials at A and B be denoted by these letters themselves, then, by Ohm's law (it being understood that *a positive current flows from a point of higher to a point of lower positive potential*),

$$C=\frac{A-B}{R} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

or

$$CR = A - B \quad . \quad . \quad . \quad . \quad . \quad (2)$$

or

$$R = \frac{A - B}{C} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

expressions which will be frequently used.

Now if a battery or other electromotor be included between A and B, its electromotive force resulting from a difference of potential E, then, presuming R to be as before,

$$C_1 = \frac{(A - B) \pm E}{R} \quad . \quad . \quad . \quad . \quad (4)$$

the sign of E depending upon the direction of the poles of the battery, or

$$C_1 R = (A - B) \pm E \quad . \quad . \quad . \quad . \quad (5)$$

in other words, the algebraical difference of the potentials at two points in a circuit is equal to the current between the points multiplied by the included resistance.

Consider next any number of currents approaching or leaving any point O (Fig. 2), then, since there is no accumulation of

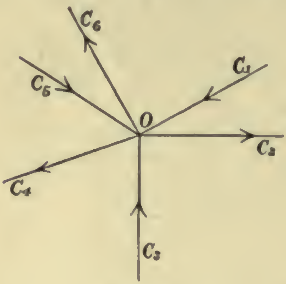

Fig. 2.

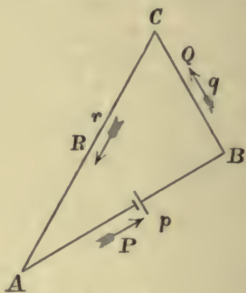

Fig. 3.

electricity at O, *the sum of the currents that leave the point is equal to the sum of those that approach the point;* hence

$$C_2 + C_4 + C_6 = C_1 + C_3 + C_5 \quad . \quad . \quad . \quad . \quad (6)$$

This is the **first law of Kirchhoff**, which may be applied to every node or point of meeting of currents. Considering that currents flowing to the point have an opposite sign to those flowing from it, this law may be thus expressed :—

Law I.—**The algebraical sum of the currents meeting at a node of a network is zero.**

2. Again, if ABC (Fig. 3) be any closed portion of any circuit whatever, and if there be an electromotive force E between A and B; P, Q, R being the resistances of the three sides, and p, q, r the currents flowing through these in the direction of the arrow heads, then, expressing the potentials at the nodes by A, B, C, we shall have by (5)

$$\begin{aligned} (A-B)+E &= Pp \\ B-C \quad &= Qq \\ C-A \quad &= Rr \end{aligned}$$

Hence by addition

$$E = Pp + Qq + Rr.$$

We may extend and interpret this result as follows :—

Law II.—**In any mesh of a network the sum of the electromotive forces is equal to the sum of the products of the resistances into the respective currents of the boundaries.**

It will be desirable to illustrate this law by a numerical example.—Let ABC (Fig. 3) be an ordinary complete circuit (the current being of course in this case the same throughout), and let the resistance of $AC = 7$, that of $CB = 4$, that of $AB = 9$. Also let a battery of E. M. F. $= 20$ be introduced between A and B, the resistance between this battery and A being $= 3$; and finally conceive the whole arrangement to be insulated. Then to the right of the point where the E. M. F. acts we may suppose the potential to be $+10$, while to the left of it this will be -10. Now in this example the current is evidently unity, for $C = \frac{E}{R} = \frac{20}{7+4+9} = 1$, and since in such a circuit the E. M. F. or difference of potential between any two points is proportional to the resistance between them the potential at A will be $-10 +$

$3 = -7$ (*i.e.* by 3 units more positive than that at the left of the point where the E. M. F. acts). In like manner the potential at B will be $+10-6=+4$, while that at C will be 7 units more positive than that at A, or 4 units more negative than that at B; in other words, it will be $=0$. Hence we see that

$$\begin{array}{llll} & & & \text{Resistance} \times \text{Current.} \\ (A-B)+E & = -7-4+20 & = & 9\times 1 \\ B-C & = 4-0 & = & 4\times 1 \\ C-A & = 0+7 & = & 7\times 1 \end{array}$$

Hence by addition

$$E=9\times 1+4\times 1+7\times 1=20;$$

or, in other words, Law II. is verified by this example. Examples will now be given of the application of these laws.

I. To find the combined resistance x of r_1, r_2, r_3, three branch resistances (Fig. 4) joining the points A and B, and forming

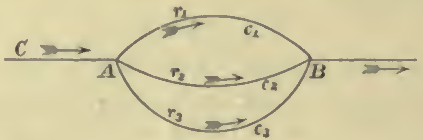

Fig. 4.

part of a circuit through which a current C is flowing. Let us call the currents in the branches c_1, c_2, c_3. Then

$$C-c_1-c_2-c_3=0 \text{ by Law I.}$$

Hence also (representing the potentials at the points by the letters there)

$$\frac{A-B}{x}=\frac{A-B}{r_1}+\frac{A-B}{r_2}+\frac{A-B}{r_3} \text{ by Ohm's law;}$$

or

$$\frac{1}{x}=\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}.$$

Hence

$$x=\frac{1}{\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}}.$$

II. To find the relation between the currents in the divided circuits AFB, ADB, AGB (Fig. 5), due to an electromotive force E in AGB.

Let us call the current in AFB $= c_1$, in ADB $= c_2$, in AGB $= C$; also let the respective resistances be r_1, r_2, R. Then

$$C - c_1 - c_2 = 0 \text{ by Law I.} \quad . \quad . \quad . \quad . \quad (1)$$
$$E = CR + c_2 r_2 \quad ,, \quad \text{II.} \quad . \quad . \quad . \quad . \quad (2)$$

But

$$E = CR + c_1 r_1, \text{ also by Law II.} \quad . \quad . \quad . \quad (3)$$

Hence from (2) and (3)

$$c_1 r_1 = c_2 r_2 \quad . \quad . \quad . \quad . \quad . \quad (4)$$

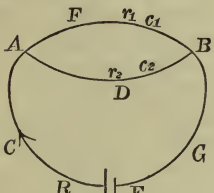

Fig. 5.

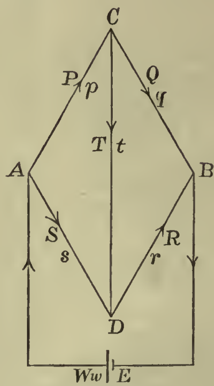

Fig. 6.

Substituting the value of c_2 derived from (4) in (1), we obtain

$$c_1 = \frac{r_2}{r_1 + r_2} C,$$

and similarly

$$c_2 = \frac{r_1}{r_1 + r_2} C.$$

III. Let us next apply these laws to the Wheatstone's net, as exhibited in Fig. 6. The resistances and the currents in the various lines are denoted by the large and small letters respectively of Fig. 6, the currents flowing in the direction

of the arrow heads, and the electromotive force of the battery being E.

By Law I. we have

$$p-q-t=0 \quad . \quad . \quad . \quad . \quad . \quad (1)$$
$$w-r-q=0 \quad . \quad . \quad . \quad . \quad . \quad (2)$$
$$r-s-t=0 \quad . \quad . \quad . \quad . \quad . \quad (3)$$
$$w-p-s=0 \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Again, by Law II., we have

For mesh	WRS	$-\mathrm{W}w-\mathrm{S}s-\mathrm{R}r=-\mathrm{E}$	(5)
,,	WQP	$-\mathrm{W}w-\mathrm{P}p-\mathrm{Q}q=-\mathrm{E}$	(6)
,,	PST	$-\mathrm{P}p+\mathrm{S}s-\mathrm{T}t=0$	(7)
,,	QTR	$-\mathrm{Q}q+\mathrm{T}t+\mathrm{R}r=0$	(8)

Let us now explain why certain products have negative signs in equations (5), (6), (7) and (8). For reasons given in note 2 § 3 we go round each mesh in a direction the opposite to that of the hands of a watch. When the current goes *with* us we reckon it *positive*, when it goes against us we reckon it *negative*.

An examination of these eight equations will show that only three in each set are independent. Thus, for instance, from (1) and (3) we derive the equation $p+s=r+q$. Hence given (2) the equation (4) will follow as a natural consequence. In like manner we may from (7) and (8) derive the equation $\mathrm{S}s+\mathrm{R}r=\mathrm{P}p+\mathrm{Q}q$; hence given (5) equation (6) will follow as a natural consequence.

We have thus six unknown quantities, p, q, r, s, t, w, and six independent equations by which to find them.

The solution of the six equations is very laborious, but by an ingenious device of Maxwell the equations may be reduced to three. This device will now be explained.

3. *Maxwell's Method.*[1]—Imagine that round each *cell* or *mesh* or *cycle* of the network *imaginary currents* flow, all directed in the same way, thus round

[1] Note on Wheatstone's bridge, p. 206 of Maxwell's *Elementary Treatise*, or vol. i. (2d edition) of his large work. See also Dr. Fleming, "Problems on the Distribution of Electric Currents in Networks of Conductors treated by the method of Maxwell," *Phil. Mag.*, September 1885.

WRS there is a current x (Fig. 7),
QTR „ „ y,
PST „ „ z,

the direction of the currents being the opposite to that of the hands of a watch.[1] Here the signs of x, y, z, etc. may be, some positive and others negative. The *real* current from

B to D is $x-y$,
C to D is $y-z$,
D to A is $x-z$.

To avoid additional letters designate the potentials at the nodes by the letters placed there. Then, by Law II.,

$$\mathrm{A}-\mathrm{B}=\mathrm{W}x-\mathrm{E} \quad . \quad . \quad (1)$$
$$\mathrm{B}-\mathrm{D}=\mathrm{R}(x-y) \quad . \quad . \quad (2)$$
$$\mathrm{D}-\mathrm{A}=\mathrm{S}(x-z) \quad . \quad . \quad (3)$$

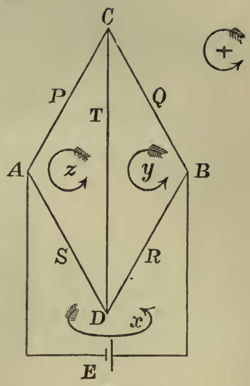

Fig. 7.

Hence adding together (1), (2), and (3) we have

$$\mathrm{E}=x(\mathrm{W}+\mathrm{R}+\mathrm{S})-\mathrm{R}y-\mathrm{S}z.$$

This is called the *equation of the x cycle.* To form the equation of any cycle we have hence the following rule :—

Maxwell's Rule.—**The effective E. M. F. (written + or − according as it is with or against the cycle current) in any cycle is equal to the product of the sum of the resistances of the boundaries into the cycle symbol (current), less the sum of the products of each neighbouring cycle symbol (current) into the resistance of the common bounding cell.**

By the aid of this rule the equations of the various cycles can be immediately written down. We thus get

[1] This way of regarding currents has been adopted by Dr. Fleming for the following reason. A current is considered to be circumnavigated positively when you walk round it inside, so as to keep the boundary on your right hand.

$$E = x(W + R + S) - yR - zS \quad . \quad . \quad . \quad (\alpha)$$
$$0 = y(Q + R + T) - xR - zT \quad . \quad . \quad . \quad (\beta)$$
$$0 = z(S + T + P) - yT - xS \quad . \quad . \quad . \quad (\gamma)$$

4. The equations that we have just written down will be applied for finding the resistance of the network from A to B. Let us call this r, then we have

$$r = \frac{B - A}{x} \text{ by Ohm's law.}$$

But

$$B - A = E - Wx \text{ by Kirchhoff's second law.}$$

Hence

$$r = \frac{E - Wx}{x} = \frac{E}{x} - W.$$

Now a little consideration will show us that the resistance of the battery branch may be anything we please without affecting the resistance of the rest of the network. Let us therefore make $W = 0$, hence

$$r = \frac{E}{x}.$$

It will therefore be necessary to determine the value of x from the equations (α), (β), (γ), first putting $W = 0$. The student will save himself much mechanical labour in solving such equations if he will master the elementary principles of determinants.

For our present purpose it will be sufficient to exhibit the general solution obtained most easily by the method of determinants of three equations of the first degree between three variables. Let these three equations be as follows :—

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

then will

$$x = \frac{d_1A_1 + d_2A_2 + d_3A_3}{a_1A_1 + a_2A_2 + a_3A_3},$$

also

$$y = \frac{d_1B_1 + d_2B_2 + d_3B_3}{b_1B_1 + b_2B_2 + b_3B_3},$$

and

$$z = \frac{d_1C_1 + d_2C_2 + d_3C_3}{c_1C_1 + c_2C_2 + c_3C_3},$$

where

$$\begin{aligned} A_1&=b_2c_3-b_3c_2, & B_1&=a_3c_2-a_2c_3, & C_1&=a_2b_3-a_3b_2\,; \\ A_2&=b_3c_1-b_1c_3, & B_2&=a_1c_3-a_3c_1, & C_2&=a_3b_1-a_1b_3\,; \\ A_3&=b_1c_2-b_2c_1, & B_3&=a_2c_1-a_1c_2, & C_3&=a_1b_2-a_2b_1. \end{aligned}$$

the student will find that the denominators of x, y, z in the above expressions are the same.

Now let us suppose that $P=1$, $Q=2$, $R=3$, $S=4$, $T=5$, (W being $=0$), then the equations (α) (β) (γ) will become

$$\begin{aligned} 7x-\ 3y-\ 4z&=E \\ -3x+10y-\ 5z&=0 \\ -4x-\ 5y+10z&=0. \end{aligned}$$

Hence

$$\begin{aligned} A_1&=10\times10-\ 5\times5\ =75 \\ A_2&=\ 5\times\ 4+\ 3\times10=50 \\ A_3&=\ 3\times\ 5+10\times\ 4=55. \end{aligned}$$

Hence also

$$x=\frac{75E}{7\times75-3\times50-4\times55}=\frac{75E}{155},$$

and

$$r=\frac{E}{x}=\frac{155}{75}=2{\cdot}07.$$

5. To get the real current in any common cell boundary it will be necessary to take the difference of the imaginary cycle currents. Thus if we require the current in T it will be necessary to find both y and z, and then take their difference. Maxwell diminishes the labour by giving the symbol y to one cell and $y+z$ to the other, then $z=y+z-y$ will give the *real* current in the boundary. This will be better understood by the following example :—

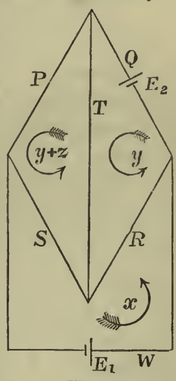

Fig. 8.

General case of Wheatstone's Net—Problem.—To obtain the equation of the currents in the most complicated case that occurs in practice, namely, when there is an E. M. F. (E_2) in one of the arms. From the diagram (Fig. 8) we obtain, by applying Maxwell's rule,

$$
\begin{aligned}
(W+R+S)x - Ry - S(y+z) &= E_1 \,. \quad . \quad . \quad (1)\\
(Q\ +R+T)y - Rx - T(y+z) &= E_2 \,. \quad . \quad . \quad (2)\\
(P\ +S\ +T)y+z) - Ty - Sx \quad &= 0 \,. \quad . \quad . \quad (3)
\end{aligned}
$$

which become

$$
\begin{aligned}
(W+R+S)x - (R+S)y - Sz \quad &= E_1 \quad . \quad . \quad (4)\\
-Rx + (Q+R)y - Tz \quad &= E_2 \quad . \quad . \quad (5)\\
-Sx + (P+S)y + (P+S+T)z &= 0 \,. \quad . \quad . \quad (6)
\end{aligned}
$$

Hence

$$z = \frac{E_1(SQ - RP) - E_2\{P(W+R+S) + WS\}}{D},$$

where

$$D = S\{R(P+S) - S(Q+R)\} - T\{S(R+S) - (W+R+S)(P+S)\} + (P+S+T)\{(W+R+S)(Q+R) - R(R+S)\} \qquad (\text{I.})$$

then if $E_2 = 0$

$$z = \frac{E_1\{QS - PR\}}{D} \quad . \quad . \quad . \quad . \quad (\text{II.})$$

If z also $= 0$, then

$$QS - PR = 0, \text{ or } \frac{P}{Q} = \frac{S}{R} \quad . \quad . \quad . \quad (\text{III.})$$

which is the principle of Wheatstone's bridge.

6. *Best Position of Battery and Galvanometer.*—Since, when the equation $\frac{P}{Q} = \frac{S}{R}$ is satisfied, $z = 0$, the current in W is therefore independent of the resistance of T.

These branches are *now* said to be **conjugate**, and it may be proved that this relation is **reciprocal**; that is to say, if the battery be placed in the arm T and the galvanometer in W, then (*the resistances of these arms remaining the same*) the current in T will be independent of the resistance of W. If the galvanometer is very delicate it will of course be quite immaterial whether it is placed in the arm W or the arm T; but if the galvanometer be not very delicate it is a matter of importance to know in which position it will give the most accurate adjustment. Let us therefore suppose that the relation $\frac{P}{Q} = \frac{S}{R}$ is nearly but not exactly fulfilled, and find by which corners of the bridge the battery and galvanometer should be connected.

The following rules, deduced from the equations already given, will be found fully proved in Maxwell's *Electricity and Magnetism* :—

Rule A.—Galvanometer Resistance > Battery Resistance	*Galvanometer* should be connected between the corner which represents the junction of the two smallest arm resistances, and that which represents the junction of the two greatest.
Galvanometer Resistance < Battery Resistance	*Battery* should be connected between the corner which represents the junction of the two smallest arm resistances, and that which represents the junction of the two greatest.

Example.—Thus if $P = Q = 1000$, $R = S = 100$, $T = 5000$, and $W = 50$, then the battery would be joined to A and B, while the galvanometer joins C to D (Fig. 7, p. 445).

7. Another point is to ascertain what is the best value of galvanometer resistance to use for a particular purpose. The following rule is proved by L. Schwendler:—[1]

Rule B.—The best value of the galvanometer resistance is when this equals the resistance of the sums of the arms on either side of the galvanometer branch, these two added resistances being placed in multiple arc.

Example.—If $P = Q = 2$, $R = S = 3$, then $P + Q = 4$ and $R + S = 6$. Hence

$$T = \frac{1}{\frac{1}{4} + \frac{1}{6}} = 2{\cdot}4.$$

[1] On "The Galvanometer Resistance to be employed in Testing with Wheatstone's Diagram," *Phil. Mag.*, May 1866.

8. The next point is the best value of battery resistance.[1] Disregarding the galvanometer resistance, since it is conjugate, here we have the following simple rule :—

Rule C.—The best value of the battery or internal resistance is when this is equal to the external resistance, not including that of the galvanometer.

Example.—$P=100$, $Q=10$, $R=50$, $S=500$,

$$W=\frac{1}{\frac{1}{110}+\frac{1}{550}}=91 \text{ nearly.}$$

9. Finally we have to study the best arrangement of arms.[2] Here we have

$$Q=\sqrt{RW\frac{R+T}{R+W}} \quad . \quad . \quad . \quad \text{Rule D}$$

$$P=\sqrt{TW} \quad . \quad . \quad . \quad . \quad \text{Rule E}$$

$$S=\sqrt{RT\frac{R+W}{R+T}} \quad . \quad . \quad . \quad \text{Rule F}$$

Here R is supposed to be the resistance to be determined, while that of the battery and galvanometer are regarded as fixed. If both battery and galvanometer may be changed, then the best arrangement is

$$P=Q=R=S=T=W \quad . \quad . \quad . \quad \text{Rule G}$$

10. *Thomson's Galvanometer Resistance Method.*—Here the galvanometer whose resistance is required is removed, say to arm Q (Fig. 9), and a key substituted in its previous place at T. Adjustment is made until on opening and closing the key the current through Q is unaffected. Now when this takes place it is clear that the current is independent of the resist-

[1] For a discussion of this the reader is referred to Cumming, *Introduction to the Theory of Electricity*, Art. 170.

[2] See Oliver Heaviside on "The Best Arrangement of Wheatstone's Bridge for measuring a Given Resistance with a Given Galvanometer and Battery," *Phil. Mag.*, July 1873; and Kempe's *Handbook of Electrical Testing* (new edition), p. 173, where an algebraical proof is given.

ance of T, and hence that the condition PR = QS must be fulfilled. It may likewise be shown that the adjustment will be the more delicate the greater the difference in the amount of current that passes through the galvanometer due to opening or closing the key, when the balance is not quite perfect, and it can be shown that this gives us the following rule:—

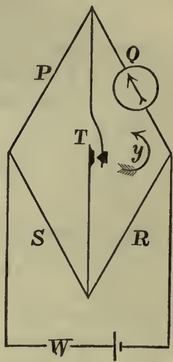

Fig. 9.

Rule H.—The proportional (P and Q) arms to differ as much as possible.—The algebraical proof of these points may be left to the student as an exercise on the use of the preceding equations.

11. *Mance's Battery Resistance Method.*—The battery whose internal resistance is required is placed in the Q arm and a key in the W arm (the galvanometer being in T), and the resistances of the arms are adjusted until no change is produced in the galvanometer by opening or shutting the key in W. Here $E_1=0$, and hence the current through the galvanometer is

$$z=-E_2\frac{P(W+R+S)+WS}{D}$$

Now when $W=0$, we shall have

$$z_1=-E_2\frac{P(R+S)}{T(P+Q)(R+S)+PQ(R+S)+RS(P+Q)},$$

and when $W=\infty$, we shall have

$$z_2=-E_2\frac{(P+S)}{T(P+Q+R+S)+(Q+R)(P+S)}.$$

Now if $z_1=z_2$, and if this condition is to hold for all values of T, it must hold for $T=0$. Hence

$$\frac{P(R+S)}{PQ(R+S)+RS(P+Q)}=\frac{P+S}{(Q+R)(P+S)}=\frac{1}{Q+R}.$$

This equation will be satisfied if $PR = QS$ as before, also the rule for the best effect will be Rule H.

12. *Resistance containing an E. M. F.*—This will be the general case where there are two electromotive forces in the circuit. Here the galvanometer is supposed to be in T, while Q is the unknown resistance containing the E. M. F. E_2. Hence we have

$$z = \frac{E_1(QS - PR) - E_2(P(W + R + S) + WS)}{D}$$

If the resistance be adjusted until $z = 0$, then

$$E_1(QS - PR) - E_2(P(W + R + S) + WS) = 0 \quad . \quad . \quad (1)$$

Suppose now that the direction of E_1 is reversed and a new adjustment made as above, R becoming R′, then we shall have

$$-E_1(QS - PR') - E_2(P(W + R' + S) + WS) = 0 \quad . \quad . \quad (2)$$

From (1) and (2) the unknown resistance Q can be found.

The simplest case will be when R is nearly equal to R′ and E_2 is small, then, since the second members of the equation may be supposed to be left out, we shall have

$$QS - PR = PR' - QS, \text{ or } 2QS = P(R + R').$$

Hence

$$Q = \frac{P}{S} \cdot \frac{R + R'}{2} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

In general, since (1) and (2) hold for all conceivable values of E_1 and E_2, we must have

$$\frac{PR - QS}{QS - PR'} = \frac{P(W + R + S) + WS}{P(W + R' + S) + WS}$$

Hence, by compounding,

$$\frac{P(R + R') - 2QS}{P(R - R')} = \frac{P(R - R')}{2(PW + PS + WS) + P(R + R')}$$

or

$$Q = \frac{P}{S} \cdot \frac{(R + R')(PW + PS + WS) + 2PRR'}{2(PW + PS + WS) + P(R + R')} \quad . \quad . \quad (4)$$

When $P = S$

$$Q = \frac{(R+R')(P+2W)+2RR'}{2(P+2W)+R+R'} \quad . \quad . \quad . \quad (5)$$

and if W is very small

$$Q = \frac{P(R+R')+2RR'}{2P+R+R'} \quad . \quad . \quad . \quad . \quad (6)$$

Equations (3), (4), (5), (6) give **Rule K,** enabling us to determine the value of a resistance which contains an E. M. F.

13. *False Zero Method.*—The rather troublesome calculation of Q from the formulæ of last article may altogether be avoided if we once for all measure the resistance of the testing battery, and arrange so that it may be substituted for a wire resistance of equal value by pressing a key. Then, if an equal deflection be obtained when E_1 is in circuit and when its equivalent resistance is substituted, we shall have the coefficient of E_1 in equation (1) of last article $= 0$. Hence

$$Q = \frac{P}{S} \cdot R \quad . \quad . \quad . \quad . \quad \text{Rule L}$$

14. *Application of Rules* K *and* L.—They have two important uses. (1.) For determining the metallic resistance of cables. Here, when a second cable is not available, the return circuit has to be made by means of the earth, which will cause the introduction of an E. M. F. into the resistance on account of earth currents and polarisation at the plates used to get connection with the earth. (2.) For ascertaining the resistance of the earth in the case of a lightning conductor. Connections would be made between the lower portions of the conductor and a *good earth,* such as a system of water pipes. For the same reason as before, the resistance under measurement would contain an E. M. F. To eliminate the error due to the use of this second earth, it is better to follow the plan of Jamieson, who uses a *third earth,* such as that of the system of gas pipes. Measurements are made between earths A and B say, then

between A and C, and finally between B and C. From the three values it is easy to find A, B, and C.

15. *Experimental Proof of Net Laws.*—It will be a very instructive exercise for the student to measure actually the current in all the branches of a Wheatstone's bridge, or the resistance from one corner to another, and thus prove the laws just given.

For this purpose a model of the net may be used (see Fig. 10). The resistances P, Q, R, S, T may be coils having their resistances in the ratio 1, 2, 3, 4, 5. For E_1 and E_2

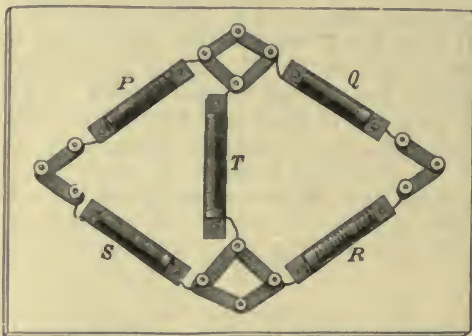

Fig. 10.

Grove's cells may be used, two cells for E_1 and one cell for E_2. The resistance of these cells should be ascertained in the usual manner. For measuring the currents a galvanometer of negligible resistance will be required. The arrangement of binding screws shown in the model will enable the galvanometer and batteries to be placed in any one of the arms by removing any of the copper strips shown clamped under the binding screws.

B.

FORCE—POTENTIAL—LINES OF FORCE—ELECTRICAL UNITS—INDUCTION.

1. The following propositions regarding forces, which vary inversely as the square of the distance, will be found in all text-books, so that while it is desirable to state them, it will not be necessary to furnish proofs.

> *Proposition I.—A uniform spherical shell exercises no attraction upon a particle placed within it.*

For an experimental proof of this in the case of electricity, the student is referred to the first chapter of this volume.

> *Proposition II.—The attraction of a uniform spherical shell upon a particle placed without it is the same as if its whole mass were collected at the centre.*

It follows from this proposition that if ρ denotes the volume-density and r the radius of the shell, of which the thickness is δ, while d is the distance of a point from the centre, and if unit of attracting matter be placed at this point, then the attraction on it of the shell will be $\frac{4\pi r^2\rho\delta}{d^2}$, in which expression the numerator represents the whole mass or attracting matter of the shell, while the denominator is caused by the law according to which the attraction varies with the distance.

2. It will be noticed that both for gravity and for electricity the attraction is proportional to the product of the attracting matter into the attracted, and in conformity with this we find in page 18 of this volume that if a quantity of electricity $= m$ attracts another quantity $= n$ the whole attraction is proportional to the product mn. This enables us, as we have seen (page 18), to define *unit quantity of electricity as that which will*

exert unit of force (one dyne) on a quantity equal to itself and placed at unit distance from it (one centimètre) in air. It follows at once that *unit surface density of electricity means unit of quantity on unit of surface, that is to say, on one square centimètre.*

3. Let us now suppose that we have concentrated in a point a unit of positive electricity, and that we have likewise an insulated conducting spherical shell charged, let us say, with negative electricity. Let ρ represent the density of the electricity on the surface, and let r be the radius of the shell, while d is the distance of the positive unit from the centre of the shell. Then by § 1 we may conceive all the electricity of the shell to be concentrated at its centre, and hence the attraction upon the unit will be $\frac{4\pi\rho r^2}{d^2}$. If the unit be brought quite close to the surface, being still without, then d will be virtually equal to r, and the above expression will become $4\pi\rho$. No doubt the unit acting inductively would somewhat alter the uniform distribution of the electricity over the surface of the shell, but for our present purpose this may be disregarded, since the journey of the unit is an imaginary one. Hence the force of the electrified system upon the unit, at or near the surface, but still without, will be $4\pi\rho$, this force tending to the centre. If we now suppose the unit continuing its journey to have pierced the surface, and to be within the enclosure, the force, according to Proposition I., will be zero. Thus the piercing of the surface has produced a change upon the force $= 4\pi\rho$.

Exactly the same reasoning will apply to a gravitating spherical surface such as that of the earth. Imagine, for instance, that the surface of the earth is a strictly spherical surface covered with water, and that we dive beneath it through a certain distance δ. We shall thus have thrown above us a certain shell of the earth's substance. Before we began to dive the attraction of this shell taken all round the earth was the same as if the matter composing it was concentrated in the earth's centre. After we have dived, however, the attraction of this shell will count for nothing, and the whole attraction will be so far diminished in consequence.

4. It will at once be seen that before we began to dive our view

was bounded by a circular horizon embracing watery particles, which we might regard without sensible error as forming a flat circular surface. The matter in this flat circular surface through a depth δ would exercise a *downward* attraction upon us before we began to dive, but after we had reached this depth the matter, being above us, would exert now an equal *upward* attraction. On the other hand, as regards the very distant particles of the shell, our position has so little changed that we may view their attraction as unaltered by the operation of diving. There is thus a portion of the whole attraction which changes sign, and another portion which remains *sensibly* constant. Precisely the same thing happens in electricity; here let us call the part which changes sign x and the constant part C. Hence we obtain

$$C + x = 4\pi\rho,$$
$$C - x = 0,$$

from which it follows that $x = 2\pi\rho$. *Hence the normal electric attraction of any plate of density ρ on unit quantity of electricity placed at a distance from it, very small as compared with its diameter, will be $2\pi\rho$.* We have already made use of this proposition in explaining the theory of Thomson's electrometer.

5. We now come to the subject of *potential.* Let us suppose that we have a centre of positive electricity at a point O_1 (Fig. 11), containing, let us say, m_1 units, and let it act on a unit of

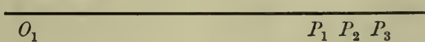

Fig. 11.

positive electricity placed at a point P_1. There will, of course, be repulsion along the line O_1P_1, and let us suppose that in course of time the unit is driven to a point P_2 on the same line (O_1P_2 being greater than O_1P_1, but the two points being very near each other). The force at the one point will be $\frac{m_1}{(O_1P_1)^2}$ and that at the other $\frac{m_1}{(O_1P_2)^2}$. These two expressions are very nearly the same, and it may be shown that in this case the average attraction is best represented by $\frac{m_1}{O_1P_1 \cdot O_1P_2}$. Now, a distance P_1P_2 is gone over in the line of action of this force, and hence an amount of

kinetic energy will be gained by the unit equal to the force multiplied by the distance gone over, *i.e.*,

$$=\frac{m_1 P_1 P_2}{O_1P_1 \cdot O_1P_2}=\frac{m_1\{O_1P_2-O_1P_1\}}{O_1P_1 \cdot O_1P_2}=m_1\left\{\frac{1}{O_1P_1}-\frac{1}{O_1P_2}\right\}.$$

Next consider another small element of the path between P_2 and another point P_3 ; here, by precisely similar reasoning, we might show that the energy gained by the unit as it proceeds through this element will be $m_1\left\{\frac{1}{O_1P_2}-\frac{1}{O_1P_3}\right\}$, and hence the whole energy through both elements of its path will be

$$m_1\left\{\frac{1}{O_1P_1}-\frac{1}{O_1P_2}+\frac{1}{O_1P_2}-\frac{1}{O_1P_3}\right\}=m_1\left\{\frac{1}{O_1P_1}-\frac{1}{O_1P_3}\right\}.$$

Here the law is obvious, so that if it go from any point P to any point Q we shall have the energy

$$=m_1\left\{\frac{1}{O_1P}-\frac{1}{O_1Q}\right\}$$

It may be easily shown that the above expression will be true even though O_1P and O_1Q should not be in the same line, provided only that the distances of the points P and Q from O_1 remain constant. For if it were not true it would be possible to create energy independently by carrying the unit along one road from P to Q, and then back again by some other road, which is manifestly absurd.

Having proved this point, let us now imagine that we have, besides O_1, other centres, namely O_2, containing m_2 units; O_3, containing m_3; and so on. Hence similar expressions will hold, and we shall have the whole energy gained by the unit going from P to Q

$$=m_1\left\{\frac{1}{O_1P}-\frac{1}{O_1Q}\right\}+m_2\left\{\frac{1}{O_2P}-\frac{1}{O_2Q}\right\}+\text{etc.}$$

This expression may be written in a general form as follows :—

$$\text{Energy}=\Sigma m\left\{\frac{1}{OP}-\frac{1}{OQ}\right\},$$

an expression which, we have seen, is quite independent of the path pursued by the unit between P and Q.

6. If the unit be carried from P to a point at an infinite distance from the repelling centres, then $\frac{1}{OQ}$ becomes $=0$, and the expression becomes

$$\text{Energy}=\Sigma\frac{m}{OP}.$$

We have thus the following definition of what we may call the *absolute potential* at a point.

If a point at which there is placed unit of positive electricity be at distances, say r_1, r_2, r_3, etc., *from centres at which quantities of positive electricity*, m_1, m_2, m_3 etc., *are placed*, then

$$\frac{m_1}{r_1}+\frac{m_2}{r_2}+\frac{m_3}{r_3}+\text{ etc. }=\Sigma\frac{m}{r}$$

will be the absolute potential at that point due to the electric system.

We see from our method of treating the subject, in which the unit is charged with positive electricity, that at a point for which $\Sigma\frac{m}{r}$ is positive, kinetic energy is gained by the unit as it recedes from the point to one at an infinite distance, that is to say, positive electricity tends to go from a place of higher to a place of lower positive potential.

7. If we define *absolute zero of potential* as that which exists at a place infinitely far removed from all electricity, it is clear that we have access to no such place. All that we can do is to take the earth, which is a large conducting body, as our standard, and consider its potential as zero of potential. A positively electrified body is therefore one of a higher potential than the earth, and *from* which positive electricity has a tendency to flow *towards* the earth, while a negatively electrified body is one having a lower potential than the earth, and *towards* which positive electricity has a tendency to flow *from* the earth.

8. But while we have no knowledge of absolute potential, we can measure differences of potential, and may define unit difference as follows: *Unit difference of electric potential exists between two points when the unit of work is spent in moving unit of electricity from the one to the other against electric repulsion.* The reader will now begin to see that differences of electric potential

mean in reality differences of electric level, and just as there is no flow of water between two points that are at the same gravitation level, so there is no flow of electricity between two points that are at the same potential or electric level. Furthermore, we may, for any electrical system, draw a surface embracing all those points that are at the same potential. Such a surface would be an *equipotential surface*, and would be similar to the surface of the ocean or of a lake in the case of gravity. There is therefore no tendency for electricity to flow from one point of an equipotential surface to another.

We have, in the case of gravity, an easy method of measuring differences of level by means of the plumb-line, which is not applicable to electricity. But the strictly scientific definition of unit difference of potential which we have given for electricity may be very easily adapted to gravity, and we may define our unit difference of gravitation potential or level to be that difference where unit of work (the foot-pound if we adopt the British system) will be spent in raising unit of mass (the pound) from the one level to the other against the force of gravity, this force being assumed to be practically constant.

9. It is usual to denote potential by V (the first letter of Volta's name), let us therefore denote by V_1, V_2 the potential due to an electric system at two points P_1, P_2, very near each other. The work done by the test unit in going from the one point to the other will be $V_1 - V_2$. But if f denote the force, the work done will likewise be $f \times P_1P_2$. Hence $V_1 - V_2 = f \times P_1P_2$, and hence

$$\frac{V_1 - V_2}{P_1P_2} \doteq f.$$

Here the left-hand term is evidently the rate of change of the potential in the direction P_1P_2, and hence *the rate of change of the potential in any direction represents the resultant force in that direction.* It follows from this that there is no resultant force along an equipotential surface, since here the rate of change of the potential is zero ; and also that this force is greatest in a direction perpendicular to such a surface. We may therefore imagine an electric system to be surrounded by a

series of surfaces of this nature, and also by a series of lines going perpendicular to these surfaces. Such surfaces would be equipotential surfaces and such lines *lines of force*, and just as the surface of the ocean is in the case of gravity an equipotential surface, so the direction of the plumb-line is that of a line of force.

10. It is desirable that we should so systematise our conception of lines of force as to get the greatest possible amount of use out of it.

Let us therefore suppose that we have an attractive system at O of such a strength that its force of attraction or repulsion upon the testing unit placed at a distance from O equal to unity, that is to say, one centimètre, will be one dyne. We may call this a *unit system*. Let Fig. 12 represent the spherical sur-

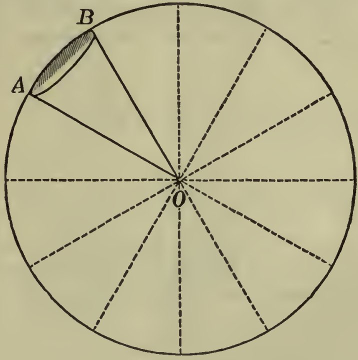

Fig. 12.

face at unit distance from O. It is clear that this and similar spherical surfaces will all be equipotential surfaces, and that radii drawn from O will be lines of force. Now we might, if we chose, draw an infinite number of such lines of force. Let us however restrict ourselves to a given number, drawn regularly outwards from O in all directions. It is clear that as these lines proceed outward they will become less closely packed together. At unit distance from the centre they will

be spread over a surface $= 4\pi$, at double the distance they will be spread over a fourfold surface, and so on. We may say that their closeness or density will vary inversely as the square of the distance. But it is also in this very way that the force varies, and hence we see how the force at any point may be represented by the closeness or density of the lines of force at that point. To make this conception useful let us imagine that *at unit distance from the centre, when the force is unity, we have one line of force for every unit of area.* Thus it is clear that the whole number of lines of force drawn from O will be 4π. It is further clear that the number must be drawn so as to be proportional to the whole mass or amount of attracting matter at O. If therefore this be m, then will m likewise denote the number of lines of force that cross unit area at unit distance, so that the whole number of such lines will now be $4\pi m$.

In the above figure let AB represent unit area at unit distance (whether the boundary be circular or otherwise is of no conse-

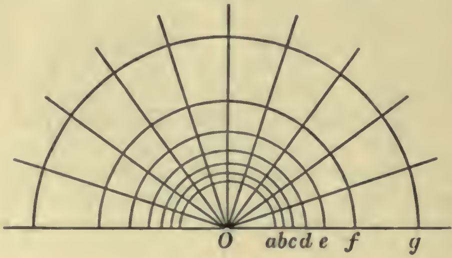

Fig. 13.

quence), then the solid angle made at O by the radii drawn to the various points of this boundary will be a *unit solid angle.*

In the above diagram the lines of force are all straight lines, but it will be seen further on that in magnetic systems, as well as in systems connected with electricity, the lines of force are by no means always straight lines.

The diagram (Fig. 13) exhibits a convenient way of graphically representing the subject. Here O is the centre of an electrified sphere, supposed to be charged with eight units, and having a

radius of one unit, and from O lines of force radiate in all directions (to save space only the semi-circle is shown). A number of equipotential surfaces are drawn around it, so as to express unit difference of potential or unit of work performed by the test unit in passing from a given surface to the one next it. Such surfaces will be clustered closely together where the force is great, and spread out where it is small. These surfaces will really be those of a series of concentric spheres, formed by the revolution of the semi-circles about the base line as axis. To find the radii of the respective spheres we observe that since

$$V=\frac{m}{r},$$

when $m = 8$ and $r = 1 = Oa$, then $V = 8$, or the inner sphere is an equipotential surface of value 8. We now require to find the radii of the surfaces having values 7, 6, 5, etc. Now

$$r=\frac{m}{V},$$

hence when $m=8$ and $V=7$, then $r=1{\cdot}14=Ob$,
,, ,, $V=6$, ,, $r=1{\cdot}33=Oc$,
,, ,, $V=5$, ,, $r=1{\cdot}60=Od$,

and so on.

We have not adhered to a strict system in drawing the lines of force.

If the unit of work were the foot-pound and the system that of the earth, we should draw round the earth at that distance from the centre where we exist, a series of surfaces, the vertical distance between any two of these being one foot. At the distance of the moon, however, where the force of terrestrial gravity is only $\frac{1}{3600}$ of what it is here, the distance between two such surfaces would be 3600 feet, or the best part of one mile, and so on.

11. From what has now been said it will not be difficult to find the electrical force acting on a positive unit of electricity placed just outside an electrified conductor at a point whose electrical density is ρ. The argument of § 4 may be adapted to this question, even although the conductor may not be a sphere, and we may infer that the force will change when the unit

passes through the surface by a quantity equal to $4\pi\rho$. But it is well known from the experiments described in the first part of this volume that the force within such a conductor is zero. Hence the force just outside will be $4\pi\rho$.

In like manner it is very easy to find the potential at any point either within or without a conducting sphere electrified with a given quantity of electricity. For let r denote the radius of the sphere, and Q the quantity of electricity. Since there is no force inside it follows that the surface and all the points within the sphere, including the centre, have the same potential. Let us therefore consider the central point, which is at the constant distance r from all parts of the electrified surface. Its potential is therefore $\frac{Q}{r}$, which is therefore likewise the potential of any point on the surface or within the sphere.

With regard to points without the sphere we may obviously regard the electricity as acting at the centre, and hence, if the distance from the centre be d, the potential will be $\frac{Q}{d}$.

Again, if we carry a quantity of electricity $= Q$ from one electrical level or potential V_1 to another V_2, the work done will be equal to $\{V_2 - V_1\}Q$. Now in a Leyden jar, if we double the quantity of electricity which we put into it, we double at the same time the difference of potential or level between the inside and the outside (the outside being always at the level of the earth), and thus the energy of the discharge (represented by the heat produced) will be four times as great in the second case as it is in the first.

12. In dynamical electricity the *unit magnetic pole* replaces the test unit of the previous articles. This is that pole of a magnet which points to the north, and it is regarded exclusively and without reference to that other pole of the same magnet which points to the south, and which we may if we choose imagine to be at an infinite distance from us, thus producing no influence. Bearing this in mind, *a unit magnetic pole is a pole which exerts unit force (one dyne) on another similar pole placed at unit distance from it.*

It will thus be seen that a unit pole may be regarded in the same light as unit mass or unit quantity of electricity, and may

be looked upon as possessing 4π lines of force. Hence if the intensity of the pole be m the number of lines of force will be $4\pi m$. In reality, however, we never have a north pole without a south pole, and in some respects instead of imagining an infinitely long magnet, it is desirable to consider the action of an extremely small one where the poles are very near together instead of being very far apart. Now, let us suppose such an exceedingly small magnet to be carried from the north to the south pole of a large bar magnet in such a way that its axis shall always lie along the direction in which it is carried. In this case it will describe in its path a line of force which will be a curved line. The lines of force of a bar magnet the student has already obtained experimentally in Chap. II. They consist of loops stretching from pole to pole. If therefore we were to imagine a gigantic bar magnet and a man to set forth from one pole of it on a dark night with an illuminated compass needle in his hand, always walking in the direction in which the needle pointed, he would ultimately be led to the other pole. It will be noticed that the lines of force are much concentrated about the poles, and it is there that the resultant force is the strongest.

13. While, however, exceedingly small magnets, such as iron filings, are useful to enable us to perceive graphically the lines of force of a large magnet, for scientific purposes we must keep to our unit pole, and it will likewise be necessary that the student should conceive the idea of a magnetic shell, which is supposed to act on this unit pole. Suppose that we have an infinite number of thin short magnets of equal strength, and that we insert them perpendicularly into a circular space in the plane of this paper in such a manner that (1.) they shall be regularly distributed throughout the area, and that (2.) the centres shall all be in the plane of the paper, the N. poles all above and the S. poles

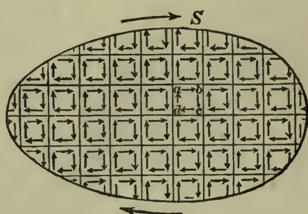

Fig. 14.

all below. This arrangement is a *simple magnetic shell* (see p. 333 of this volume).

Again, we may imagine a great number of such shells to be placed, the one behind the other, so as to form a pile, all the N. poles of the one coinciding in position with all the S. poles of the next, and so on. It is clear that the internal shells will not exert any influence on our pole, inasmuch as the N. poles are all cancelled by the S. poles. The influence of the system will thus depend on the bounding external magnets.

14. Let us now endeavour to find the potential of a small circular thin magnetic shell, of thickness AB (supposed exceedingly small), upon a unit pole placed at O, and let us first (Fig. 15) suppose that O is a point in continuation of the normal

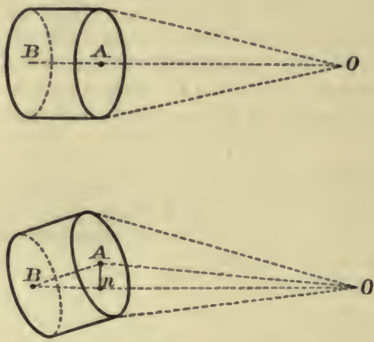

Fig. 15.

BA. Let the small circular surface of the shell be σ, and let $\pm\rho$ be the strength of the pole of the shell for unity of surface; the strength for the whole surface will thus be $\pm\rho\sigma$.

Hence the *positive* potential of the surface at A on the unit pole at O will be $\frac{\rho\sigma}{\text{AO}}$, while the *negative* potential of the other side of the shell will be $\frac{\rho\sigma}{\text{BO}}$. Hence the whole potential will be $\rho\sigma\left\{\frac{1}{\text{AO}} - \frac{1}{\text{BO}}\right\} = \rho\sigma\left\{\frac{\text{OB}-\text{OA}}{\text{OA}\cdot\text{OB}}\right\}$. If AB be small this will become

$\rho\sigma\frac{\cdot BA}{OA^2}$ nearly. Now $\rho\cdot BA$ is the magnetic moment of the shell for unit of surface, while $\frac{\sigma}{OA^2}$ represents (§ 10) the solid angle subtended at the point O by lines drawn from the circumference of the shell. We may therefore assert that in this case the potential is represented by the moment of the shell per unit of area multiplied by the solid angle at O embraced by the boundary of the shell, or in other words, is proportional to the apparent size of the shell as viewed by the eye at O.

If, however (Fig. 15), O be not a point in prolongation of the normal, then the potential will be reduced to $\rho\sigma\ \frac{Bn}{OA^2}$, B$n$ being the difference between AO and BO. That is to say, the potential will be reduced in the proportion of AB to Bn. But a little consideration will show that the solid angle at O is reduced exactly in the same proportion. It follows therefore that in this case likewise we may assert that the potential is represented by the magnetic moment of unit of area of the shell multiplied by the solid angle, so that we may sum up for all the various elements of the shell, and assert that the *whole potential of the shell on the unit pole at O is equal to the magnetic moment of unit of area of the shell multiplied by the whole solid angle at O embraced by the boundary of the shell.* The potential will therefore be ρAB × solid angle.

15. Suppose now the unit pole to be placed at a point in the plane of the paper beyond the rim of the shell. Here the solid angle, and hence the potential, will be zéro. Now let the unit pole be carried above the plane of the paper and towards the shell. In the course of this carriage the solid angle and hence the potential will increase, and the potential being positive, there will be a force resisting the carriage of the pole. Imagine that this goes on until the unit pole reaches the shell and is made to go through it, and that on the other side of the paper it performs below a journey precisely similar to that which it performed above, returning back to the point from which it started. It is clear that as soon as it has crossed the shell the sign of the potential and the direction of the force is reversed, so that it will now perform the second portion of its journey against an attractive force. It will thus in fact be resisted all through

the journey, and energy will have to be spent upon it all the way. We have thus a state of things very different from what we considered in the beginning of this Appendix, for since work has been done upon the pole in carrying it back to the point from which it started, we cannot say that the potential is the same. In this case the potential of the point is said to have *discontinuous values* at the surface of the shell.

If, however, in its journey the pole does not pass through the attracting system, but goes round it, returning back to the point from which it started, on the whole no work will either be done by or upon the pole, and the potential of the point has thus only one value. It is in fact now similar to the case that we considered in statical electricity.

It is clear that when the unit pole, travelling towards the shell, arrives at A the solid angle will be 180° all round, and in accordance with the method of estimating such angles (§ 10) its value will be 2π. If, therefore, the moment of the magnetic shell for unit of area be unity its potential will be $+2\pi$ when the unit pole touches it on one side, and when it passes through to the other side this will become -2π.

Now, if instead of a unit N. pole we were to carry a unit S. pole through the shell in the same way, we should have work done *by* the pole instead of *upon* it throughout the journey. Is there therefore in the one case a destruction and in the other a creation of energy? By no means. The reader must bear in mind that a unit N. pole apart from a corresponding S. pole is not a reality, and that what we should do in fact would be to carry a magnet through the shell, in which case the work done upon the one pole would be counterbalanced by the work done by the other. There would therefore be no creation or destruction of energy. In fine, when a magnet is passed through the shell from a point without and goes back again to the point, as we have supposed, without again passing through the shell, there is work done *by* the one pole and as much done *upon* the other. If, however, on its journey the magnet does not pass through the shell at all, there is on the whole no work done by or against either pole.

16. Instead of finding the potential of the shell upon the

magnet, we may wish to know the equal and opposite potential of the magnet upon the shell (see § 24 of this Appendix). Here it will be seen that the solid angle made at O by the boundary of the shell represents and is equal to the number of lines of force of the magnet which are enclosed by the shell, and as the number of such lines will likewise vary with the strength of the pole at O, we may finally assert that the *potential of the magnet at O upon the shell will be equal to the magnetic moment of unit of area of the shell multiplied by the whole number of lines of force intercepted by the shell.*

17. Let us now consider the lines of force of a current, and let us imagine that we have near us an indefinitely long vertical wire conveying a current, the other parts of the circuit (which must be a closed one) being infinitely remote. Magnets will tend to set themselves at right angles to such a current, so that the lines of force will be horizontal circles, and the equipotential surfaces planes passing through the current.

18. Next, with regard to a closed current, which, for the sake of simplicity, we may suppose to be circular, it is clear from p. 333 of this volume that such a current is fully represented by a simple magnetic shell filling up the area of the circle, and having a moment per unit of area proportional to the strength of the current. A little consideration will show that this statement affords us at once a method of defining current strength. It would be convenient to regard *that current strength as unity where the representative magnetic shell has likewise a moment per unit of area equal to unity.* But how will this definition agree with that which we have virtually accepted in the text (Art 118)? The definition adopted in the text is as follows: —*Unit current is one which, in a wire of unit length bent so as to form an arc of a circle of unit radius, would act upon a unit pole at the centre of the circle with unit force.* Suppose, for instance, that we have a current whose strength is unity flowing in a circle of unit radius. It will be seen from Art. 118 that its force upon a unit pole at the centre will be 2π. Now this will at the same time denote the force of a circular magnetic shell of unit radius, whose magnetic moment per unit

of area is unity. It thus appears that the two definitions agree together perfectly.

19. Most of the results contained in this volume contemplate a steady current, the energy of which is spent upon the circuit—generally taking the shape of heat. We must, however, remember that the current has a field as well as a circuit, and that the first effect of making contact is to establish *the current and its field.* Then we have the period of steady current and steady field; and finally, when contact is broken, we have the withdrawal of *the current and its field.* Now when we make the current to do work in its field, since there can be no creation of energy, this will be done at the expense of the current strength in the circuit. This lessening of the current in the circuit, or opposing current, is in reality what is known as an *induced current.*

20. In order to render our conception definite, let us imagine that we have a circular current of strength i, also let R denote the resistance of the circuit. Furthermore, let there be a unit pole in the field of this circuit, so placed as to be attracted by the current. Suppose also that this pole is being moved with a uniform velocity towards the current during the very small time τ for which we are regarding the current. Work will evidently be done *by the pole.* Let N denote the number of lines of force of the pole which pass through the circuit at the beginning of this motion, and let N′ denote the same at the end, then the work done will be by § 16 (since a current may be treated as a magnetic shell) $= (N' - N)i$. For the sake of simplicity let us imagine that our circuit is 1000 miles long, and that the battery is at one end, while the inductive action takes place at the other. Now clearly the amount of zinc burnt in the battery will always be proportional to the current i, *whether or not there be induction* at the other end of the circuit. The energy which is caused by the burning of the zinc will therefore be proportional to i, and this energy will be represented by $Ei\tau$, where E denotes the normal electromotive force of the battery when there is no induction. But while we assert that the whole energy is $Ei\tau$, *we do not assert that E represents the*

electromotive force when induction is taking place. There will, on the contrary, be a back current and a back electromotive force. Thus (p. 263) during the time τ the whole energy of the battery is $\mathrm{E}i\tau$. Also the amount of energy expended in heat in the circuit is $\mathrm{R}i^2\tau$. Now during this movement the battery has both to heat the circuit and to do work in the field. Hence

$$\mathrm{E}i\tau = \mathrm{R}i^2\tau + (\mathrm{N}' - \mathrm{N})i,$$

or

$$\mathrm{E}\tau = \mathrm{R}i\tau + (\mathrm{N}' - \mathrm{N}).$$

Let i_0 denote the steady current in the circuit when no work is being done in the field, then, by Ohm's law, $\mathrm{E} = \mathrm{R}i_0$. Hence

$$\mathrm{R}i_0\tau = \mathrm{R}i\tau + (\mathrm{N}' - \mathrm{N}).$$

Hence also

$$\mathrm{R}(i - i_0)\tau = -(\mathrm{N}' - \mathrm{N}),$$

and

$$(i - i_0)\tau = -\frac{\mathrm{N}' - \mathrm{N}}{\mathrm{R}},$$

or

$$i - i_0 = -\frac{\mathrm{N}' - \mathrm{N}}{\tau} \times \frac{1}{\mathrm{R}}.$$

Now $i - i_0$ is the strength of the induced current, and thus it will be seen that *this induced current is represented in strength by the number of lines of force added in unit of time divided by the resistance of the circuit.*

21. Let us see what will happen if there be no current in the circuit towards which the magnetic motion is made. This means that $i_0 = 0$, and we might expect from the above expression that we shall still have an induced current. It will, however, be in the opposite direction to that of the current which produces attraction of the pole, and hence it will repel the pole. In other words, we shall have to do work in bringing the magnetic pole towards the circuit, and the mechanical equivalent of this work will take the form of an induced current produced in the circuit by the motion of the magnet.

22. In discussing this question we have taken a unit pole as one of our two systems, but a little consideration will show

that the results may be extended to the lines of force proceeding from any two systems. For let us consider two systems, A and B, and take the lines of force from the first system A (whose potential upon the second system B we wish, let us say, to find), limiting ourselves to those lines which proceed to a very small area of the magnetic shell which represents the second system. Let these lines of force, which we may regard as parallel to each other, be equal to n, and let the magnetic moment for unity of area of the second system be ρAB, as in § 14. It is clear that $\rho \mathrm{AB} \times n$ will denote the potential produced by the first system on this element of the second, and that it is a matter of no consequence whether these lines of force proceed from a single pole or from a complicated system. They are parallel to each other, and are in number equal to n, and this determines their effect upon this element. Hence the whole effect of the first system on the second will be $\rho \mathrm{AB} \times \{n_1 + n_2 + \text{etc.}\} = \rho \mathrm{AB} \times \mathrm{N}$. But ρAB is the intensity of the current in the second system. Hence the potential of the first system upon the second is represented by the number of lines from the first system which the second intercepts, multiplied by the intensity of the current in the second system.

23. Let us now suppose that, as in Fig. 16, we have a rectangular system consisting of thick metallic rails, of which the resistance may be neglected. Let this system be open at one end, but capable of being completed by a rail AB (of resistance = R), which may slide perpendicularly between the rails. Suppose the system to be horizontal, so that the lines of force due to the vertical component of the earth's magnetism will pass through it perpendicular to the plane of the paper. Call the intensity of the force V. Now let AB be made to move to the right with a uniform velocity BB′ = v, so that at the end of unit of time it will be found to occupy the position A′B′. Also let the distance AB between the bars be denoted by d. It is thus clear that the number of vertical lines

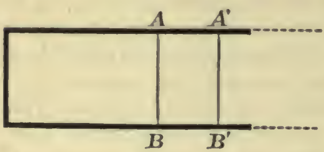

Fig. 16.

of force that will be added to the circuit in unit of time will be those comprised in the space $ABB'A' = Vdv$, and hence by Art. 20 the current produced in the circuit will be $C = \frac{Vdv}{R}$.

Let us here pause for a moment and attempt to realise the significance of this result. By means of the definition of § 18 we can at once measure current strength.

Suppose now that we proceed with our definitions, and define our units of electromotive force and of resistance as follows :—

Unit electromotive force *is that which must be maintained between the ends of a conductor in order that unit current may do unit work in a second.* Again

Unit resistance *is that of a conductor in which unit current is produced by unit electromotive force between its ends.*

Now in Ohm's law there are three things—current, electromotive force, and resistance, and when any two of these are known we can find the third. We know how to measure the current, but we require also to know how to measure the electromotive force or the resistance—one of the two. The above definitions show us that in order to do this we should require to know the work done by the current in one second. Now this work takes the shape of heat, so that in order to estimate it exactly we should not only require an accurate calorimetric method, but an exact knowledge of the mechanical equivalent of heat. The result which we obtained above will, however, enable us to measure either the E. M. F. or the resistance without the necessity of employing calorimetric methods. For we have $C = \frac{Vdv}{R}$. Hence $CR = Vdv = E$ (by Ohm's law). Now V, d, v, and C can be easily measured, and hence both E and R become known. We may therefore adopt the following definition :—

Unit electromotive force *is that which is created in a conductor moving through a magnetic field at such a rate as to cut one unit line per second.*

In practice, however, the method of the sliding rail, which is exceedingly useful to give us a simple conception of the problem, is not the one adopted, but we use instead a revolving coil, such as that described in the text.

24. Hitherto we have for the most part regarded the subject

of potential by the aid of a test unit, representing the potential of a system at a point P by the work done by or upon the unit as it is carried from P to an infinite distance from the system. But in § 22 we extended our view so as to embrace the whole potential of one system upon another. Continuing this method of treatment, let there be two systems A and B, then we may define the potential of A upon B to be the work done, let us say, *by* B as it is carried from its present position to an infinite distance from A ; and we may define the potential of B upon A in similar terms. It is easy to see that the potential of A upon B is equal to that of B upon A, for we may represent any current system A by a magnetic shell which may be supposed to consist of a series of centres at which quantities of attracting and repelling matter, m_1, m_2, m_3, etc. are placed ; and we may in like manner represent a current system B by centres at which quantities of similar matter, n_1, n_2, n_3, etc. are placed. Now any small component of the potential exerted by A on B will be represented by $\frac{m_1 n_1}{r_1}$, and this will also denote the small corresponding component of the potential exercised by B on A. Hence the whole potential of A on B is equal to the whole potential of B on A.

Again, it will be seen from § 22 that if we have a current of intensity i_1 in the circuit A, the potential of the system B upon it will be represented by i_1 multiplied by the number of lines of force of B intercepted by A ; and if we have a current i_2 in B the potential of the system A upon it will in like manner be represented by i_2 multiplied by the number of lines of force of A intercepted by B. If we bear in mind that the number of lines of force which proceed from a circuit is proportional to the intensity of the current in that circuit, it will be seen that the mutual potential between A and B is represented by a quantity which is proportional to the product $i_1 \times i_2$ and that the result will be the same whether the current i_1 be in A and i_2 in B, or *vice versa*.

25. Now let the current in B be unity, and let there be no current in A, and suppose that in a very small time τ the system B is carried to an infinite distance from A. We shall

have by § 20 an induced current in A, which may be represented thus :—

$$\begin{array}{c}\text{Mean induced current} \times \tau\text{, or} \\ \text{total quantity of current}\end{array} = \frac{\left\{\begin{array}{c}\text{Number of lines of force of} \\ \text{B which pass through A}\end{array}\right\}}{\text{Resistance of A}},$$

inasmuch as these lines of force have been withdrawn in the time τ. Furthermore, the same result will follow if the circuit at B is simply broken, the operation of disestablishing the current lasting for a small time τ.

It follows from what we have said that if there be unit current in A and none in B, and if the current in A be broken, we shall have precisely the above number of lines of force withdrawn from B. Let us call the total induced current M when the resistance is unity, *then M is called the coefficient of mutual induction between A and B*, so that if i denote the primary current in A while there is none in B, and if the current in A be broken, the total induced current in B will be $\frac{iM}{\text{Resistance of B}}$, and if i denote the primary current in B while there is none in A, the total induced current in A will be $\frac{iM}{\text{Resistance of A}}$.

C.

VARIOUS SYSTEMS OF ELECTRICAL UNITS.

(a) *Electrostatic Units* (C. G. S.)

(1.) The *unit quantity* of electricity is that which exerts the unit of force (one dyne) on a quantity equal to itself, at a distance of one centimètre across air.

(2.) The *unit difference of potential* exists between two points when unit of work (one erg) is spent by unit of electricity in moving from the one to the other against electric repulsion.

(3.) When there is a distribution equivalent to unit quantity of electricity per square centimètre on any small portion of a charged conductor, then *unit density* of electricity exists at that place. In most cases the density changes continuously from one portion of a charged conductor to another.

(4.) The *capacity* of a conductor is the quantity of electricity necessary to give it unit difference of potential.

(5.) The coefficient by which the capacity of an air condenser must be multiplied in order to give the capacity when another dielectric is used, is called the *specific inductive capacity* of that dielectric.

(β) *Electro-Magnetic Units* (C. G. S.)

(1.) A *unit magnetic pole* is that which repels a similar pole at unit distance (one centimètre) with unit force (one dyne).

(2.) A *unit current* is one which, in a wire of unit length bent so as to form an arc of a circle of unit radius, would act upon a unit pole placed at the centre of the circle with unit force.

(3.) *Unit quantity* of electricity is the quantity which a unit of current conveys in unit of time.

(4.) *Unit electromotive force* is that which must be maintained between the ends of a conductor in order that unit current may do unit work in a second.

(5.) *Unit resistance* is that of a conductor in which unit current is produced by unit electromotive force between its ends.

(6.) *Unit capacity* is that of a condenser which will be at unit difference of potential when charged with unit quantity.

(γ) *Practical System of Units.*

(1.) The *coulomb* or practical unit of quantity $= 10^{-1}$ C. G. S. units.

(2.) The *volt* or practical unit of electromotive force $= 10^{8}$ C.G.S. units.

(3.) The *ohm* or practical unit of resistance $= 10^{9}$ C. G. S. units.

(4.) The *farad* or practical unit of capacity $= 10^{-9}$ C. G. S. units. The *microfarad*, or one millionth of a farad, is frequently used.

(5.) The *ampère* or practical unit of current is that which passes one coulomb per second. The ampère is equal to the current of electricity transmitted through one ohm by one volt.

D.

DETERMINATION OF E. M. F.

1. This appendix will be devoted to a description (1.) of some standards of electromotive force, and (2.) of some special methods of comparing electromotive forces.

2. The difficulty of producing a constant standard of E. M. F. is so great that no official standard has as yet been issued. The best substitutes are certain forms of Daniell's cell and the mercury cell of Latimer Clark.

3. *Standard Forms of Daniell's Cell.* — Daniell's cells are fairly constant as long as copper from the copper sulphate is prevented from being deposited on the zinc. The various forms of standard cells exhibit different ways of retarding this deposit. The chief forms are :—

(1.) *The Post Office Form.*—A box is made with three compartments, each of which contains a glass battery jar. The middle cell is called the *working cell*, the lateral cells are the *idle cells.* The working cell contains a semi-saturated solution of zinc sulphate, and at its bottom there are scraps of zinc. The idle cells contain water only. When the cell is not in use, in one idle cell lies a plate of zinc, and in the other a porous pot containing a saturated solution of copper sulphate, in which is immersed a copper plate. When the cell is required to be in action the zinc and the porous pot with its contents are brought into the working cell. After the test has been made they should be again removed to the idle cells. The purpose of the zinc scraps is to decompose any copper sulphate that may have been diffused through the porous pot. As long as the zinc plate is kept clean and the cell is in good condition the E. M. F. of the cell is assumed to be about 1·07 volt. The cell is only adapted for rough commercial purposes. Lord Rayleigh found that two

specimen cells differed by 2·5 per cent, the mean values being 1·081 and 1·056 *true* volt.

(2.) *Raoult's Cell.*—This consists essentially of two separate cells, one containing the zinc and zinc sulphate, and the other the copper and copper sulphate. Connection is made between these cells by an inverted U tube containing zinc sulphate solution. The ends of the U tube are tied over with pieces of bladder. III. of Fig. 17 shows a modified form of this type. Here, when one cell is required to be connected with the other, some of the zinc sulphate is made to fill the connecting limb. This may be done by blowing down the straight tube.

(3.) *Sand Cells and the Cells of Beetz.*—I. of Fig. 17 shows one

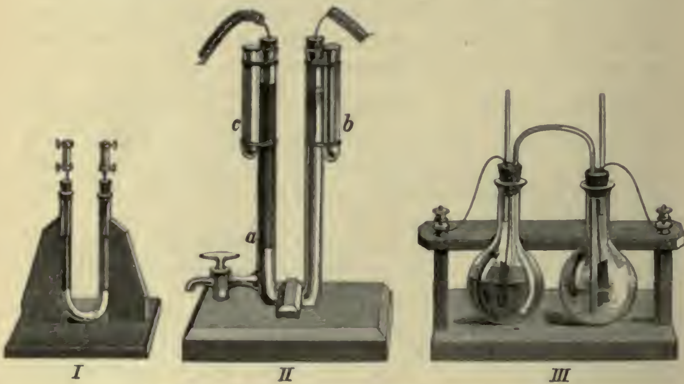

Fig. 17.—Forms of Standard Daniell's Cells.

of these cells made from a U tube. In the sand cells a layer of clean sand is placed in the bend, and is saturated with zinc sulphate solution, which is also contained in one limb, whilst in the other limb is a solution of copper sulphate. Here the sand retards for a long time the diffusion of the copper sulphate. The process of diffusion may be still more retarded by following the process of Beetz, who mixes the liquids with plaster of Paris so as to form a paste, which may be poured into the limbs, where it will harden.

(4.) *Fleming's Cell.*—We are indebted[1] to Dr. Fleming for making a thorough examination of the best conditions of constancy in a Daniell's cell. He has devised a form of cell which we have found convenient to modify into the form shown in II., Fig. 17. It consists of a U tube having a stopcock at the bottom of one of the limbs. The open ends of both limbs are fitted with perforated india-rubber corks. Through the left-hand one passes a copper rod, and through the right-hand one a zinc rod. The rods are made of the purest materials procurable ; the zinc is a rod of twice-distilled zinc that has been amalgamated with pure mercury, the copper is formed by electro-deposition on fine copper wire. Strapped by elastic bands to the two limbs are two idle tubes, *b* and *c*, for the reception of the rods when the cell is not in use. The solutions for charging the cell consist of pure zinc sulphate and pure copper sulphate. The absolute density of the solutions is not a matter of importance within certain limits, provided that both solutions are of the *same density.* In filling the cell first pour a layer of zinc sulphate into the bend of the U tube up to the level *a*, and then pour simultaneously down the right and left limbs the solutions of zinc and copper sulphate. There should be a well-marked line of demarcation between the two liquids at *a*. In order to secure this the cell should be occasionally tilted to the right and a portion of the copper sulphate run off. On again placing the tube vertical the layer of zinc sulphate will be above *a*. In a cell of this form carefully made up, the solutions must not mix at the place of contact, and the copper must have been freshly electrotyped and must be free from spots of oxide. Under these circumstances the E. M. F. is very nearly 1·1 *true* volt. The cell has a small temperature coefficient, generally negligible for the usual range of temperature in a laboratory.

(5.) *Lodge's Cell.*—This is convenient for many experiments and is easily put together. A wide-mouthed bottle (Fig. 18) is provided with a cork. Through it passes the wide tube T, in which is the rod of zinc. A small test tube *t* is fastened to the end of T by an elastic band. At the bottom of *t* are crystals of

[1] On the use of Daniell's cell as a standard of E. M. F., by J. A. Fleming, see *Proc. Phy. Soc.*, vol. vii. p. 161.

sulphate of copper. The copper electrode is the bared end of a gutta-percha covered wire which passes through a hole in the cork to the bottom of *t*. The bottle contains a solution of sulphate of zinc.

(6.) *Sir William Thomson's Cell.*—Here (Fig. 19) a zinc plate is placed at the bottom of the cell. On it is poured a solution of dense sulphate of zinc (density 1·2). Above the zinc is the

Fig. 18.—Lodge's Standard Daniell.

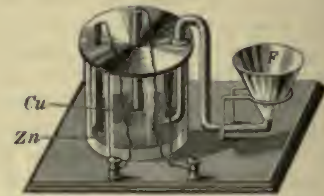

Fig. 19.—Sir Wm. Thomson's Standard.

copper plate. By using the funnel F a solution of half-saturated sulphate of copper is carefully poured on the top of the lower dense solution. The E. M. F. is 1·07 *true* volt.

4. *The Normal Element of Latimer Clark.*—Since this cell was originally devised it has been subjected by different experimenters to a most rigorous examination, to ascertain how far it may be employed for standard purposes. Lord Rayleigh, more especially, has recently[1] not only worked out the best practical method of making this cell, but also redetermined its value in absolute measure. Fig. 20 shows one of the many forms that the cell may take. Through the bottom of a stoppered weighing tube a piece of platinum wire is inserted and fused. The stopper is also perforated, and platinum wire is passed through it and fused. An ebonite tube E, divided nearly into two portions, mounted on a wooden block, serves as a stand for the cell. The platinum wires are connected to the binding screws fixed to the

[1] See *Phil. Trans.*, 1884, part ii.; and 1885, part ii.

ebonite. The bottom of the cell contains mercury (H*y*), and on this is a layer of paste (S*u*), made by mixing mercurous sulphate with a solution of zinc sulphate. The paste is sufficiently liquid to form on standing a layer of liquid at the upper part, P. In the paste is immersed a rod of zinc *Zn* that has been soldered to the platinum wire passing through the stopper.

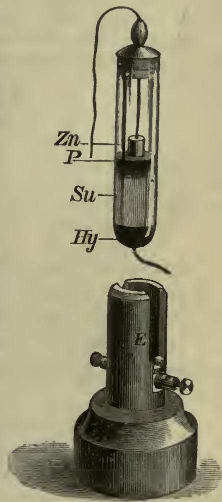

Fig. 20.—The Normal Element of Clark.

The cell just described is only suitable for special experimental purposes. Where a large number has to be made up it is better to use simply a tube, such as a small test tube, sealed at the top with marine glue. The method of charging the cells, with the several essential precautions, will now be given, mainly in Lord Rayleigh's own words :—

(1.) Pour in sufficient mercury to cover the platinum wire sealed through the bottom of the tube. The best mercury for the purpose is that which has been distilled *in vacuo*.

(2.) The paste should next be introduced, care being taken not to soil the sides above the proper level. To prepare the paste it is first necessary to make a solution of zinc sulphate. Mix in a flask distilled water with about twice its weight of crystals of pure zinc sulphate, and add a little pure zinc carbonate to neutralise free acid. Effect the solution by *gentle* heat. Allow the mixture to stand, in order to precipitate any iron that may be present. Filter the solution in a warm place into a stock bottle. When it is intended for use, expose the solution to a gentle heat for some time, and draw off the solution from near the crystals at the *bottom of the bottle*, in order that there may be certainty of the liquid being saturated. To prepare the paste rub together in a mortar 150 grammes of pure mercurous sulphate,[1] 5 grammes of zinc carbonate, and as much of the *saturated*[2] zinc sulphate as is required to make a thick paste.

---

[1] In purchasing the mercurous sulphate some discretion should be exercised, for mercuric sulphate or turpeth mineral may be substituted. The mercurous sulphate should be *white*.

[2] Great care should be used not to have the solution *supersaturated*.

It is best to leave the paste in a mortar for two or three days, rubbing it at intervals with additions of zinc sulphate until all carbonic anhydride has escaped. The paste should then be transferred to a well-stoppered bottle, where it should keep good for several months. Before pouring out for use the bottle should be well shaken.

(3.) The zincs are cut from rods sold as *redistilled zinc.* A copper wire, insulated with gutta-percha, should be soldered to the top of each zinc, after which the zinc should be cleaned by dipping it in sulphuric acid, and then washed with distilled water and dried by filtering paper. To support the zinc centrally in the tube, it is passed through a ring of cork (nicked to allow the escape of air) that just fits within the tube. The cork is pushed down until its lower surface nearly touches the paste, in order that as much air as possible may be excluded. Above the cork a layer of marine glue should be poured in order to seal the cell.

A Clark cell prepared according to the above rules, when (1.) protected from large fluctuations of temperature, (2.) never used in short-circuit, and (3.) only used with exceedingly weak currents, has an E. M. F. expressed by $E = 1{\cdot}435\{1 - {\cdot}00077(t - 15)\}$, where E is the number of *true* volts and t is the temperature centigrade.

5. *Comparison of Standard Cells.*—Owing to the difficulties introduced by polarisation, standard cells can only be accurately compared under three conditions—either when (1) they are in open circuit, (2) when in a circuit of very high resistance, or (3) when one cell is balanced against another. If we use the electrometer or condenser the cells are being compared under the first condition, and the tests are independent of the internal resistance of the cells. Hence we could compare directly, say, a Grove of ·2 ohm internal resistance against a Beetz Daniell of over 1000 ohms. These methods have, therefore, an important advantage over the ordinary galvanometer methods, where means must be adopted to eliminate the battery resistance. However, by using a galvanometer of high sensibility, in whose circuit a high resistance is included, we may, by adopting the method of sum and difference (p. 240), compare cells with tolerable accuracy. The last mentioned method, as well as that of the electrometer and that of the condenser, are direct deflection methods, which are inferior to the methods in which one cell is balanced against another. Of the latter kind are the compensation methods of

Bosscha and Poggendorff, possessing, as they do, all the advantages associated with zero methods.

6. *Bosscha's Compensation Method.*[1]—The connections shown

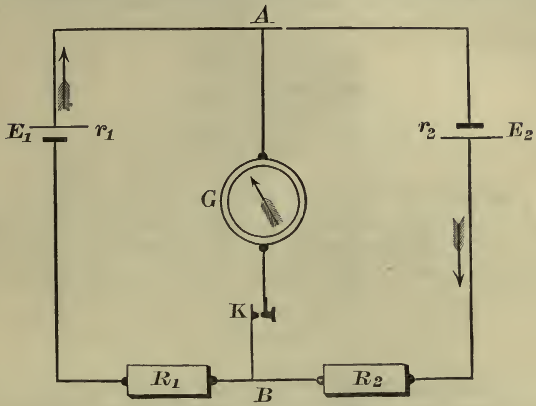

Fig. 21.—Bosscha's Method.

in Fig. 21 being made, the resistances R_1 and R_2 are adjusted until the galvanometer is undeflected, then

$$\frac{E_1}{E_2}=\frac{R_1+r_1}{R_2+r_2} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

In this equation (which may be proved by applying Kirchhoff's laws) r_1 and r_2 are the internal resistances of the cells of electromotive force, E_1 and E_2, under comparison. By finding a fresh pair of values, R'_1 and R'_2, which will also give equilibrium, we shall have, as before,

$$\frac{E_1}{E_2}=\frac{R'_1+r_1}{R'_2+r_2} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

From (1) and (2) we obtain

$$\frac{E_1}{E_2}=\frac{R_1-R'_1}{R_2-R'_2} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

[1] Often called in England Lumsden's Method.

an equation in which the resistances of the batteries do not appear. This forms a very convenient and accurate method of comparison. In making the test the galvanometer should be one of great sensibility, and the adjustable resistances should be high.

Example.—

$$R_1=3231,\ R_2=775.$$

$$R'_1=6600,\ R'_2=1600.$$

$$\frac{E_1}{E_2}=\frac{3231-775}{6600-1600}=\frac{2456}{5000}=\cdot 4912.$$

7. *Poggendorff's Compensation Method.*—The different modifications of this method are based on the principles explained in Lesson XXXVIII., from which it will be seen from equations (3) and (4) that instead of eliminating E and e we may eliminate x, with the result,

$$\frac{E}{e}=\frac{a-a'}{b-b'}.$$

This equation thus furnishes a method of comparing E and e. By replacing e by a second cell e', and obtaining a new balance, we can compare e with e' under the same conditions, namely, when both are compensated. Here

$$\frac{E}{e}=\frac{x+a}{b},$$

and

$$\frac{E}{e'}=\frac{x+a}{b'},$$

hence

$$\frac{e}{e'}=\frac{b}{b'},$$

or the electromotive forces are simply as the resistances included between P and Q in the two cases, PS being kept constant. If PS be a uniform wire provided with a linear scale, it would only be necessary to read off the position of Q in the two cases. This last method is often adopted; but the best arrangement is that adopted by Lord Rayleigh, and used by him in comparing the Clark cells. The compensating battery L (Fig. 22) consists of two Leclanché elements. It is connected in series with two

resistance boxes, R and R′, joined by a short thick wire. The derived circuit consists of the cell T under test, a high resistance galvanometer, a coil of high resistance S (to prevent accidental strong currents passing through the cell), and a key K_2. R and R′ are adjusted until a balance is obtained, *but the resist-*

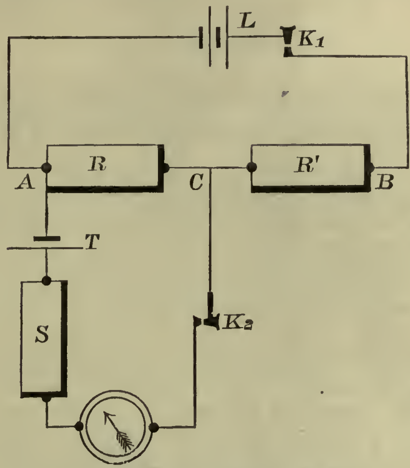

Fig. 22 – Rayleigh's Compensation Method.

ance between A and B is always kept = 10,000 *ohms.* The cell T is then replaced by a second cell T′, and a new balance is obtained. If we call the resistances between A and C in two cases r and r', then

$$\frac{T}{T'}=\frac{r}{r'}.$$

Example.—S was about 200,000; it consisted of a pencil of prepared carbon. T consisted of one Daniell's cell, and T′ of two cells of the same kind.

$$r=3660, \ r'=7210.$$

$$\frac{T}{T'}=\frac{3660}{7210}=\cdot 5076.$$

With the reflecting galvanometer used it was possible to adjust r easily to within 2 ohms, hence this result may be considered accurate to within $=$ ·0002.

E.

METHOD OF DETERMINING THE TEMPERATURE AND INDUCTION COEFFICIENTS OF A MAGNET.

1. In determining the temperature coefficient of a magnet it is firmly fixed in a water-tight wooden box provided with a thermometer—being placed therein upon a frame in such a manner that its axis shall lie in the same horizontal plane with the needle of a unifilar magnetometer a short distance off. It is adjusted so that the axis of the two magnets shall be at right angles to one another—the axis of the magnet in the box passing through the centre of the suspended magnet.

Water at 85° F. is then poured into the box, and when the magnet has become heated to this temperature the unifilar magnet is brought to rest and its position accurately noted. A similar observation is made at 60° F. and 35° F. After this another series of similar observations is made, beginning at 85° F., and this is again repeated once or twice. Let us assume that the readings obtained are first of all corrected for changes of the earth's magnetic force during the progress of the experiment, ascertained by means of simultaneous readings of the curves of self-recording instruments, or by some other method. When this has been done the following demonstration will show how the temperature coefficient is determined from the corrected observations.

Let t_0 denote a standard temperature, and let t denote the temperature of observation, then we may with much probability assume the following expression for the magnetic moment:—

$$\text{Magnetic moment at } t = m\{1 - q(t - t_0) - q'(t - t_0)^2\} \qquad . \quad (1)$$

where q and q' are coefficients which we wish to determine.

Now let $m = \dfrac{\mathrm{H}r^3}{2}\sin u$ be the normal equation of equilibrium of the unifilar magnet at temperature t_0, then at t this equation will become

$$m\{1 - q(t - t_0) - q'(t - t_0)^2\} = \frac{\mathrm{H}r^3}{2}\sin(u - du) \qquad . \quad . \quad (2)$$

In this equation substitute the above value of m, and we have

$$\frac{\mathrm{H}r^3}{2}\sin u\{1-q(t-t_0)-q'(t-t_0)^2\}=\frac{\mathrm{H}r^3}{2}\sin(u-du) \quad . \quad (3)$$

Hence

$$\sin u - q(t-t_0)\sin u - q'(t-t_0)^2\sin u = \sin(u-du).$$

Now let $q\sin u=x$ and $q'\sin u=y$, and we have

$$x(t-t_0)+y(t-t_0)^2=\sin u-\sin(u-du) \quad . \quad . \quad (4)$$

from which x and y may be found, and q and q' easily determined.

There is one remark regarding this method which it is desirable to make. It seems possible that, at least in most cases, when a magnet is raised from a lower to a higher temperature and then lowered again to its first temperature, there is a loss of permanent magnetism caused by the process. Suppose now that the magnet was vibrated at temperature t, and then immediately used as a deflector at the same temperature, we may suppose that m in the one case is identical with m in the other. But if between the two observations the temperature be first raised to $t'>t$ and then lowered to t, it is possible that the moment in the one case will not be the same as that in the other.

But this difference would not be indicated by the usual formula for temperature corrections.

For hollow cylindrical magnets Whipple finds that the mean value of q for 1° F. is 0·000161 and of q' 0·00000043.

2. The induction coefficient is determined from observations of deflection made after the method of Lamont, the magnet being alternately placed with its north pole upwards and downwards, but always at the same distance from a suspended needle, the difference in the amount of deflection of the latter in the two positions determining the effect of the earth's inductive action upon the magnet. The following formula, differing slightly from Lamont's, is due to the late John Welsh :—

In the first place it is assumed that the induction produced by the earth's action is distributed in the same manner throughout the magnet as its permanent magnetism.

Now let μ = the increase of the magnetic moment m, caused by the action of an inducing force = unity, also let H, V denote the horizontal and vertical components of the earth's force, and let i denote the inclination. Further, let ϕ, ϕ' denote the angles of deflection north end downwards and north end upwards. Finally let u = angle of deflection produced when the magnet is used as an ordinary deflector at distance unity. Then, first of all, we have $\frac{\sin u}{2}=\frac{m}{\mathrm{H}}$. Next, since the line joining the centres of the two magnets is in every part of the observation approximately at right angles to the suspended needle, it will

follow that the attraction of the deflecting bar will be proportional to its magnetism.

Hence, if c be a constant, we may state the equation of equilibrium north pole downwards thus—

$$c(m+V\mu)=H\sin\phi \quad . \quad . \quad . \quad . \quad (5)$$

and with north pole upwards thus—

$$c(m-V\mu)=H\sin\phi' \quad . \quad . \quad . \quad . \quad (6)$$

By subtraction and addition of (5) and (6) we derive

$$2cV\mu=H\{\sin\phi-\sin\phi'\} \text{ and } 2cm=H\{\sin\phi+\sin\phi'\}$$

Hence, dividing the first of these equations by the second,

$$\frac{V\mu}{m}=\frac{\sin\phi-\sin\phi'}{\sin\phi+\sin\phi'}=\frac{\tan\frac{1}{2}(\phi-\phi')}{\tan\frac{1}{2}(\phi+\phi')},$$

and since $V=H\tan i$ we have

$$\mu=\frac{m}{H\tan i}\ \frac{\tan\frac{1}{2}(\phi-\phi')}{\tan\frac{1}{2}(\phi+\phi')}.$$

But since $\frac{m}{H}=\frac{\sin u}{2}$, we have finally,

$$\mu=\frac{\sin u}{2\tan i}\ \frac{\tan\frac{1}{2}(\phi-\phi')}{\tan\frac{1}{2}(\phi+\phi')} \quad . \quad . \quad . \quad (7)$$

Whipple finds that on an average $\mu=0{\cdot}000207$.

F.

ADDITIONAL PRACTICAL DETAILS.

1. *Manipulation of Gold Leaf.*—To provide the electroscope with gold leaves is a comparatively easy process when we are provided with the following materials as used by the gilder :—(1.) A *cushion*, this consists of a board 8 inches by 5 inches, which is first covered with baize and then with buff leather,

tightly stretched. At one end is a raised edge of parchment to protect the cushion from accidental winds. (2.) A *gilder's knife*, which is a kind of palette knife with a long flexible blade, having an edge not sufficiently sharp to cut the leather of the cushion. (3.) A *tip* or large flat brush of squirrel's hair for taking up and placing the gold leaf. (4.) A powder of *burnt talc*, which is dusted upon the cushion to prevent the gold leaf from sticking to it.

The leaf is transferred by means of the knife to the cushion, and then cut by pressure of the knife. The cut leaf may then be lifted by the tip, very slightly greased.

2. *Switch for Battery.*—Fig. 23 shows the general arrangement of a switch for one or two cells. A metal bar SS_1, pro-

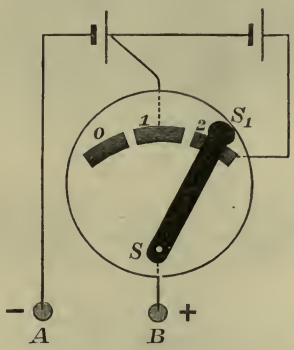

Fig. 23.—Battery Switch.

vided with a handle at S_1 and pivoted at S, may be placed in contact with any one of three metal segments, 0, 1, and 2, that are fixed to a wooden or ebonite block. When the switch is at 0 both cells are out of the circuit that connects A and B, but according as the switch is at 1 or 2 one or two cells are in circuit. Instead of a pivoted switch a plug switch is often used.

3. *Silk for Suspension of Galvanometer Needles.*—The best silk

is obtained from the middle of a good cocoon. The cocoon should be steeped in tepid water, and the silk wound off it on to a simple reeling machine. Fig. 24 shows such a machine, in which the reel R is made of a number of glass rods that connect

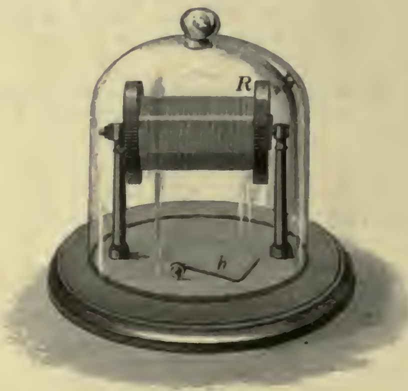

Fig. 24.—Reel for Silk.

the two wooden ends of the reel. When the silk has been wound, the handle *h* should be removed, and the whole covered by a glass shade to protect the silk from dust. See also Vol. I. Appendix C.

4. *Clamp and Binding Screws.*—The various patterns of these are figured (Fig. 25).

1 is of the ordinary French pattern.

1*a* is a special pattern of the same, with a second binding screw at the end of its shank.

2 is an ordinary telegraphic binding screw.

2*a* is the same with a lock nut.

2*b* is the same with a double-screw for use with two separate wires.

3 and 3*a* are common clamp screws for connecting two wires.

3*b* is the telegraphic pattern that is also useful for connecting plates.

4 is a battery clamp.

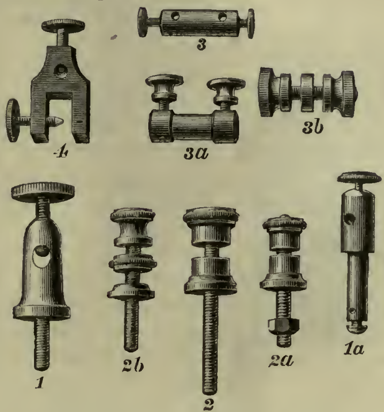

Fig. 25.—CLAMP AND BINDING SCREWS.

5. *Turned Wood Reels.*—These may be obtained made of boxwood, the wall of the reel being so thin as to be semi-transparent. They are very useful for making electro-magnets, etc.

6. *Manufacture of a Paraffin Paper Condenser.*—In making the condenser the process is much facilitated by having two operators. They should be provided with good thin paper of foolscap size, thin tinfoil in sheets 48 cm. by 32 cm., and a shallow bath of clean paraffin. (1.) Each sheet of tinfoil should be thrice folded. At least twenty of these eight-paged sheets should be piled on the top of each other and well pressed together. They should then be taken to a bookbinder, who should be directed to cut away the edges and one corner, as shown at *h g* of Fig. 26. In this way 160 separate sheets will be prepared. (2.) Divide each sheet of foolscap into four equal parts, and examine each one for pin-holes by holding it between the eye and a gas jet in

a dark room. Make a pile of 170 perfect sheets, and let them be cut by a bookbinder into the form *a b c d e f* (Fig. 26). (3.) Melt the paraffin by two Bunsen burners, and take care that the melted wax does not boil or burn. Well warm the sheets at the fire. (4.) Fasten a string across the room, on which are placed a number of wooden clips. Whilst one operator is keeping the wax at the right temperature, the other should take a sheet of paper and dip it in the wax for a short time, taking care not to allow it to be singed. On removing it out of the bath, if the wax is hot enough, the air bubbles will run off. If not, the paper must be again immersed and the process repeated. Suspend the paper by one corner by a wooden clip until the wax sets hard. Proceed in this manner until all the sheets are waxed. (5.) Commence to build up the condenser. Place a piece of waxed paper, *a b c d e f* (Fig. 26), at the bottom, on this place a sheet of tinfoil *g h k l m*, with the corner *m* projecting. On this place another sheet of paper, upon which a second sheet of foil must be made to lie with the left-hand corner instead of the right-hand one projecting. The piling up of the paper and tinfoils is continued, with the corners of the latter projecting first to the right and then to the left, until all the tinfoils are exhausted. At the top of the pile put several sheets of paraffin paper. Bind the whole together by elastic bands, and fold the whole in paraffin paper. The condenser should now be well pressed at a bookbinder's. (6.) A shallow wooden box, into which the condenser will just fit, will now be required. It should have a lid fitted with screws and a strip of ebonite let into one of its narrower ends. The insulation of the box is improved by coating it internally with paraffin. The exposed ends of the tinfoil are to be well pressed together and put in connection with two binding screws, whose shanks pass through the

Fig. 26

strip of ebonite into the box. To ensure a good connection fragments of tinfoil must be packed between the shanks and the tinfoil. Finally, the condenser may be fixed in position in the box by pouring melted paraffin in at the edges; paraffin paper should be put at the top to fill the box and the lid screwed on. (7.) It will now be well to test for leakage by the method of p. 192. The insulation resistance at first should be found to be high, but owing to some reason not altogether explained it gradually falls so as to make a paraffin paper condenser very inferior to one of mica.[1]

7. *Construction of Tangent Galvanometer.*—For the information of any one who wishes to construct a tangent galvanometer of the type described in Lesson XXXIX., the working drawings of the

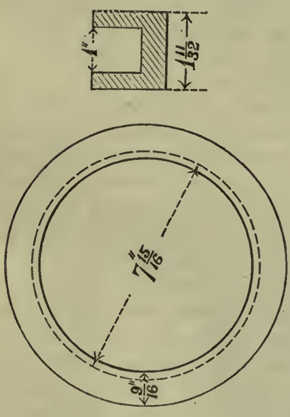

Fig. 27.

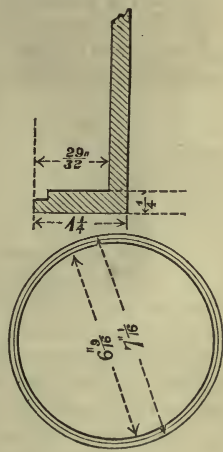

Fig. 28.

reel and compass box are given. Fig. 27 shows the reel and Fig. 28 the compass box in plan and section. The small recess

[1] Condensers, with insulating qualities equal to mica but far cheaper, are now made by the *Société d'Exploitation des Cables Electriques* Systéme Berthoud, Borel & Co., Cortaillod, Switzerland.

shown in the latter is for the reception of the glass that covers the box. It will be found of great convenience to make the base of two separate pieces, one above the other, the upper one being pivoted so as to enable the instrument to be turned in azimuth.

8. *Soldering.*—Perhaps no operation in the electrical laboratory is so important or requires to be performed so often as soldering; hence a few details relating to it will be useful. The materials requisite are a small soldering iron, soft solder, and a means of heating the iron. We find the soldering-iron heater of Fletcher very useful for the purpose. There will further be necessary either powdered resin or chloride of zinc for enabling the solder to make good contact. The former material is much to be preferred for electrical apparatus, but it is more difficult to solder by its means than by the chloride of zinc. When the latter is used the soldered place should afterwards be washed, otherwise galvanic corrosion will take place at the joint.

9. *Substitutes for Wire Resistance.*— Resistance coils of German silver being very expensive, it is desirable that the student should know of several simple substitutes. (1.) *Hittorf's Solution.*—This has already been mentioned (p. 192). It is a solution of amylic alcohol, containing 10 per cent of cadmium iodide. The electrodes are of cadmium obtained by melting the metal in a glass tube and inserting a platinum wire in the molten metal, which is allowed to set, and then the glass is broken away. The tube containing the solution should be kept vertical, and it is advisable that the lower electrode should be connected with the *negative* pole, since the solution is apt to concentrate there, and otherwise liquid currents would be formed. The prepared tube should show no polarisation current. (2.) *Carbon Resistances.*— The specific resistance of carbon varies very greatly according, to its method of preparation. That of the open-grained charcoal sticks used by artists is very high, whilst that of the dense carbon employed in electric lighting is comparatively low. Thus by selecting specimens of carbon of varying denseness a series of useful resistances may be made. The carbon is best mounted by copper-plating its ends,

which are then soldered to wires in connection with binding screws. Resistances may also be made by rubbing pencil lines on ground glass, or using graphite paper (*i.e.* paper in which graphite has been incorporated with the pulp). Finally, ordinary lead pencils and incandescent lamps containing prepared carbon filaments are useful.

G.

ADDITIONAL TABLES.

Table P.

Resistance of Hard-Drawn Pure Copper Wires, According to the New Standard Wire Gauge.

Temperature 15° Cent.

Descriptive No. S.W.G.	Diameter.		Resistance.		Weight. (Density=8·95)		Nearest B.W.G.
	Ins.	Cms.	Ohms per Yard.	Ohms per Mètre.	Lbs. per Yard.	Grms. per Mètre.	
7/0	·500	1·270	·000126	·000137	2·285	1134·0	
6/0	·464	1·179	·000146	·000159	1·970	976·3	
5/0	·432	1·097	·000168	·000184	1·706	846·3	
4/0	·400	1·016	·000196	·000215	1·463	725·6	
3/0	·372	·945	·000227	·000248	1·265	627·6	
2/0	·348	·884	·000259	·000283	1·107	549·6	0
0	·324	·823	·000299	·000327	·960	476·1	
1	·300	·762	·000349	·000381	·823	408·1	
2	·276	·701	·000412	·000451	·696	345·4	
3	·252	·640	·000494	·000541	·581	288·0	
4	·232	·589	·000583	·000638	·492	244·1	
5	·212	·538	·000698	·000764	·411	203·8	
6	·192	·488	·000851	·000931	·337	166·8	
7	·176	·447	·00101	·00111	·283	140·5	
8	·160	·406	·00123	·00134	·234	116·1	
9	·144	·366	·00151	·00166	·190	94·0	9
10	·128	·325	·00192	·00210	·150	74·3	11

TABLE P (*continued*).

RESISTANCE OF HARD-DRAWN PURE COPPER WIRES, ACCORDING TO THE NEW STANDARD WIRE GAUGE.

Temperature 15° Cent.

Descriptive No. S.W.G.	Diameter.		Resistance.		Weight. (Density=8·95)		Nearest B.W.G.
	Ins.	Cms.	Ohms per Yard.	Ohms per Mètre.	Lbs. per Yard.	Grms. per Mètre.	
11	·116	·295	·00233	·00255	·123	61·0	
12	·104	·264	·00290	·00317	·0989	49·0	
13	·092	·234	·00371	·00406	·0774	38·4	14½
14	·080	·203	·00490	·00536	·0585	29·0	
15	·072	·183	·00606	·00662	·0474	23·5	
16	·064	·163	·00760	·00838	·0374	18·6	16
17	·056	·142	·0100	·0109	·0287	14·2	
18	·048	·122	·0136	·0149	·0211	10·4	
19	·040	·102	·0196	·0215	·0146	7·26	
20	·036	·0914	·0242	·0265	·0118	5·88	20
21	·032	·0813	·0307	·0335	·00936	4·64	21
22	·028	·0711	·0400	·0438	·00717	3·56	
23	·024	·0610	·0545	·0596	·00526	2·61	
24	·022	·0559	·0649	·0709	·00443	2·19	24
25	·020	·0508	·0786	·0858	·00366	1·80	26
26	·018	·0457	·0969	·106	·00296	1·47	27
27	·0164	·0417	·117	·128	·00246	1·22	
28	·0148	·0376	·143	·157	·00200	·893	
29	·0136	·0345	·170	·185	·00169	·839	
30	·0124	·0315	·204	·223	·00141	·697	31
31	·0116	·0295	·233	·255	·00123	·610	
32	·0108	·0274	·269	·294	·00107	·529	
33	·0100	·0254	·314	·343	·000914	·453	32
34	·0092	·0234	·371	·406	·000774	·384	
35	·0084	·0213	·445	·486	·000645	·320	
36	·0076	·0193	·544	·594	·000548	·262	36½
37	·0068	·0173	·679	·742	·000423	·210	
38	·0060	·0152	·872	·954	·000329	·163	
39	·0052	·0132	1·16	1·27	·000247	·123	
40	·0048	·0122	1·33	1·49	·000211	·104	

Table Q.

Vapour Tension of Sulphuric Acid and Water.

Temp. °C.	For Acid of 25 per cent (Regnault).	For Acid of 10 per cent (Bertin ; approximate).
	m.	mm.
13	9·37	10·09
14	9·99	10·78
15	10·64	11·50
16	11·33	12·27
17	12·05	13·08

END OF VOL. II.

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